ABOUT AEROSERVOELASTICITY CRITERIA FOR ELECTROHYDRAULIC SERVOMECHANISMS SYNTHESIS

Ioan Ursu, Senior Researcher, **Mihai Vladimirescu,** Scientific Researcher, and
Felicia Ursu, Senior Researcher
"Elie Carafoli" - National Institute for Aerospace Research,
Iuliu Maniu 220,
Bucharest, sect. 6, Romania

Abstract

The evolution of modern day aircraft, characterised by ideas as: CCV, Active Control, Mechatronic Systems, Innovative Control Strategies, etc., impose that the design problem of electro-hydraulic servomechanisms can not be separated, even in the early stage of design, of the control strategy of the aircraft. Our paper presents two important criteria of servomechanism's design:

1) a criterion based on the impedance function of the electro-hydraulic servomechanism;

2) a criterion determined by the active control of the flexible body modes of the aircraft.

Usually, this type of criteria is not explicitly taken in account in the design phase of the electro-hydraulic servomechanisms. Our main result is a method of servomechanism synthesis according to the just mentioned criteria. Firstly, we have introduced a new impedance function idea for an electro-hydraulic servomechanism. In the process of satisfying the second criterion, we noticed that superimposing the servomechanism dynamics on the aircraft control channels resulted in a loss of controllability.

Consequently, another result of our paper consists in the introduction of an inner adaptive controller to obtain a quasi-proportional behaviour of the electro-hydraulic servomechanism.

Finally, we used a LQG control strategy to damp the flexible body modes of the aircraft without any loss of handling qualities.

1. Introduction

During the past two decades, the active control of the flexible body modes of an aeroelastic aircraft has focused the attention of the researchers in the field. In connection with this, increased the role of new concepts such as: CCV, Active Control, Mechatronic Systems, Motion Control, Innovative Control Strategies, etc. These impose that the design problem of electro-hydraulic servomechanisms' design can not be separated of the aircraft control strategy, even in the early stage of design. This is due to the fact that not any limit of the performance can be surpassed later by the use of a proper control law.

Our paper presents two important criteria of servomechanism's design:

1) a criterion based on the impedance function of the electro-hydraulic servomechanism;

2) a criterion determined by the active control of the flexible modes of the aircraft.

In order to show the use of these criteria we used an aeroelastic aircraft model derived by M. R. Anderson [5] for B1 aircraft. This mathematical model ignores the dynamics of the electro-hydraulic servomechanisms. When this dynamic is superimposed, the extended systems lose its controllability and furthermore it became unstabilizable.

We used an inner adaptive control strategy to transform the servomechanism in a quasi-proportional system and to surpass, this way, the difficulty just mentioned.

To fulfil the second criterion we employed an LQG type controller to improve the damping of the aircraft flexible body modes without deteriorating the handling qualities.

2. A Representative Linear Mathematical Model for the Electro-Hydraulic Servomechanism in an Aeroservoelastic Context

In one of our recent papers [1], we have presented a detailed approach of the problems emerging in the process of mathematical modelling of hydraulic servomechanisms used in aircraft control chains.

For the purpose of this paper, the mathematical model of the electro-hydraulic servomechanism, in the general context of an application in the field of the aeroservoelastic active control, must fulfil the following two conditions:

a) It must be sufficiently accurate to show the influence of the aeroservoelastic elements connected with the hydraulic actuator (i.e. the stiffness of the servomechanism-structural and rod-control surfaces' links) in the case of the whole synthesis problem.

b) It must not exceed a certain degree of complexity - using a very complicated system, one may easily lose the essence of the problem.

Consequently, one should neglect the structural damping as well as the dynamics of the electro-mechanical converter element and obtain the following model equations (see Figure 2.1 - variables introduced are functions of time, t):

$$\begin{cases} S\cdot(\dot{z}'-\dot{z}'')+k_C\cdot\Delta\dot{p}_m=-K_Q\cdot(y-z'')-K_{QP}\cdot\Delta p_m \\ y=k_\Delta\cdot\varepsilon \\ -E\cdot z''=S\cdot\Delta p_m=N\cdot(z'-z)=m\cdot\ddot{z}+f\cdot\dot{z}+r\cdot z \end{cases} \quad (2.1)$$

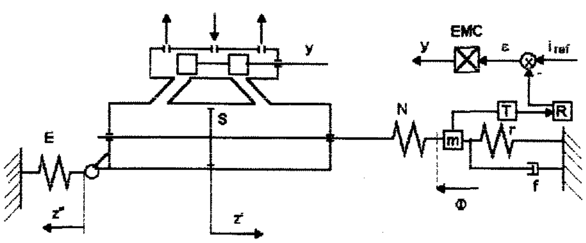


Fig. 2.1. The simplifyed physical model of an electro-hydraulic servomechanism
EMC - electro-mechanical converter; R - regulator; T- transducer

In addition to the notations used in fig. 2.1 we denoted by: K_Q, K_{QP} - the coefficients of the linearised flow rate characteristic [(2)] and k_Δ - the gain coefficient of the converter.

We will further introduce the following derived parameters:

$$\begin{cases} \dfrac{1}{r_h}=\dfrac{C}{2\cdot B\cdot S^2};\quad \dfrac{1}{r_d}=\dfrac{1}{N}+\dfrac{1}{E}+\dfrac{1}{r_h};\quad k_C=\dfrac{C}{2\cdot B} \\ \alpha_{ES}=\dfrac{K_Q}{E}+\dfrac{K_{QP}}{S} \end{cases} \quad (2.2)$$

where: C - volume of the actuator's chambers with the piston in the middle of the stroke;
B - bulk modulus of the fluid;
α_{ES} - quasi-hydraulic parameter;
r_h - hydraulic stiffness;
r_d - "dynamic" stiffness;
Δp_m - actuator differential pressure.

Assuming null initial conditions, one can obtain the open loop transfer function of the servomechanism as:

$$-\frac{z(s)}{\varepsilon(s)}=\frac{b_0}{a_0+a_1\cdot s+a_2\cdot s^2+a_3\cdot s^3} \quad (2.3)$$

where s is the Laplace variable and:

$$\begin{cases} b_0=K_Q\cdot k_\Delta;\quad a_0=\alpha_{ES}\cdot r;\quad a_1=\alpha_{ES}\cdot f+S\cdot(1+\dfrac{r}{r_d}) \\ a_2=\alpha_{ES}\cdot m+\dfrac{f}{r_d}\cdot S;\quad a_3=\dfrac{m\cdot S}{r_d} \end{cases} \quad (2.3')$$

We write the system in the time domain with the state matrix in the "companion" form:

$$\dot{x}(t)=A_1\cdot x(t)+B_1\cdot u(t) \quad (2.4)$$

$$\begin{cases} A_1=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\dfrac{a_0}{a_3} & -\dfrac{a_1}{a_3} & -\dfrac{a_2}{a_3} \end{bmatrix};\quad B_1=\begin{bmatrix} 0 \\ 0 \\ \dfrac{\alpha_S\cdot b_0}{a_3} \end{bmatrix} \\ \alpha_S\cdot z=\delta_C;\ x_1=\delta_C;\ x_2=\dot{\delta}_C;\ x_3=\ddot{\delta}_C;\ u\equiv\varepsilon \end{cases} \quad (2.4')$$

We assumed a direct proportionality between the linear displacement z and the angular displacement δ_C of the control surface.

3. The Aeroservoelastic Criterion of the Impedance Function

According to the AvP 970 Regulation, vol. 1, Leaflet 500/7, issue Nov. 1979, impedance is defined as "*the inverse of the ratio of the complex output displacement response to an externally applied sinusoidal force when input is held fixed and when there is no output inertia*". The structure of the impedance function, shortly the impedance, derived from the intrinsic design properties of the servomechanism, influences its behaviour in the presence of flutter generated oscillations: attenuation or amplification, as the impedance function determines a positive or negative damping. The regulation just mentioned mark this way the beginning of a new discipline: **Aeroservoelasticity**.

The impedance of a mechanic-hydraulic servomechanism is presented in some important works in the field [3, 4]; the ideal shape of the positive damping impedance (which is used against the flutter), in the complex plane, is a semicircle situated in the upper semiplane, with its base on the positive real axis, evolving, for the increase of the frequencies, from the left to the right (the intersections of this curve with the real axis represents the static stiffness, r_S ($\omega = 0$), and respectively the dynamic stiffness, r_d ($\omega = \infty$), with $r_S < r_d$).

The negative damping impedance (which enhances the flutter) has its ideal shape as a semicircle situated in the lower semiplane of the complex plane, with $r_S(0) > r_d(\infty)$. Even this impedance model is well known, a rigorous mathematical proof of it is not available.

A mathematical argument of the influence of the impedance models presented, on the flutter, in the case of a mechanic-hydraulic servomechanism may be developed using an energetic balance method [1].

One of the present paper's results is the introduction of an impedance function model in the case of an electro-hydraulic servomechanism too (we were not able to find such an impedance model in the available literature of the field). This type of servomechanisms is very important for the active control systems SAS (**S**tability **A**ugmentation **S**ystem) or SMCS (**S**tructural **M**ode **C**ontrol **S**ystems). According to the approach we have previously proposed [1], one may start from the equations' system (2.1), rewritten in a form suitable to the impedance calculus (see figure 2.1):

$$\begin{cases} \Phi = N\cdot(z - z') = -S\cdot\Delta p_m = E\cdot z'' \\ -k_T\cdot z = \varepsilon; \quad -k_\Delta\cdot\varepsilon = y \\ S\cdot s\cdot(z' - z'') + k_C\cdot s\cdot\Delta p_m = -K_Q\cdot(y - z'') - K_{QP}\cdot\Delta p_m \end{cases} \tag{3.1}$$

k_T is the gain of the transducer T; Φ is the disturbance of force at the servomechanism rod, at the level of the controlled load m, in the case of the input I_{ref}=0.

One can easily derive the impedance $I(i\cdot\omega)$ (i - imaginary unit) :

$$I(s) \stackrel{\Delta}{=} \frac{\Phi(s)}{z(s)} = r_d\cdot\frac{s + \dfrac{K_Q\cdot k_T\cdot k_\Delta}{S}}{s + \dfrac{K_{QP}\cdot r_d}{S^2}} = r_d\cdot\frac{s + a_1}{s + a_2} \tag{3.2}$$

This relation is analogous with the impedance relation derived in the case of the mechanic-hydraulic servomechanism, with the notations:

$$a_1 = \frac{\lambda_1\cdot K_Q}{S}; \quad a_2 = r_d\cdot\left(\frac{\lambda_2}{\lambda_1}\cdot\frac{a_1}{E} + \frac{K_{QP}}{S^2}\right) \tag{3.3}$$

where λ_1, λ_2 are the kinematics coefficients of the stiff feedback loop of the servomechanism.

The (positive) damping criterion of the flutter oscillations by the electro-hydraulic servomechanism takes the form of the following *Aeroservoelastic Impedance Criterion*:

$$\frac{K_Q\cdot k_T\cdot k_\Delta}{r_d} < \frac{K_{QP}}{S} \tag{3.4}$$

as a consequence of the condition [1] :

$$a_1 < a_2$$

4. The Aeroservoelastic Active Control Problem Formulation

This problem is connected with the high performance use of the electro-hydraulic servomechanisms in systems of the SAS and SMCS type [5, 6]. M. R. Anderson [5] presents a composite system:

$$\begin{cases} \dot{x}(t) = A\cdot x(t) + B\cdot u(t) \\ y(t) = C\cdot x(t) + D\cdot u(t) \end{cases} \tag{4.1}$$

derived by coupling the rigid body modes and the flexible body modes, for the longitudinal control of a B1 aircraft. The matrices [5] A, B, C, D are presented in table no. 1.

The first step in formulating the aeroservoelastic control problem consists in integrating the system (4.1) (considering its controls as state variables of an extended system) by taking also in account the electro-hydraulic actuators' dynamics. Usually, this is neglected from insufficiently motivated reasons (we believe).

The control vector [5]:

$$u=[\text{elevator command, vane command, plunge gust}]^T$$

is identified with three of the nine state variables of the three servomechanisms (see 2.4):

$$\dot{x}_j(t) = A_j \cdot x_j(t) + B_j \cdot i_j(t) \ , \ j=\overline{1,3} \ , \ x_j \in R^3$$

$i_j \equiv \varepsilon_j$ are the control electric currents of the hydraulic servovalves (EMC), and A_j and B_j can be put in relations similar to (2.4'), $B_j=[0,0,b_j]^T$.

The extended system takes the form:

$$\dot{x}_E(t) = A_E \cdot x_E(t) + B_E \cdot u_E(t), \quad u_E = [i_1, i_2, i_3]^T$$

$$x_E = \left[\tilde{x} \mid x_1 \mid x_2 \mid x_3 \right]^T \tag{4.2}$$

and may be easily obtained.

The matrices from table no. 1. use as units of acceleration g, unit of angles rad., unit of length in. and unit of time sec. In the subsequent chapters of our paper we used a scale transform of these values to obtain results in: centimetres for length, daN for force, sec. for time, cm/s^2 for acceleration.

The second step in formulating the aeroservoelastic control problem consists in verifying the controllability of the extended system. Table no. 2. shows the controllability modal measures, derived according to the Popov-Belevitch-Hautus indicators [7] in the case of the system (4.1), and table no. 3. shows the same data for the system (4.2)..

The data from the table no. 3. show the loss of controllability of the extended system, taking into account the third order dynamics of the actuators. This can be seen also with the use of the controllability grammian.

Furthermore, the extended system is not even stabilizable.

Assuming this as a *de facto* situation, we may formulate the active control of the aircraft flexible body modes for the longitudinal control as follows:

For the given structure of the eigenvalues (frequencies and damping) of the matrix A, to synthesise a control law to improve the damping of the flexible body modes without deteriorating the handling qualities of the aircraft.

The modal systems (open loop systems) of the matrices A and A_j, j=1,2,3 are shown in table no. 4.; (A_j, B_j) pairs, j=1,2,3 defining the hydraulic actuators for the elevator command, vane command and, respectively, plunge gust are:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1.3281\cdot10^5 & -1.7549\cdot10^5 & -0.9\cdot10^2 \end{bmatrix}; \ B_1 = \begin{bmatrix} 0 \\ 0 \\ 6.4388\cdot10^3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3.5012\cdot10^5 & -2.5410\cdot10^5 & -1.6\cdot10^3 \end{bmatrix}; \ B_2 = \begin{bmatrix} 0 \\ 0 \\ 9.24\cdot10^3 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1.0798\cdot10^6 & -0.7996\cdot10^6 & -0.3\cdot10^3 \end{bmatrix}; \ B_3 = \begin{bmatrix} 0 \\ 0 \\ 3.8650\cdot10^5 \end{bmatrix}$$

The matrix system presented has only an illustrative purpose. We must add that the problem is meaningful (because the extended system is not stabilizable) only after formulating another problem about the electro-hydraulic actuators.

5. Formulation and Solution of an Adaptive Compensator Problem for the Electro-Hydraulic Servomechanism

The difficulty linked with the loss of controllability of the extended system, including hydraulic actuators' dynamics, can be surpassed by formulating the following synthesis problem:

One should design an inner adaptive controller in the feedback loop of the electro-hydraulic servomechanism in such a way to obtain a close loop system quasi-proportional (i.e. proportionality between the input - servovalve command current and the output - rod displacement).

The solution of this problem is inspired by the algorithm proposed by D. E. Miller and E.J. Davison [8] (in a recent paper we proposed a solution based on an internal model method [10]).

TABLE No. 1. (A,B,C,D) System

A=

-10.1192	0	0	0	0	0	0	0
0	-50	0	0	0	0	0	0
-0.2529	-0.0035	-0.9850	0.9539	-0.0128	0.0049	-0.0001	-0.0005
-15.7550	0.2905	-9.9701	-1.8641	-0.4931	0.4754	-0.0149	-0.0015
0	0	0	0	0	0	1.0	0
0	0	0	0	0	0	0	1.0
-2971.7	-57.0	-194.1	-173.4	-380.6	37.2	-4.6	1.9
313.9	-31.0	-1470.4	-89.7	-9.7	-1578.	-0.4	-4.9

B=

10.0	0	0
0	50.0	0
0	0	-0.0010
0	0	-0.0106
0	0	0
0	0	0
0	0	-0.2061
0	0	-1.5609

D=

0	0	0
0	0	-0.0205
0	0	0.0215
0	0	0.0171

C=

0	0	0	1.0	0	0	0.0047	0.0057
-27.9860	-0.1558	-19.2971	-3.1952	-2.3450	-8.3805	-0.0388	-0.0209
8.4571	-0.1110	20.2640	0.6177	-1.0197	-3.3898	-0.0119	-0.0018
-1.1958	2.7961	16.1391	-0.7979	6.2202	22.1817	0.0396	0.0134

TABLE No. 2. Controllability Measures
(A,B) Pair

mode	first control	second control	third control
1	0.0081286	0.0005601	0.0248375
2	0.0081286	0.0005601	0.0248375
3	0.3054247	0.0023942	0.0090971
4	0.3054247	0.0023942	0.0090971
5	0.3496891	0.0020044	0.0451017
6	0.3496891	0.0020044	0.0451017
7	1.0000000	0.0000000	0.0000000
8	0.0000000	1.0000000	0.0000000

TABLE No. 3. Controllability Measures
(A_E, B_E) Pair

mode	first control	second control	third control
1	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000000	0.0000000
3	0.0000009	0.0000000	0.0000000
4	0.0000009	0.0000000	0.0000000
5	0.0000042	0.0000001	0.0000000
6	0.0000042	0.0000001	0.0000000
7	0.0000042	0.0000000	0.0000000
8	0.0000000	0.0000029	0.0000000
9	0.0000057	0.0000000	0.0000000
10	0.0019033	0.0000000	0.0000000
11	0.0019033	0.0000000	0.0000000
12	0.0000000	0.0000039	0.0000000
13	0.0000000	0.0011649	0.0000000
14	0.0000000	0.0011649	0.0000000
15	0.0000000	0.0000000	0.0000013
16	0.0000000	0.0000000	0.0006654
17	0.0000000	0.0000000	0.0006654

Table no. 4. Open Loop Poles

mode		open loop poles		damping	frequency [Hz]
Bending 2	1	-2.4220	+39.6549i		
	2	-2.4220	-39.6549i	0.0610	6.32
Bending 1	3	-2.4342	+19.3090i		
	4	-2.4342	-19.3090i	0.1251	3.09
Pitch	5	-1.3183	+3.1089i		
	6	-1.3183	-3.1089i	0.3904	0.53
	7	-10.1192		-	1.61
	8	-50.0000		-	7.95
Elevator actuator	9	-0.7571		-	0.12
	10	-42.8239	+416.6412i		
	11	-42.8239	-416.6412i	0.1022	66.66
Vane actuator	12	-1.3791		-	0.22
	13	-81.6242	+497.2065i		
	14	-81.6242	-497.2065i	0.1620	80.19
Gust actuator	15	-1.3511		-	0.21
	16	-157.2999	+880.0187i		
	17	-157.2999	-880.0187i	0.1760	142.28

We will show the use of this algorithm on the system characteristic for the elevator actuator: A_1, B_1, $C_1=[k_T\ 0\ 0]$, $k_T=20/3.5$ mA/cm. This system fulfils the minimum phase condition and has a relative degree three [8].

The structure of the adaptive controller synthesis problem is as follows:

$$\begin{cases} \dot{x}(t) = A_1 \cdot x(t) + B_1 \cdot i_1(t) \\ y(t) = C_1 \cdot x(t) \\ i_1(t) = r(t) + 2 \cdot \dot{r}(t) + \ddot{r}(t) \\ r(t) = k(t) \cdot J \cdot v(t) \\ \dot{v}(t) = k^2(t) \cdot (G \cdot v(t) + H \cdot e(t)), \quad v(t_i^+) = 0 \\ e(t) = i_{1,ref}(t) - y(t) \\ k(t) = 0.1 \cdot (2.5)^i, \quad t \in (t_i, t_{i+1}], \quad i = 1,2,\dots \\ G = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}; \ H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \ J^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{cases} \tag{5.1}$$

In fact, the adaptive controller consists of a (p-1)th order high gain linear invariant compensator (p is the relative degree of the system), parametrized by a single parameter that is adjusted by a tuning mechanism at discrete points in time, i.e. it is a switching-type controller. The tuning mechanism is itself parametrized by some constants defining the transient and steady-state response; the switching times, t_i, are chosen according to these constants.

The charts disclosed in Fig. 5.1, 5.2 , 5.3 show a performant behaviour of the proposed algorithm according to the formulating problem of a quasi-proportional system. Figure 5.1 shows the ability of the algorithm to ensure both an arbitrarily small error with a quick transient. This has been tested by various numerical simulations that are not presented here of space reasons. Figure 5.2 and figure 5.3 shows the error evolution (the imposed error was 0.1 mA), respectively the evolution of the compensator gain k(t).

It is of course a price to be paid for this performant behaviour consisting in the control saturation.

A less performant but realistic type of controller can be developed by taking in account the proportional controller structure:

$$\begin{cases} \dot{x}(t) = A_1 \cdot x(t) + B_1 \cdot i_1(t) \\ y = C_1 \cdot x(t) \\ i_1(t) = K(t) \cdot e(t) \\ e(t) = i_{1.ref}(t) - y(t) \end{cases} \tag{5.2}$$

The gain K(t) may be chosen as piecewise constant function, but situated in a region to ensure the stability in the parameter's space. An example of a robustness-type analysis may consider α_{ES}, m, r_d as the parameters defining the uncertainties of the servomechanism model.

As the result of a Routh-Hurwitz stability analysis, performed for the just mentioned parameters, we have retained the charts disclosed in figure 5.4, figure 5.5, figure 5.6.

The gain K(t), for the variation range of the parameters presented in figure 5.4 - 5.6, may be chosen in the domain [1, 20], as a function of the admissible attenuation. For a sinusoidal-type reference signal we noticed that the error cannot be minimised less than 20% (for a usual frequency range less than 5 Hz). The attempts to do this may rend the system unstable.

6. An LQG Type Algorithm for the Active Control of the Flexible Body Modes of the Aircraft

The intermediate designs stage, just presented, is a necessary step to use a second general criterion for analysis and synthesis of the electro-hydraulic actuators: a criterion determined by the active control of the flexible body modes of the aircraft. We used the standard LQG [9] algorithm to achieve this goal.

The pair system - Kalman compensator is as follows:

$$\begin{cases} \dot{x}(t) = A \cdot x(t) + B \cdot u(t) + \xi(t) \\ y_0(t) = C \cdot x(t) + D \cdot u(t) + \theta(t) \\ y_p(t) = C_p \cdot x(t) \end{cases} \tag{6.1}$$

$$\begin{cases} \hat{\dot{x}}(t) = (A - B \cdot k_R - k_E \cdot C_0) \cdot \hat{x}(t) + k_E \cdot y_0 \\ \hat{y}(t) = u(t) = -k_R \cdot \hat{x}(t) \end{cases}$$

where:

$$\begin{cases} k_E = \tilde{Y} \cdot C_0^T \cdot R_\theta^{-1}; \; k_R = R_j^{-1} \cdot B^T \cdot \tilde{X} \\ A^T \cdot \tilde{X} + \tilde{X} \cdot A + C_p^T \cdot Q_j \cdot C_p - \tilde{X} \cdot B \cdot R_j^{-1} \cdot B^T \cdot \tilde{X} = 0 \\ A \cdot \tilde{Y} + \tilde{Y} \cdot A^T + Q_\xi - \tilde{Y} \cdot C_0^T \cdot R_\theta^{-1} \cdot C_0 \cdot \tilde{Y} = 0 \\ E\{\xi(t') \cdot \xi^T(t)\} = Q_\xi \cdot \delta(t' - t) \\ E\{\theta(t') \cdot \theta^T(t)\} = R_\theta \cdot \delta(t' - t) \\ J = \lim_{T \to \infty} E\left\{ \frac{1}{2 \cdot T} \cdot \int_0^T \left(x^T \cdot C_p^T \cdot Q_j \cdot C_p \cdot x + u^T \cdot R_j \cdot u \right) \cdot dt \right\} \\ C_p = \left[O_{(4 \times 4)} \;\vdots\; I_{(4 \times 4)} \right] \end{cases} \tag{6.2}$$

We denoted by E{.} the mathematical expectation, by δ(.) the Dirac function and by J the performance index to be minimised; k_R represents the controller gain and k_E the Kalman-estimator gain.

The C_p matrix imposes a constraint on the flexible body modes using the weighting matrix Q_j. The damping of the flexible body modes obtained this way should not deteriorate the handling qualities of the aircraft. The weighting matrix Q_j satisfying these requirements can be derived in a trial-and-error process.

The intensity noise matrices both for the state noise Q_ξ and the measurement noise R_θ together with the control weighting matrix R_j have been assumed unitary matrices.

The results obtained for the Q_j matrix as follows:

$$Q_j = [100 \quad 1 \quad 1 \quad 100]$$

are excellent and are presented in table no. 5.

Fig.5.1 Input i1ref - output y evolution

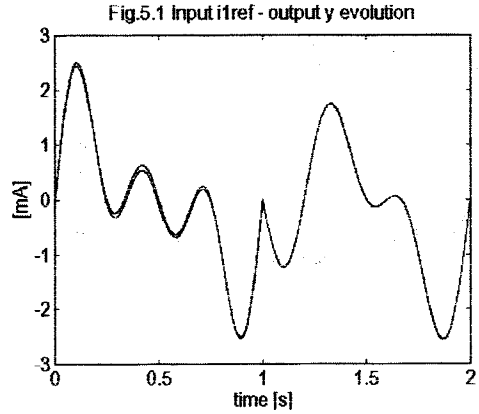


Table no. 5. The Increase in the Damping of Flexible Body Modes

(The weighting matrix Qj=diag[100,1,1,100])

	Open loop system		System together with the LQG compensator	
mode	damping	frequency [rad/s]	damping	frequency [rad/s]
1/2	0.0610	39.73	0.1225	40.0519
3/4	0.1251	19.49	0.3641	22.4721
5/6	0.3904	3.37	0.3910	3.3157
7	-	10.119	-	9.6914
8	-	50.00	-	49.9515

The efficiency of the LQG algorithm is superior to the method proposed by F. Kubica and T. Livet [(6)] consisting in a LQR controller with output feedback. It worth to mention that the results presented in table no. 5 are obtained with a supplementary weighting factor on the controls of 0.8.

This factor can be assimilated with a steady-error of the servomechanism.

Fig.5.2 Error e evolution

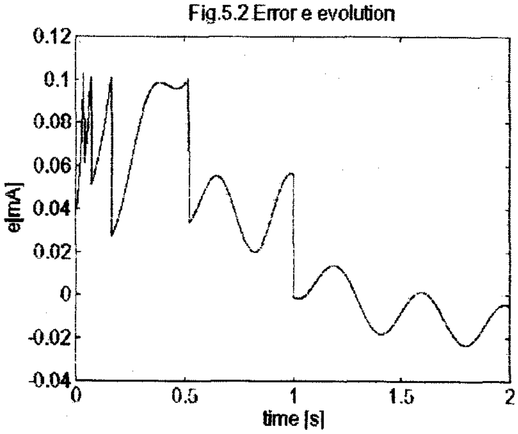


Fig.5.3 K-switching gain evolution

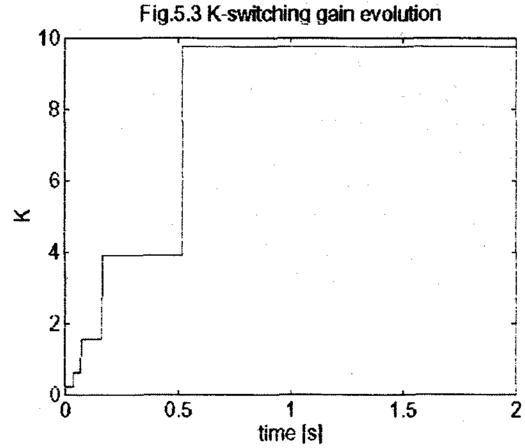
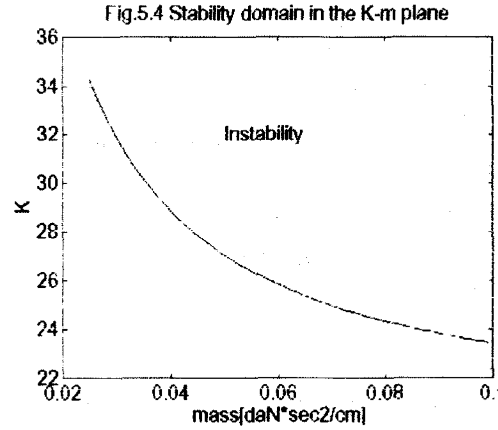
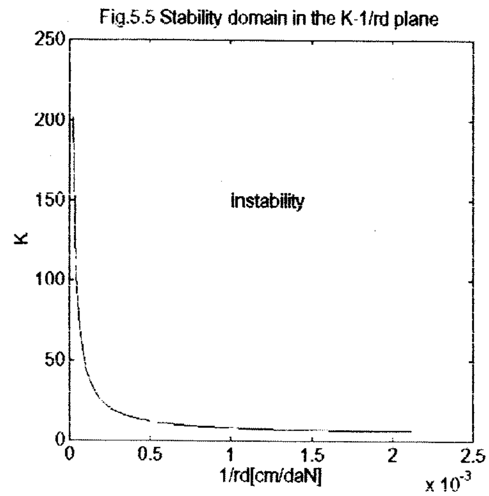
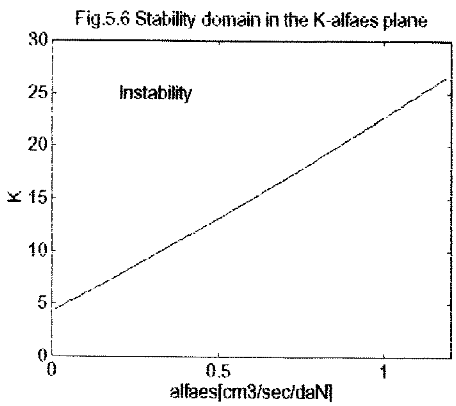


Fig.5.4 Stability domain in the K-m plane







7. Concluding Remarks

Modern aviation evolution imposed a new discipline, **Aeroservoelasticity**, which studies the interactions between the aeroelastic dynamics of the aircraft and the control and power elements (i.e. electro-hydraulic actuators). The importance and the role of the electro-hydraulic actuators' dynamics as integrated sub-systems in the large control systems of the aircraft are usually ignored in the preliminary calculus stages both for the synthesis and for the analysis of these systems. The explanation of this situation is perhaps due to an insufficient co-ordination between the electro-hydraulic systems designers, the control engineers, the aerodynamicists, etc.

Our paper presented only some aspects in the design of an electro-hydraulic servomechanism as a controlled system for the case of its use in SAS, SMCS, etc., systems.

The results presume a trade-off between the two criteria proposed, which may open a way to further research.

References

[1] Ursu, F., Ursu, I. and Vladimirescu, M. *About mathematical modelling of hydraulic servomechanism*, to be published in Studii si cercetari de mecanica aplicata, 1996, Bucharest

[2] Ursu, I., Popescu, F., Vladimirescu, M. and Costin, R. *Linearization methods of generalized flowrate characteristic of the hydraulic servomechanisms*, Revue Roumaine de Mecanique Appliquee, **39**, no.2,1994

[3] Gaminin, N., S. Hydraulic power control systems, Masinostroenie, Moscow, 1972

[4] Guillon, M., L'Asservissement hydraulique et electro-hydraulique, Dunod, Paris, 1972

[5] Anderson, M., R., *Robustness evaluation of a flexible aircraft control system*, J. Guidance, **16**, no. 4,1993

[6] Kubica, F., Livet, T., *Design of flight control system for an aeroelastic aircraft*, ECC 93, Glasgow

[7] Hamdan, A., M., A., Nayfeh, H., *Measures of modal controllability and observability for first-and-second order systems*, J. Guidance, **12**, no. 3., 1989

[8] Miller, D., E., Davison, E., J., *An adaptive controller which provides an arbitrarily good transient and steady-state response*, IEEE Trans., Automat. Contr., **36**, no.1, 1991

[9] Kwakernaak, H., Sivan, R., Linear optimal control systems, (Russian edition), Mir, Moscow, 1977

[10] Popescu, F., Ursu, F., Ursu, I., Vladimirescu, M., *Mathematical modelling of hydraulic servomechanism: robust synthesis and simulation,* Proc. of EUROSIM'95, Vienna, 1995.