COMPUTER ANALYSIS AND SIMULATION OF TRANSIENT STATE AND PRESSURE RECOVERING IN FAST CYCLIC HYDRAULIC ACTUATORS

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ABSTRACT

In the paper is presented dynamic model of high cyclic hydraulic servo-actuator. Effects of cyclic control including the changes of fluid flow direction and pressure discontinuities are also included in the model. For determination performances and abilities of fast cyclic hydraulic servo-actuator computer analysis of actuator transient state between its engage and full pressure recovering is needed. Hydraulic actuator dynamic model is assumed usually in different mathematical forms without any geometrical and physical discontinuities and ambiguity of initial pressure conditions. Fast cyclic hydraulic actuator can be assumed with two serial connected compressible fluid flows controlled by supply and return variable restrictors enclosed in control servo-valve and separated by actuator piston, expressed by equivalent mass, viscous damping and arbitrary external force. Presented mathematical model includes transient state of fast cyclic hydraulic actuator, which can be described and determined by following effects:

a) ambiguity of initial pressure conditions in actuator chambers as result of final state of its previous operations and existing fluid leakage, which can produce arbitrary value of initial pressure condition;

b) hydraulic pressure drop and surge caused by damped hydraulic expansion and compression shock waves, respectively, arising in the moment of actuator engage if exists finite pressure difference between pipelines and actuator cambers;

c) relatively small pressure surge in the moment of change direction of piston motion as result of geometric and flow asymmetry of actuator and its control servo -valve.

INTRODUCTION

Usually, hydraulic actuator is assumed with compressible fluid flow excluding the effects of shock waves propagation and fluid viscosity. Fluid compressibility is assumed as quasi-static change of its density depending on static pressure. For fast hydraulic actuators it is of interest to involve the effects of shock wave propagation along the pipelines. Reverse propagation of expansion wave produces corresponding pressure drop, depending on pressure drop on actuator servo valve. These effects increases with velocity of control servo-valve throttle and can be suppressed by its partially closing. Fast hydraulic actuators are relatively sensitive of wave propagation effects through the pipelines. In difference with classic hydraulic actuator modeling in which fluid is assumed as incompressible or quasi static compressible, wave effects introduces pressure disturbances during its propagation. These effects are very fast and slightly damped by fluid viscosity, but for digitally controlled actuators it cannot be neglected.

Each of the mentioned effects produces local pressure drop or surge and corresponding actuator operational time delay, which is the limiting factor of its cyclic velocity. Dominant influence on actuators time delay (less than 3% of unit step discrete control piston stroke time) is caused by effect of fluid volumetric compressibility.

Pressure drop caused by propagation expansion waves in the source pipeline of fast high cyclic hydraulic actuator produces possible anomalies in its function. To prevent pressure drop it is possible to minimize wave effects by active control of actuator servo-valve throttle leakage. In the paper is also presented synthesis of discrete active control of hydraulic actuator and its servo-valve for prevention expansion wave pressure drop. Control synthesis is based on effects of static pressure increasing by decreasing of fluid flow velocity, which can be done by partial closing throttle leakage of servo-valve. Some effects of assumed active control are shown on corresponding diagrams of servo-valve throttle motion, actuator piston displacement and its corresponding linear velocity.

Dynamic model of fast hydraulic actuator transient state can be expressed in a different forms. Two of them are of spatial interest. The first one is based on the finite element analysis of compressible fluid flow and the second one is founded on the method of characteristics. Finite element analysis can be applied only on linearised mathematical problem formulation, in which fluid velocity is assumed as a small one. In that case problem is presented in the form of acoustic wave equation. Its numerical solving leads to relatively high dimensional linearised system form with expected numerical problems and oscillatory form of its solution. These kind of solution forms exists for the problems of liquid spring (free piston oscillations with locked actuator chambers). For the problem treated in this paper effect of liquid spring is not of practical influence, but it must be encountered for the cases of forced vibrations of high cyclic hydraulic servo-actuators.

Method of characteristics is more aplicative for this problem. Problem formulation need not to be linearised. Oscillatory solutions are completely excluded in the solution by assuming corresponding number of points in which system will be evaluated. System dimension is to less for the similar computational precision. Corresponding model form is presented in the form of algebraic linear equations, which is equal to the finite element interpolation for both system coordinates, along the streamline and during time. Wave propagation is more visible in the solution form of characteristics and corresponds to the real physical effects.

In the paper following problems are treated:
-pressure discontinual changes in reverse of piston motion directions;
-nonlinear effects of actuator behavior and its linearisation;
-shock wave and pressure drop effects.

EFFECTS OF SYSTEM DISCONTINUITIES

Any change of direction of actuator motion produces pressure discontinuity in its both pipelines. This discontinuity is caused by inversion of fluid flow which produce the change of connection between supply and return pipelines and both actuator chambers. In the moment of change of flow direction each of actuator chambers changes connections with system pump and return pipeline, producing corresponding discrete changes of pressure in actuator chambers. Possible pressure drop or surge is also caused by geometric asymmetry of servo valve. These effects are explained on the following figures.

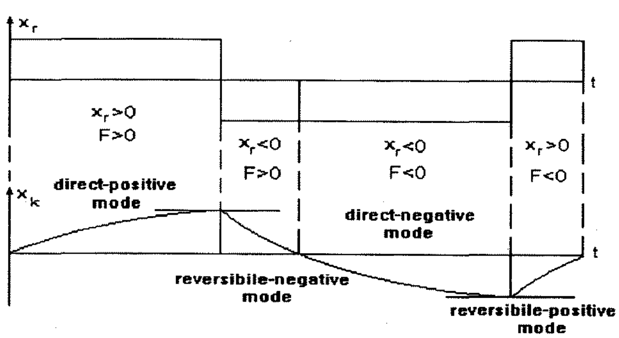

Fig.1

On figure 1 is shown actuator motion asymmetry between direct and reverse modes. This asymmetry is result of pressure distribution along supply and return streamlines, shown on diagrams a) and c) on figure 3. Diagram a) corresponds to direct mode of actuator function represented by symmetrical pressure drops at supply and return branches of its servo-valve. The third step of pressure drop at actuator piston position corresponds to the applied external force. On diagram c) is shown pressure distribution for actuator reverse mode. Main difference between these modes is in opposite directions of external force related to the streamline of fluid flow. For reverse mode external force support system pump as additional serial connected system source. This fact appears on figure 1 as different curve gradient for direct and reverse modes. However, absolute value of gradient is greater for reverse mode. More data about gradient value will be shown on figure 6.

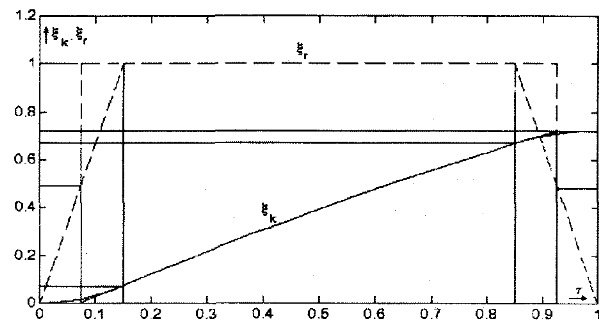

Fig. 2

On figure 2 is presented actuator output on servo-valve control input assumed as unit step function. This approximation is very close to the real situation for digitally controlled actuators.

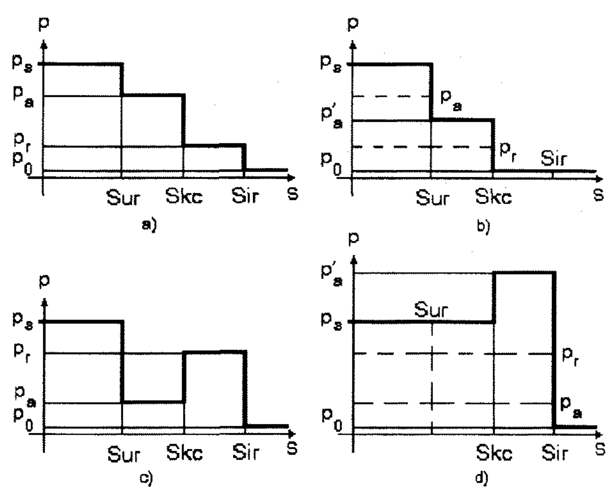

Fig.3

On diagrams b) and d) of figure 3 are shown, respectively, equivalent pressure drops for direct and reverse actuator modes, corresponding to the conventional mathematical modeling of hydraulic actuator, with total pressure drop on servo-valve by neglecting effects at its supply and return parts as two separated fluid flows. This usual approximation cannot be accepted if there are included effects of hydraulic pressure drop and surge caused by fluid compressibility.

CONVENTIONAL SYSTEM MODELING

Following symbols are used in the further text: μ flow coefficient, b^r equivalent geometric wide, x_r position of control valve throttle, p_s supply pressure of hydraulic system pump, p_a static pressure in supply chamber of actuator cylinder, p_r static pressure in return chamber of actuator cylinder, p_0 static pressure in return pipeline, A_k area of actuator piston, β coefficient of fluid compressibility, c coefficient of fluid leakage, H_{cil} piston stroke, θ coefficient of parasite volume of connected pipeline to actuator cylinder, x_k position of piston and F applied external force (including inertial forces) to the actuator piston. Both signs in equations corresponds to the direct and reverse modes of actuator function.

Real state is described by pressure drops at supply and return throttles of control servovalve and corresponding pressure difference caused by external force:

$$\Delta p_{ul} = p_s - p_a \qquad \Delta p_{iz} = p_r - p_0 \tag{1}$$

$$p_a - p_r = \frac{|F|}{A_k} sgn(F x_r) \tag{2}$$

Equations (1) and (2) defines basic formulation of system dynamic model. Enclosed formulation of mentioned pressure drops cannot be determined without additional approximations. If we assume that fluid flow through servo-valve is a turbulent, approximate expressions of corresponding equivalent pressure differences in accordance to the figures b) and d) are defined for incompressible fluid flow by following relations:

$$Q_s = Q_0 = \pm A_k \dot{x}_k = \pm\mu^{\pm} b_r^{\pm} x_r \sqrt{\frac{2}{\rho}(p_s - p_a^{'})}$$
$$Q_s = Q_0 = \pm A_k \dot{x}_k = \mu^{\pm} b_r^{\pm} x_r \sqrt{\frac{2}{\rho}(p_r^{'} - p_0)} \tag{3}$$

where equivalent pressure values for direct and reverse modes are defined in the form:

$$p_r^{'} = p_F + p_s$$
$$p_a^{'} = p_F + p_0$$

SEPARATE FLOW MODELING

Previous relations (1) and (2) can be expanded for symmetric supply and return flow characteristics of servo-valve in the following form (with assumed value of p_0=0) for direct and reverse modes:

$$p_a = p_s - \Delta p = \frac{1}{2}(p_s \pm \frac{F}{A_k})$$
$$p_r = p_0 + \Delta p = \frac{1}{2}(p_s \mp \frac{F}{A_k}) \tag{4}$$

Expressions (4) needs more attention on its meaningless. Corollary of expressions (4) is that nominal system pressure for zero external load is defined as

$$p_{sr} = \frac{p_s + p_0}{2} \tag{5}$$

It means that hydraulic system pressure for zero load must be equal to the p_{sr}. In the cases of continuous actuator function previous relation holds. In addition, this expression are satisfied for all regimes in which effects of pressure surge can be neglected. In opposite cases, presented mathematical system formulation does not hold. Then subjected mathematical model is not compatible. Corresponding to the relations (4) pressure drop can be determined in the form:

$$Q_s = Q_0 = A_k \dot{x}_k = \mu^{+} b_r^{+} x_r \sqrt{\frac{2}{\rho}(p_s - p_0 - p_F)}$$
$$Q_s = Q_0 = -A_k \dot{x}_k = -\mu^{-} b_r^{-} x_r \sqrt{\frac{2}{\rho}(p_s - p_0 + p_F)} \tag{6}$$

where p_F represent pressure drop corresponding to external load:

$$p_F = \pm\frac{F}{A_k} \tag{7}$$

Finally, static pressure in supply and return branches of actuator streamline can be expressed in expanded form:

$$p_a = \frac{1}{2}\left(p_s + p_0 + \frac{F}{A_k} sgn\, x_r\right)$$
$$p_r = \frac{1}{2}\left(p_s + p_0 - \frac{F}{A_k} sgn\, x_r\right) \tag{8}$$

Relations (8) are similar to the relations (4). In previous discussion hydraulic system pump is assumed as strong one. It means that the system pump is able to takes up system pressure to the maximal nominal value. This assumption is valid except for existence of system model incompatibilities. This problem can be solved by assuming system pump as a weak one at the initial moment of actuator engage. It follows that any regime of small external load must be assumed as of weak pump. This statement arise from the fact that static pressure in hydraulic system at any moment of its function is caused by external load and pressure loses. As consequence of previous statements, corresponding boundary conditions at actuator pipeline inlet must be determined at initial moment as maximal pump flow. Caused value of system static pressure exists till the moment of pressure upgrading to its nominal system value. In that moment boundary conditions changes to the determined inlet pressure (equal maximal nominal value) and caused value of fluid flow (second part of flow cross the relive valve), which corresponds to the strong system pump.

If reduced valve is built in hydraulic system at actuator supply branch mentioned effects decreases. But in initial moment of actuator engage they can't vanish completely. It means that system pump can't be strong one for each of possible regimes, spatially at its initial moment.

BOUNDARY CONDITIONS

Corresponding actuator structural block diagrams for the cases of weak and strong system pump are presented on following figure.

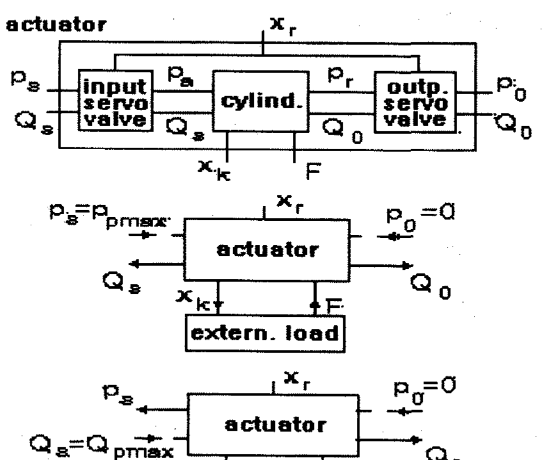

Fig.4

For geometric and flow symmetry of control servo-valve previous relations can be expressed in the form

$$A_k \dot{x}_k = \mu^+ b_r^+ x_r \sqrt{\frac{2}{\rho}(p_{p\,max} - p)}$$
$$A_k \dot{x}_k = \mu^- b_r^- x_r \sqrt{\frac{2}{\rho}(p_{p\,max} + p)} \qquad (9)$$

or in condensed form:

$$A_k \dot{x}_k = \mu^s b_r^s x_r \sqrt{\frac{2}{\rho}\left(p_{p\,max} - \frac{F}{A_k}\, sgn\, x_r\right)} \qquad (10)$$

where index s denotes parameters of symmetric servo-valve. Relation (10) is final mathematical model form of control servo-valve pressure drop for the case of actuator symmetry. This formulation is well known. It must be noted that p_{pmax} is correct term only for strong system pump. For the cases of weak pump p_{pmax} becomes equal p_s, with corresponding changes of boundary conditions formulation, which gives following model:

$$Q_{p\,max} = A_k \dot{x}_k$$
$$Q_{p\,max} = \mu^s b_r^s x_r \sqrt{\frac{2}{\rho}\left(p_{p\,max} - \frac{F}{A_k}\, sgn\, x_r\right)} \qquad (11)$$

ANALYSIS OF INCOMPRESSIBLE MODELS

All of the exposed diagrams are related to undimensional ratio system coordinates, where are: ξ_{k1} piston position, ξ_{pa} static pressure in actuator supply chamber, ξ_{pr} static pressure in actuator return chamber, ξ_r control servovalve throttle position, φ ratio of power reserve corresponding to applied external load.

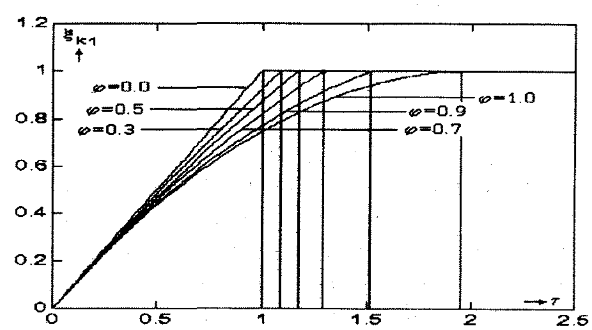

Fig.5

Corresponding to discussion about strong and weak regimes of pump function, it is of interest to determine actuator abilities for suppression external load. Corresponding coefficient φ is defined as ratio between maximal external acting load and maximal possible load which can be suppressed by the system pump. It is also shown correlation between nonlinear and linearised actuator models with incompressible fluid flow.

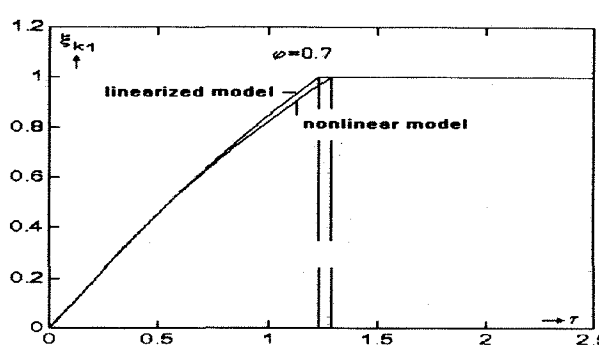

Fig.6

Effects of model nonlinearities and its corresponding linearisation for the case of incompressible flow are presented on figure 6. We can conclude from diagram that for usual reserve of actuator ability these effects are practically similar. This conclusion indicates that other effects are of higher influence than the effects of model linearisation.

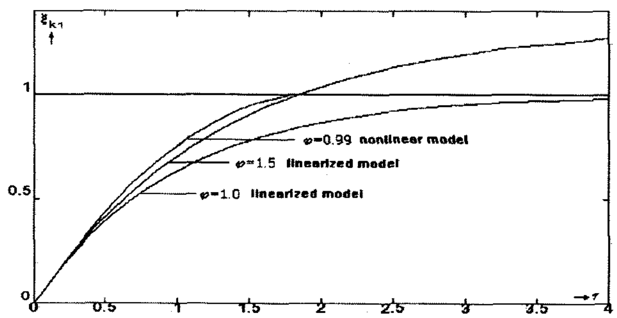

Fig.7

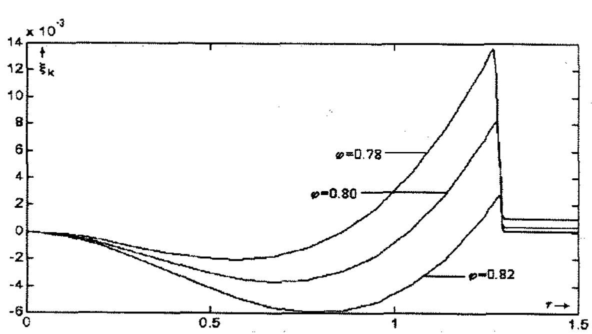

Fig.8

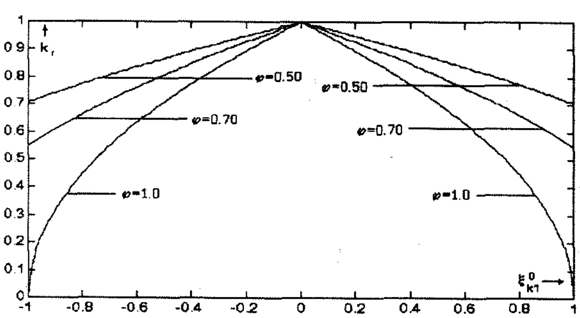

Fig.9

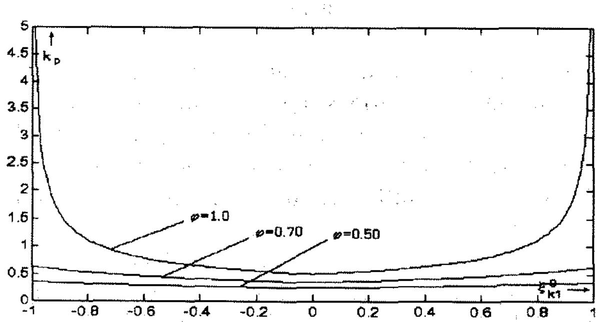

Fig.10

On figure 7 is compared nonlinear and linearised approximation on the whole domain of actuator piston stroke. Corresponding differences of actuator output between these models are presented on the figure 8. On figures 9 and 10 are shown diagrams of nondimensional coefficients of model linearisation (linearised form of equation (10) with coefficients k_r and k_p in respect to x_r and F) corresponding to its nominal regimes.

ACTUATOR TRANSFER CHARACTERISTICS

Actuator simulation is presented for step and harmonic input of actuator servo-valve. Step input is used for getting actuator quasi stationary characteristics and parameters identification. Harmonic impute can be used for high cyclic actuator identification.

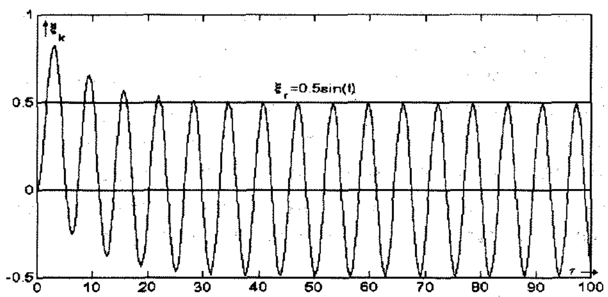

Fig.11

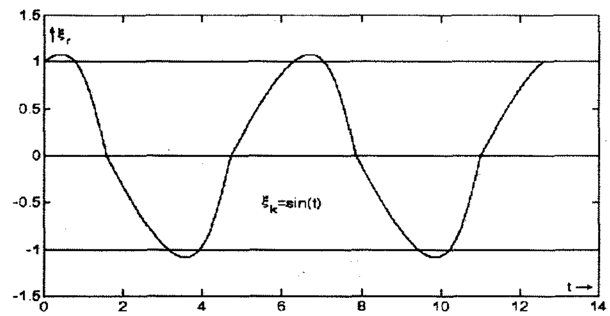

Fig.12

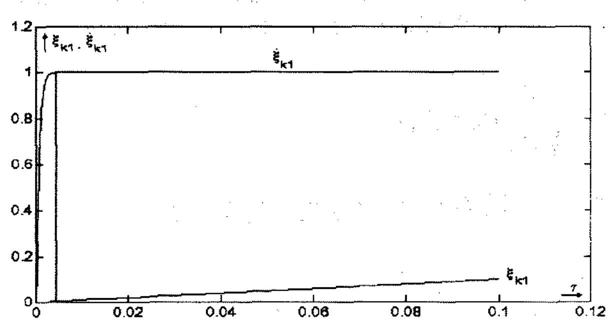

Fig.13

On figure 11 is presented simulation of nonlinear periodic actuator output piston stroke for harmonic control input, both in nondimensional form. If corresponding relative amplitude of servo-valve throttle is equal 0,50, depending of relative time equal 1 for total piston stroke, approximate harmonic output is recovered approximately after 5 cycles. System nonlinearity is shown on figure 12, which represents inverse simulation of system relative control input for harmonic relative output.

On figure 13 are presented simulation curves of relative actuator piston stroke and its corresponding relative velocity as actuator system outputs for unit step relative input if exists only inertial actuator load. Corresponding actuating relative time delay is less than 0,004.

ACTUATOR MODELING WITH ASSUMED QUASI-STATIC FLUID COMPRESSIBILITY

Pressure drop in hydraulic systems can be caused by small external load or local increasing of fluid flow to be greater than maximal possible pump source flow. To prevent this it is needed to separate corresponding branch of actuator supply by reduction valve. In these cases possible pressure surge are not of high influence and is determined by equivalent actuator stiffness together with potential external load. Pressure increases proportionally with increasing of piston displacement corresponding to its velocity. This pressure increasing is too slower than for the cases of pressure surge caused by fluid compressibility.

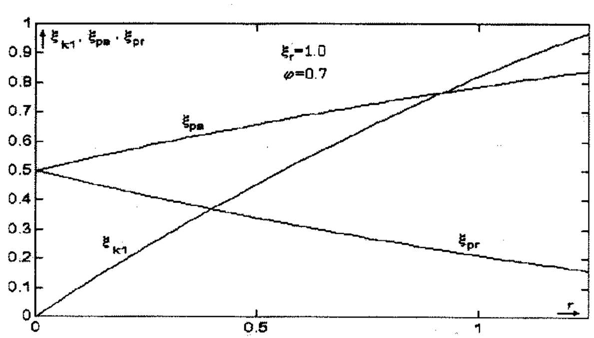

Fig.14

If actuator is assumed with quasistatic compressible fluid flow, system model can be presented for symmetric supply and return branch of fluid flow in the following form:

$$\mu b^r x_r \sqrt{\frac{2}{\rho}(p_s - p_a)} =$$

$$= A_k \dot{x}_k \pm \beta A_k (H_{cil}\theta + x_k)\dot{p}_a + c(p_a - p_r)$$

$$\mu b^r x_r \sqrt{\frac{2}{\rho}(p_r - p_0)} = \tag{12}$$

$$= A_k \dot{x}_k \mp \beta A_k (H_{cil}\theta - x_k)\dot{p}_a + c(p_a - p_r)$$

$$F = A_k(p_a - p_r)$$

On figure 14 are presented simulation of actuator piston relative stroke and relative static pressure in supply and return actuator chambers for usual mathematical form of actuator dynamic model. As it is explained, initial relative pressure values (equal 0.5) can exists in ideal model only. In real cases, initial values of relative pressure in actuator chambers is the result of actuator history and fluid leakage, which produces its ambiguity. Extreme case corresponds to the zero values of initial relative pressures. Corresponding simulation of supply and return relative pressures are presented on figures 16 and 17.

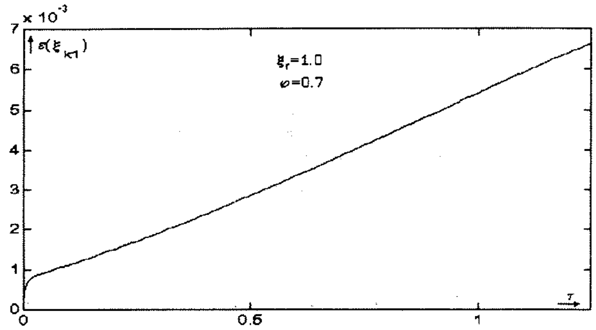

Fig.15

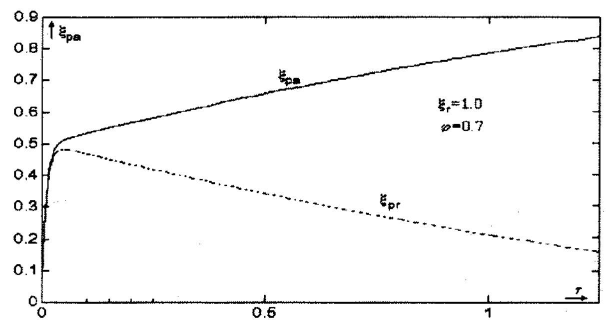

Fig.16

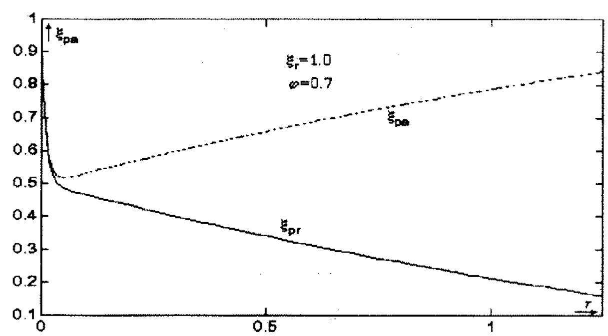

Fig.17

Presented system model enables its compatibility corresponding to the various initial conditions. On figure 14 are shown system simulation for incompressible fluid flow and corresponding model noncompatibility of initial conditions. Pressure difference between supply and return actuator chambers (which is not compensated by external force) produces piston shock motion, which can't be described by incompressible flow system modeling. Pressure difference can be caused by various effects which produces fast changes of fluid static pressure and/or external load. Actuator locked position for long time also is the reason of described effects. Pressure drop or surge caused by fluid compressibility and initial condition discontinuity as result of closed control servo-valve throttle position are shown on the diagrams on figures 16 and 17. Piston position difference in relation with its incompressible model motion is defined on figure 15. Initial piston acceleration produces piston shock motion, expressed in the later as increasing static error of its position (less than 1% for usual types of fluids). Mentioned effects are of high interest for direct digitally driven actuators. Supply and return pressure surge is presented on figure 16. Supply and return pressure drop are presented on figure 17. Because presented system model does not include previously explained real wave effects it is important to establish model in which corresponding effects are fully and compatible involved.

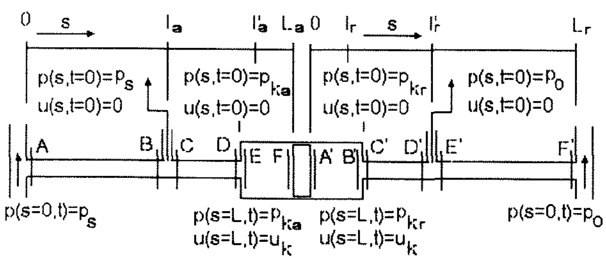

Fig.18

ACTUATOR MODEL INCLUDING WAVE PROPAGATION EFFECTS

Described effects can be evaluated by partial differential equations of continuity and momentum with additional fluid compressibility law for one dimensional flow. Effect of fluid viscosity can be entered as a friction between fluid streamline and pipeline wall. Local viscous effects at control servovalve and flow inlet and outlet of actuator cylinder represented by corresponding pressure lose coefficients are involved as boundary conditions. Equations of continuity and momentum for one dimensional fluid flow including the effects of wall friction are presented in the following form:

$$\begin{aligned} &\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-c\frac{\partial \mu}{\partial x}+\frac{1}{2}u^2\frac{\partial \xi}{\partial x} \\ &\frac{\partial \mu}{\partial t}+u\frac{\partial \mu}{\partial x}=-c\frac{\partial u}{\partial x} \\ &\mu=\int_{\rho_0}^{\rho}\sqrt{\frac{dp}{d\rho}}\frac{d\rho}{\rho} \end{aligned} \tag{13}$$

where are: u-fluid velocity, p-static pressure, ρ-density, ξ-local pressure drop caused by flid viscosity, x-coordinate along streamline and t-time. Presented system of nonlinear partial differential equations can be numerically evaluated by using several methods. Finite element method is very useful for linear problems. Method of characteristics can be successfully applied in numerical solving of presented partial differential equations. Presented equations can be also linearised because the fluid velocity is a small value.

Complete actuator system (points B through E'), connected into hydraulic system into points A (supply) and F' (return) to the hydraulic system, is presented on figure 18. Static pressure in supply and return pipelines are presented as p_s and p_0, respectively.

FINITE ELEMENT SOLUTION

In fluid dynamic problems method of characteristics is usually used for solving presented partial differential equations (13). We must note that problem formulation by method of characteristics can't be used for synthesis actuator dynamic model in the form of ordinary differential equations. Finite element problem formulation is too useful, but it makes some computational difficulties, expressed by oscillation of got solution. Performances of FEM solution needs high dimension of nonlinear problem formulation. For that purpose linearisation of presented partial equations can be more effective. This system form can be effectively reduced, but only on first corresponding system modes. It is well known problem with reduction of systems with distributed parameters.

Problem formulation of one dimensional fluid flow can be approximated in discrete form. One of possible methods is finite element method (FEM). Total number of model (differential and algebraic) equations are 12+2n, where n corresponds to the each of the assumed nodes which represents cross section of pipelines and both actuator chambers. It must be noted that FEM model formulation is not too useful for actuator mathematical description, because its high dimension and relatively low computational accuracy. Method can be used for synthesis dynamic model which includes only slow system modes. For that purpose model dynamic reduction is needed.

In comparison with method of characteristics, FEM is more applicable for compressible fluid flow formulation in the form of ordinary differential equations. For actuator simulation it is better to use model formulation in the form of method of characteristics, because its higher accuracy. In this paper comparison of methods performances is shown on the following diagrams.

For usual regimes, partial differential equations (13) can be linearised because fluid velocity can be assumed as relatively small. From the other side, nonlinear FEM problem formulation makes more computation in its simulation. It is not usable for system dynamic reduction, except model approximation in the form of matrix power series (it makes also to much computational troubles). Linearised form of continuity and momentum equations can be presented in the form:

$$\frac{A}{\rho_0}p' + \dot{Q} = 0$$
$$\frac{N}{A}Q' + \dot{p} = 0 \tag{14}$$

where are Q-fluid flow, ρ_0-normal density and A-area of pipeline cross section. Finite element formulation for low compressible fluid can be presented by interpolation functions $\varphi_i(s)$ as follows:

$$u(s,t) = \sum_{i=1}^{\infty} \varphi_i(s)u_i(t) + u_0(t) + u_L(t)$$
$$\mu(s,t) = \sum_{i=1}^{\infty} \varphi_i(s)\mu_i(t) + \mu_0(t) + \mu_L(t)$$

By using presented approximation, partial equations (13) can be transformed into system of nonlinear ordinary differential equations (j=1,2,...,n) in the form:

$$a_{ij}\dot{u}_i + \sum_{m,n=1}^{\infty} b_{mnj}u_m u_n = -cd_{ij}p_i + e_{mnj}u_m u_n + f_j$$
$$a_{ij}\dot{p}_i + \sum_{m,n=1}^{\infty} b_{mnj}u_m p_n = -c^2\rho_0 f_{ij}u_i + g_j \tag{15}$$

or in linearised form:

$$a_{ij}\dot{u}_i = -cd_{ij}p_i + f_j$$
$$a_{ij}\dot{p}_i = -c^2\rho_0 d_{ij}u_i + g_j \tag{16}$$

where are corresponding coefficients:

$$a_{ij} = \int_0^s \varphi_i(x)\varphi_j(x)dx$$
$$d_{ij} = \int_0^s \varphi_i'(x)\varphi_j(x)dx$$
$$b_{mnj} = \int_0^s \varphi_m(x)\varphi_n'(x)\varphi_j(x)dx$$
$$e_{mnj} = \frac{1}{2}\int_0^s \frac{\partial \xi}{\partial x}\varphi_m(x)\varphi_n'(x)\varphi_j(x)dx$$

One of possible problem approximations is shown on figure 19. This model includes only wave propagation trough supply and return branches of actuator pipeline between inlet and outlet of servo-valve. This approximation does not include expansion wave propagation inside the supply pipeline and corresponding pressure drop as its result. These effects will be explained for method of characteristics.

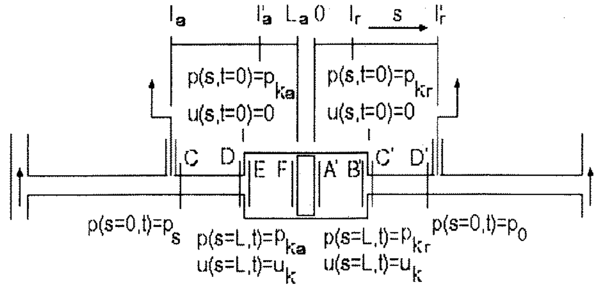

Fig.19

Complete linearised actuator dynamic model evaluated by finite element method formulation can be expressed in the following matrix form:

$$KT_{ap} + M'\dot{T}_{aQ} = E_a p_{ul} + F_a\dot{Q}_{iz}$$
$$KT_{aQ} + M''\dot{T}_{ap} = G_a Q_{iz} + H_a\dot{p}_{ul}$$
$$KT_{rp} + M'\dot{T}_{rQ} = E_r p_{ul} + F_r\dot{Q}_{iz}$$
$$KT_{rQ} + M''\dot{T}_{rp} = G_r Q_{iz} + H_r\dot{p}_{ul}$$
$$Q_s = \mu_a b_a x_r\sqrt{\frac{2}{\rho}(p_{p\,max} - T_{apo})} \tag{17}$$
$$Q_0 = \mu_r b_r x_r\sqrt{\frac{2}{\rho}(T_{rpn} - p_o)}$$
$$T_{rQn} = A_k\dot{x}_k - \beta V_r\dot{T}_{rp}$$
$$T_{aQ0} = A_k\dot{x}_k + \beta V_a\dot{T}_{ap}$$
$$A_k(T_{apn} - T_{rpo}) = k_a x_k$$

Corresponding partial results are shown on the following diagrams. It is easy to see that given solution has usual relatively large oscillating computational error. This error can be reduced, but not completely excluded, by assuming higher order of system approximation. Corresponding distribution of fluid velocity (or flow) are presented on the following diagrams.

On figure 20 it is presented fluid velocity propagation at supply actuator pipeline after actuator engage. Oscillating error arises on the beginning of wave propagation. Velocity distribution at later moments are presented on the figure 21. Zero value of velocity is the result of actuator piston area (333 times greater than area of pipeline cross section) assumed from 0.5 -1.0 of relative streamline coordinate ξ_x.

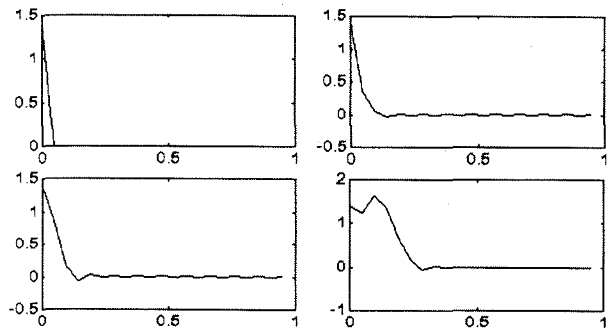

Fig.20

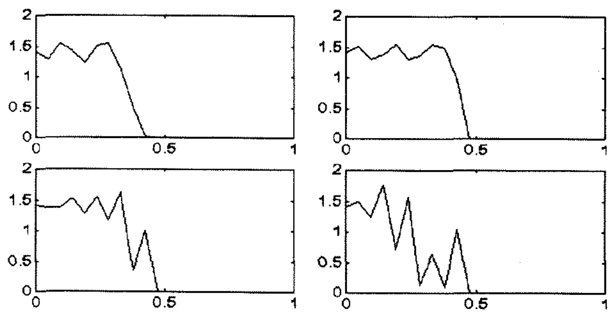

Fig.21

METHOD OF CHARACTERISTICS

Corresponding equations of characteristics of (13), excluding wall friction, takes the following form:

$$\frac{\delta_+}{\delta t}(\mu + u) = \left[\frac{\partial}{\partial t} + (u + c)\frac{\partial}{\partial x}\right](\mu + u) = 0$$

$$\frac{\delta_-}{\delta t}(\mu - u) = \left[\frac{\partial}{\partial t} + (u - c)\frac{\partial}{\partial x}\right](\mu - u) = 0 \quad (18)$$

where is c-velocity of sound. Effects of wall friction practically can be neglected because its influence are less than 1% of total pressure loses. Friction also produces lower pressure discontinuities. If the corresponding wall friction effects are neglected and external disturbances are small, linearised equations of characteristics (18) takes form of acoustic wave equation. Any system disturbance produces wave effects, which propagates trough the whole system. They travels with corresponding relative velocity of sound c on both sides of pipeline and produces piston time delay and pressure drop effects. Because in the wave equation are not included viscous effects, it describes undamped wave propagation.

Full problem formulation includes equations of characteristics with corresponding boundary and initial conditions for each of assumed system subdomains. These domains corresponds to the inlet and outlet pipelines, supply and return fluid flow sections between servovalve and actuator piston. Wave effects of return flow are to small and can be neglected. Wave effects from the both sides of the actuator piston can be assumed as completely separated. Corresponding nominal boundary conditions are assumed as symmetric ones (fig. 14). System is approximated as on the figure 22. Point A (assumed as inlet of the actuator streamline) corresponds to the fluid flow separation from the main hydraulic system bus. Actuator supply branch of servovalve is located between points B and C. Effects of pressure local lose at the inlet of fluid flow to the actuator cylinder is attached to the points D and E. Point F represents position of actuator piston as movable boundary.

Nominal supply pressure of hydraulic system can be assumed as a boundary condition at point A. In points B and C corresponding boundary conditions are defined as flow continuity between B and C, and pressure lose between B and C caused by control servovalve throttle leakage. In points D and E boundary conditions are of same type as for the points B and C with only difference in pressure lose caused by fluid viscous effects at cylinder flow inlet. At point F are defined boundary conditions of static pressure and piston velocity equal to the fluid velocity, whose relation is defined by equivalent actuator stiffness.

Initial conditions are, of course, defined at each of domains (A-B, B-C and E-F). Initial conditions of fluid flow are assumed as zero fluid velocity. Initial conditions of fluid static pressure are assumed as supply static pressure (A-B) and cylinder static supply pressure (C-D and E-F). Initial and boundary conditions compatibility is complete for the exposed system model.

ACTUATOR SIMULATION MODEL

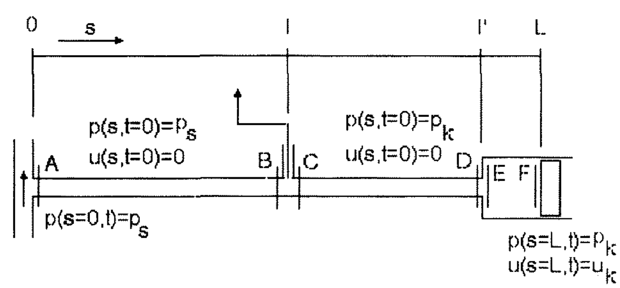

Fig.22

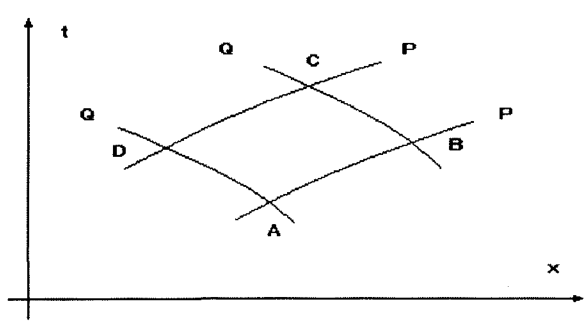

Fig.23

Initial and boundary conditions are presented by following relations corresponding to the figure 22. Initial conditions of flow velocity and static pressure are defined by following relations:

$$
\begin{aligned}
&0 \le s \le l: \Rightarrow u(s,0) = 0 \\
&l \le s \le l': \Rightarrow u(s,0) = 0 \\
&l' \le s \le L: \Rightarrow u(s,0) = 0 \\
&0 \le s \le l: \Rightarrow p(s,0) = p_s \\
&l \le s \le l': \Rightarrow p(s,0) = p_c \\
&l' \le s \le L: \Rightarrow p(s,0) = p_c
\end{aligned}
\tag{19}
$$

Boundary conditions are defined at four points: point A of separation fluid flow to actuator, control valve B-C, fluid inlet into actuator chamber D-E and movable piston F. Corresponding boundary conditions are defined in the form of continuity and Bernoulli equations or piston momentum equation in addition with pressure and velocity conditions, presented respectively as:

$$
\begin{aligned}
&s = 0: \Rightarrow p_A = p_s \\
&s = l: \Rightarrow \rho_B A_B u_B = \rho_C A_C u_C \\
&s = l: \Rightarrow \\
&\frac{p_B}{\rho_B} + \left[1 - \left(\frac{A_B}{\eta_B dx}\right)^2\right]\frac{u_B^2}{2} = \frac{p_C}{\rho_C} + \left[1 + \left(\frac{A_C}{\eta_C dx}\right)^2\right]\frac{u_C^2}{2} \\
&s = l': \Rightarrow \rho_D A_D u_D = \rho_E A_E u_E \\
&s = l': \Rightarrow \frac{p_D}{\rho_D} + \frac{u_D^2}{2} = \frac{p_E}{\rho_E} + [1 + \xi_E]\frac{u_E^2}{2} \\
&s = L: \Rightarrow p_F = p_k \\
&s = L: \Rightarrow u_F = v_k
\end{aligned}
\tag{20}
$$

For numeric solving presented problem is accepted method of characteristics. Left boundaries are determined with Q-characteristics, and right ones by P-characteristics, shown by relations (24). At the boundaries B-C and E-F corresponding values of μ and u must be determined by interpolation. At point F must be applied some of numeric integration methods for determination corresponding pressure value on the movable piston surface. Applying method of characteristics corresponding pressure and velocity at nodal points are defined:

$$
\begin{aligned}
&\mu_A + u_A - \mu_B - u_B = 0 \\
&\mu_D + u_D - \mu_C - u_C = 0 \\
&\mu_A - u_A - \mu_D + u_D = 0 \\
&\mu_B - u_B - \mu_C + u_C = 0
\end{aligned}
\tag{21}
$$

with corresponding coordinates of nodal points:

$$
\begin{aligned}
&s_B - s_A = \left(\frac{2c + u_A + u_B}{2}\right)(t_B - t_A) \\
&s_D - s_A = \left(\frac{-2c + u_A + u_D}{2}\right)(t_D - t_A) \\
&s_C - s_D = \left(\frac{2c + u_C + u_D}{2}\right)(t_C - t_D) \\
&s_C - s_B = \left(\frac{-2c + u_B + u_C}{2}\right)(t_C - t_B)
\end{aligned}
\tag{22}
$$

Relations (22) can be performed to define corresponding computational network:

$$
\begin{aligned}
&s_B - s_A = \left(\frac{2c + u_A + u_B}{2}\right)(t_B - t_A) \\
&s_{D'} - s_{A'} = \left(\frac{-2c + u_{A'} + u_{D'}}{2}\right)(t_{D'} - t_{A'}) \\
&t_B = t_{D'} \\
&s_B = s_{D'}
\end{aligned}
\tag{23}
$$

in addition with boundary conditions on corresponding P and Q characteristics:

$$
\begin{aligned}
&-u_A + \mu_A = Q_A \\
&-u_C + \mu_C = Q_C \\
&-u_F + \mu_F = Q_F \\
&u_B + \mu_B = P_B \\
&u_D + \mu_D = P_D \\
&u_F + \mu_F = P_F
\end{aligned}
\tag{24}
$$

Piston position is defined by following equation as result of numeric integration:

$$
\dot{p}_F = \frac{k_a}{A_k} u_F \tag{25}
$$

Relations (19)-(21) and (23)-(25) defines complete mathematical model of hydraulic actuator.

ACTIVE CONTROL OF PRESSURE DROP

In the following are shown some of the results of simulation of hydraulic actuator motion in undimensional form with following values of assumed system parameters: -relative position of servo -valve: $-f_l = 0{,}5$; -position of actuator cylinder flow inlet: $-f_{lp} = 0{,}7$; -ratio between cross section area of piston and pipeline: $-A_2/A_1 = 50$; -supply pressure: $-\xi ps=1{,}0$; -pressure on the supply piston surface: $-\xi p_k=0{,}3$; -coefficient of pressure lose of servo valve: $-\xi_{sir}=0{,}05$; -coefficient of relative pressure lose on flow inlet into actuator supply chamber: $-\xi_a=0{,}05$.

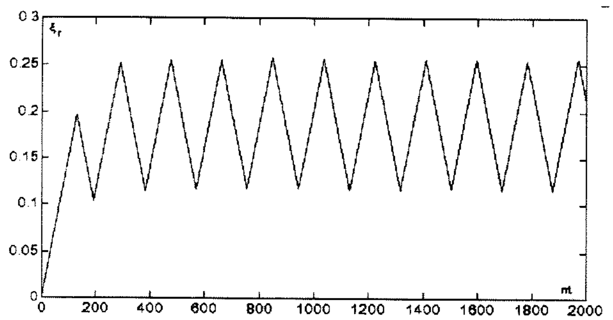

Fig.24

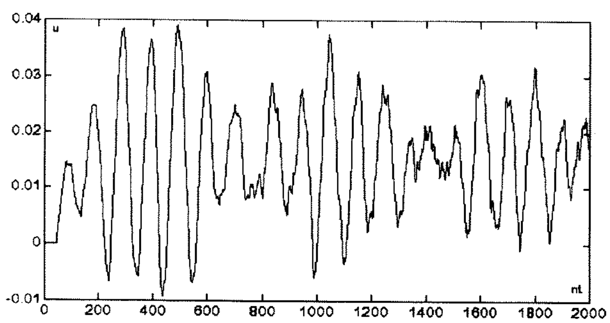

Fig.25

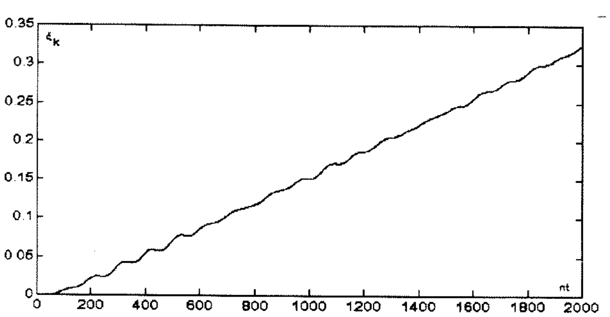

Fig.26

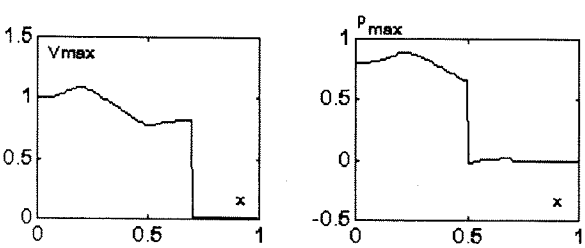

Fig.27

Control law of closed loop is assumed in the following form of relative coordinates with additional fluid velocity constraint defined by u_{lim} :

$$\dot{\xi}_r = -k\, sgn(u - u_{lim}) \qquad (26)$$

where k is assumed as equal 0.14. From the figure 24 control valve throttle moves between 12% and 26% percents of maximal control valve throttle leakage. This throttle control law produce oscillating piston motion (fig. 26) with its small reveres at the beginning caused by the effects of piston starting acceleration. After stabilization whole process of actuator starting, flow velocity is always great than zero and produces uniform piston motion, presented on the figure 25. Maximal piston acceleration is determined by arising pressure drop. Static pressure and flow velocity distribution along actuator supply pipeline of excluded actuator servo-valve throttle control for zero external load is presented on figure 27. Pressure drop exists in points with relative velocities greater than 1.0.

CONCLUSION

Computer simulation of fast cyclic hydraulic servo-actuator is very effective tool in procedure of its synthesis. Effect of fluid compressibility introduces additional wave propagation problems, which can be solved by some of numerical methods. It is shown that FEM technique has priority in actuator identification problems. For simulation purposes, it is recommended method of characteristics for system modeling. Pressure drop effects in fast hydraulic actuators must be suppressed by additional throttle active control of servo-valve. Generally, this control slows actuator motion and limits its performances, but it suppress any reverse motion of mentioned or any other actuator engaged in the hydraulic system at the same moment.

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