MINIMUM LANDING-APPROACH DISTANCE FOR A VUK-T SAILPLANE

Zoran Stefanović , Dragan Cvetković,
Faculty of Mechanical Engineering University of Belgrade
Aeronautical Engineering Department, 27.marta st.80
11000 Belgrade, F.R. of Yugoslavia

Abstract

Because of the extremely low drag associated with modern high-performance sailplanes, the landing approach trajectory can become critical if aerodynamic deceleration devices are not used. In the paper it is assumed that the sailplane approaches the landing strip head-on in still air with too much speed, altitude, or both, to allow a conventional approach glide. It is also assumed that the initial altitude is too low for any kind of go-around maneuver. The treated problem is then formulated as an optimal control problem, and this problem is one of transferring the sailplane from a prescribed initial state to a prescribed terminal state in minimum distance. The flight is confined to a vertical plane, sideslip or other lateral maneuvers are not allowed (treated in the analysis). It is shown that the most obvious, and perhaps startling, feature of the optimal trajectory is its highly oscillatory nature. Presented solution provides useful qualitative information for high L/D of sailplanes, but also the light aircraft. The lower bound on speed is necessary for the problem to have a solution. The altitude inequality constraint provides a realistic solution. All calculations were performed on modern high-performance Yugoslav sailplane VUK-T.

Introduction

Over the last few decades, the recreational use of sporting gliders or seaplanes has accelerated dramatically. The application of sophisticated analysis techniques to the many interesting and sometimes unique problems encountered in soaring flight lags considerably behind that found in other areas of flight dynamics. However, this lack of theoretical attention appears to be changing rapidly. The objective of the research presented here is to apply a modern optimum control algorithm to a simplified class of a sailplane landing-approach trajectories and to deduce the basic features of a corresponding minimum distance.

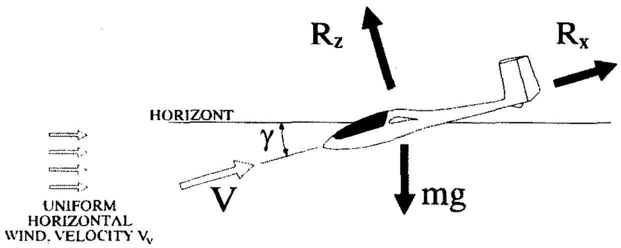
Because of the extremely low drag associated with modern high-performance sailplanes, the landing approach trajectory can become critical if aerodynamic deceleration devices are not used. For the problem treated in this paper, it is assumed that the sailplane approaches the landing strip head-on in still air with too much speed, altitude or both to allow a conventional approach glide. It is also assumed that the initial altitude is too low for any kind of go-around maneuver. The problem can then be formulated as an optimal control problem, in which one seeks the lift coefficient time history which provides the shortest possible landing-approach distance. Alternately, the problem is one of transferring the sailplane from a prescribed initial state to a prescribed terminal state in a minimum distance. Furthermore, the flight is confined to a vertical plane. Sideslip or other lateral maneuvers are not allowed. The landing approach must also be made without benefit of spoilers, drag brakes, drag chutes, or other deceleration controls. Rotation dynamics are neglected. Finally, it is necessary to impose minimum speed and altitude path constrains on the problem.

Statement of the Problem

Since the final time (t_k) is not specified, a control parameter, a = t_k is introduced via the time transformation:

$$t = \alpha \tau \qquad 0 \leq t \leq t_k \qquad 0 \leq \tau \leq 1$$

Thus, the variable end time problem will be transformed into a fixed end time problem with independent variable t.



Using inertial Cartesian coordinates the equations of motion for a sailplane becomes:

$$m\frac{dV_x}{dt} = -R_x(V,C_z)\cos\gamma - R_z(V,C_z)\sin\gamma$$

$$m\frac{dV_h}{dt} = -mg + R_x(V,C_z)\sin\gamma + R_z(V,C_z)\cos\gamma$$

$$\frac{dx}{dt} = V_x \qquad \frac{dh}{dt} = V_h \qquad \tan\gamma = \frac{V_h}{V_x + V_v}$$

The point mass equations of motion are written with respect to the usual wind or trajectory axes. Since the final range is to be minimized and since the range variable does not appear in the other dynamics equations, the range equation is simply incorporated into the performance index and is not required as a part of the optimization process. The three remaining state variables are: speed V, flight path angle **g** and altitude **h**.

The optimal control problem can be formally stated in terms of non dimensional variables as follows: Find the control function *u*(t) in domain 0< t <1 and the control parameter a which minimize the performance index:

$$I = \alpha\int_0^1 V\cos\gamma d\tau + k_1^{-1}\int_0^1\left[(gX)^{1/2}\frac{V}{V_{stol}} - 1\right]^{-1} d\tau + k_2^{-1}\int_0^1 h^{-1} d\tau \qquad (1)$$

subject to the dynamics constrains and subject to the initial (0) and terminal (1) state constrains:

$$\frac{dV}{d\tau} = -\alpha\left[\eta C_x(u)V^2 + \sin\gamma\right] \qquad (2)$$

$$\frac{d\gamma}{d\tau} = \alpha\left[\eta C_z(u)V^2 - \cos\gamma\right]/V \qquad (3)$$

$$\frac{dh}{d\gamma} = \alpha V\sin\gamma \qquad (4)$$

$$\eta = \frac{1}{2}\frac{\rho g X}{(mg/S)} \qquad (5)$$

$$h(0) = h_{start}/X \qquad h(1) = h_{end}/X \qquad (6)$$

$$V(0) = V_{start}(gX)^{-1/2} \qquad V(1) = V_{end}(gX)^{-1/2} \qquad (7)$$

$$\gamma(0) = \gamma_{start} \qquad \gamma(1) = 0 \qquad (8)$$

The characteristic values are taken form the Ref.4: $\mathbf{h_{start}}$ altitude at the beginning of the approach process h_{start}= 50 m; $\mathbf{h_{end}}$ altitude at the end h_{end}= 5 m; $\mathbf{V_{start}}$ speed at the beginning of the approach process V_{start}= 22 m/s; $\mathbf{V_{end}}$ speed at the end V_{end}= 20 m/s; $\mathbf{V_{stall}}$ stalling speed which for a VUK-T glider is V_{stall}= 15.3 m/s; $\mathbf{g_{start}}$ flight path angle at the beginning of the approach process g_{start}= -0.02 rad, and X=1000 m is an arbitrary characteristic length used in the nondimensionalization.

Control inequality constraints of the lift coefficient C_z during the process:

$$|C_z(t)| \leq C_{z_{max}}$$

due to Ref.2 can be presented as:

$$C_z(u) = C_{z_{max}}\left(2\sin^2 u - 1\right)$$

This transformation insures lift coefficient magnitude to be less then stalling value C_{Zmax}. Taking into a count aerodynamic characteristics of a VUK-T glider (drag polar and a C_{Zmax}) it follows:

$$C_z(u) = 1.78(2\sin^2 u - 1) \qquad (9)$$

$$C_x(u) = 0.01756 - 0.0095C_z + 0.021C_z^2 \qquad (10)$$

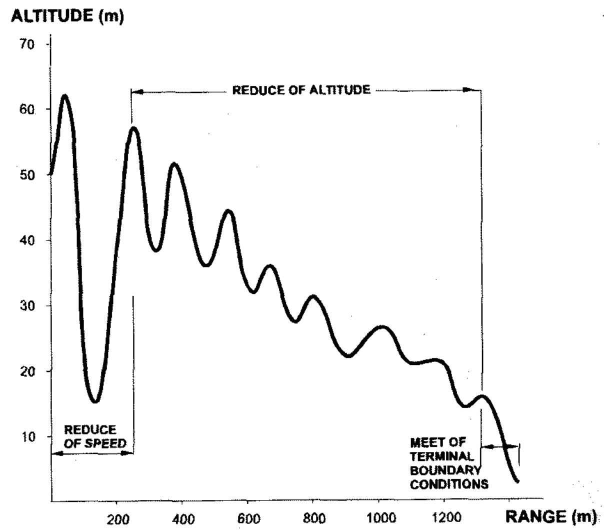
Second term of the equation (1) represent integral interior penalty function for the speed constraint, and limits the speed above the stalling value (V_{stall}). Third term of the equation (1) represent integral interior penalty function for the altitude constraint, and enforces positive values.

As with any penalty function scheme, it is necessary to solve a sequence of unconstrained sub-problems. Equations (2) to (10) with fixed positive penalty constants k1 and k2. This penalty constants are then increased between successive sub-problems to allow the solution point to move closer to the active constraint surfaces. With the use of these interior penalty functions, it is necessary to begin computations with a nominal control which generates a trajectory satisfying both state inequality constraints.

Numerical Results

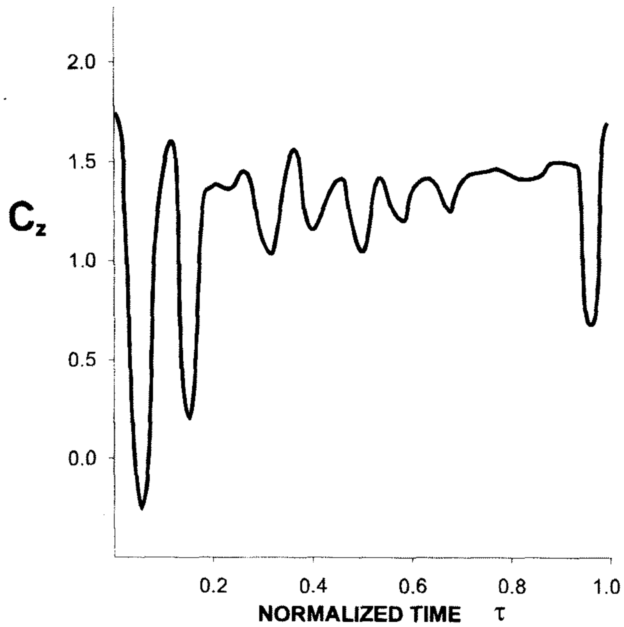
The stated optimal control problem has been solved using a gradient projection algorithm which incorporates conjugate directions of search for a rapid convergence. The purpose of the projection operator in this algorithm is to obtain satisfaction of the terminal state constraints, equations (6) to (8) at each iteration. The method is a direct gradient method in that the control function *u*(t) and the control parameter a are altered simultaneously on each iteration in an attempt to reduce value of I in equation (1) and satisfy the optimal condition.

All calculations were performed on PC486 computer using Fortran77 compiler and double-precision arithmetic. The required integrations were carried out using a standard fourth-order Runge-Kutta method with 100 fixed uniform integration steps. Three sub-problems were solved. For each of these sub-problems, the penalty constants k_1 and k_2 were each set equal to 200, 1000, and 5000 respectively. A final refinement run was made with k_1=5000, k_2=10000 and 400 integration steps. Calculations were performed for the sailplane mass of 320 kg and sailplane wing surface of 12 m^2.



The optimal landing approach trajectory of the VUK-T glider is shown at the figure. The minimum landing-approach distance is 1420 m. The optimal final time is 67.1 s. The terminal state values for this trajectory agree with the values prescribed by equations (6) to (8) to at least six significant figures. Each peak on the optimal trajectory is associated with a near stall regime.

As may be noted from the figure, the minimum-distance glide consists of three relatively distinct portions. Initially, the sailplane climbs. This is immediately followed by a step dive and a pull-up to approximately the initial altitude. At this point, the speed has been reduced to almost the stall speed. There then follows a succession of shallow, but rapid, dives and climbs. These damped oscillations appear to converge to a straight-line trajectory with a glide slope of approximately 1/37, which in turn is $(L/D)_{max}$ = 37 glide slope for VUK-T sailplane. A short final dive is required to mach the specified terminal boundary conditions. The corresponding optimal lift coefficient time history is presented at the figure.



Discussion

The most obvious, and perhaps startling, feature of the optimal trajectory is its highly oscillatory nature. In practical terms, it is even questionable whether the trajectory can be flown, since the period of oscillation is only 7-8 s. Still, the solution provides useful qualitative information for high L/D aircraft. If the problem is viewed basically as one of energy dissipation in a viscous medium, then it seems reasonable to increase the total path length as much as possible. If sufficient lift is available, this implies an oscillatory trajectory. However, if the lift capability is severely reduced by the use of spoilers, for example, then one could anticipate that oscillatory trajectories would no longer be possible.

It should also be mentioned that this optimal control problem is a rather difficult one because of the multiple speed-constrained arcs present in the optimal trajectory. The lower bound on speed is necessary for the problem to have a solution. The altitude inequality constraint provides a realistic solution; without it, a solution with an impressive underground dive results.

References

(1) J.Roskam: Airplane Flight Dynamics and Automatic Flight Controls, University of Kansas, 1982.

(2) B.L.Pierson: Maximum Altitude Sailplane Winch-Launch Trajectories, *Aeronautical Quarterly*, Vol.28, May 1977.

(3) L.S.Lasdon,A.D.Waren,R.K.Rice: An Interior Penalty Method for Inequality Constrained Optimal Control Problems, *IEEE Transactions on Automatic Control*, Vol AC-12, Aug. 1967.

(4) D.E.Metzger, J.K.Hedrik: Optimal Flight Pats for Soaring Flight, *Journal of Aircraft*, Vol 12, No 11, Nov 1975.

(5) B.L.Pierson: Panel Flutter Optimization by Gradient Projection, *Internaional Journal for Numerical Methods in Engineering*, Vol 9, 1975.

(6) Z.Stefanović, D.Cvetković: Minimalna dužina prilaza pri sletanju jedrilice VUK-T, *International Symposium Vazduhoplovstvo '95*, 1995.