AERODYNAMIC STABILIZATION SYSTEM FOR SMALL SATELLITES

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Abstract. For the satellites launched into relatively low circular orbit (height of the orbit less then 400 km) aerodynamic satellite stabilization can be used. It orients the axis of symmetry of the satellite along the tangent to the orbit. Aerodynamic restoring torque in pitch and yaw is provided by a special very light stabilizer.To damp transient oscillations of the satellite one uses two single-degree-of-freedom gyros connected with the satellite body through a viscous-spring restraint. The gyro rotor is linked to the satellite through a damping device, and the oscillations of the satellite cause its precession producing dissipation of energy of oscillations of the satellite-gyros system. Moreover, the single degree-of-freedom gyros produce additional restoring torques in yaw and roll when the system deviates from its equilibrium orientation. The aerodynamic and gyroscopic restoring torques are used to provide three-axis aerodynamic stabilization of the satellite. A similar aerodynamic stabilization system was installed, for example, in the Cosmos - 149 and Cosmos-320 russian satellites. Semipassive aerodynamic stabilization system can be used in small satellites to investigate physical processes in the atmosphere and to determine atmospheric parameters. Here general investigation of the attitude motion of small satellites with aerodynamic system is given. Main directions of study: nonlinear equations of satellite attitude motion, equilibria, stability of equilibria, optimal parameters.

Equations of motion

We consider the attitude motion of a satellite under the action of gravitational and aerodynamic torques. Equations of the attitude motion of the satellite can be written in such a way [(1)]:

$$\begin{aligned} A\dot{p} + (C-B)qr &= M_{gx} + M_{ax}, \\ B\dot{q} + (A-C)rp &= M_{gy} + M_{ay}, \\ C\dot{r} + (B-A)pq &= M_{gz} + M_{az}; \end{aligned} \qquad (1)$$

$$\begin{aligned} p &= (\dot{\alpha}+\omega)a_{21} + \dot{\gamma}, \\ q &= (\dot{\alpha}+\omega)a_{22} + \dot{\beta}\cos\gamma, \\ r &= (\dot{\alpha}+\omega)a_{23} + \dot{\beta}\sin\gamma; \end{aligned} \qquad (2)$$

Here

$$\begin{aligned} M_{gx} &= 3\frac{\mu}{\rho^3}(C-B)a_{32}a_{33}, \\ M_{gy} &= 3\frac{\mu}{\rho^3}(A-C)a_{33}a_{31}, \\ M_{gz} &= 3\frac{\mu}{\rho^3}(B-A)a_{31}a_{32}; \end{aligned} \qquad (3)$$

$$\begin{aligned} M_{ax} &= b_g Z_a - c_g Y_a, \\ M_{ay} &= c_g X_a - a_g Z_a, \\ M_{az} &= a_g Y_a - b_g X_a; \end{aligned} \qquad (4)$$

$$\begin{aligned} X_a &= -Q\left(\frac{V_X}{V}a_{11} + \frac{V_Y}{V}a_{21} + \frac{V_Z}{V}a_{31}\right), \\ Y_a &= -Q\left(\frac{V_X}{V}a_{12} + \frac{V_Y}{V}a_{22} + \frac{V_Z}{V}a_{32}\right), \\ Z_a &= -Q\left(\frac{V_X}{V}a_{13} + \frac{V_Y}{V}a_{23} + \frac{V_Z}{V}a_{33}\right); \end{aligned} \qquad (5)$$

	x	y	z
X	a_{11}	a_{12}	a_{13}
Y	a_{21}	a_{22}	a_{23}
Z	a_{31}	a_{32}	a_{33}

$$
\begin{aligned}
a_{11} &= \cos\alpha \cos\beta, \\
a_{12} &= \sin\alpha \sin\gamma - \cos\alpha \sin\beta \cos\gamma, \\
a_{13} &= \sin\alpha \cos\gamma + \cos\alpha \sin\beta \sin\gamma, \\
a_{21} &= \sin\beta, \\
a_{22} &= \cos\beta \cos\gamma, \\
a_{23} &= -\cos\beta \sin\gamma, \\
a_{31} &= -\sin\alpha \cos\beta, \\
a_{32} &= \cos\alpha \sin\gamma + \sin\alpha \sin\beta \cos\gamma, \\
a_{33} &= \cos\alpha \cos\gamma - \sin\alpha \sin\beta \cos\gamma;
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
V_X &= \omega_0 p_*(1+e\cos\theta) - \Omega\rho\cos i, \\
V_Y &= \Omega\rho \sin i \cos u, \\
V_Z &= \omega_0 p_* e \sin\theta, \\
V^2 &= V_X^2 + V_Y^2 + V_Z^2, \\
u &= \omega_\pi + \theta, \\
\omega &= \omega_0(1+e\cos\theta)^2, \\
\frac{\mu}{\rho^3} &= \omega_0^2(1+e\cos\theta)^3, \\
\omega_0^2 &= \frac{\mu}{p_*^3}, \\
Q &= \frac{1}{2}\rho_* V^2 S C_x,
\end{aligned} \tag{7}
$$

α, β, γ are the angles of pitch, yaw, roll (Fig. 1), defining the orientation of the central moments of inertia axes of the satellite Ox, Oy, Oz with respect to the orbital coordinate system $OXYZ$ (OY is the normal to the orbital plane, OZ coincides with the local vertical); A, B, C are the central principal moments of inertia of the satellite; p, q, r are the components of the absolute angular velocity of the satellite in the frame $Oxyz$; M_{gx}, M_{gy}, M_{gz} are the components of the gravitational torque; M_{ax}, M_{ay}, M_{az} are the components of the aerodynamic torque; a_g, b_g, c_g are the coordinates of the center of pressure in the frame $Oxyz$; V_X, V_Y, V_Z are the components of the velocity of the satellite mass center relative to the atmosphere in the frame $OXYZ$; ρ is the distance between the mass centers of the Earth and the satellite; θ is the true anomaly; e is the eccentricity; i is the inclination of the orbit; ω is the angular velocity of the orbital motion; p_* is the parameter of the orbit; μ is the Earth's gravitational parameter; ω_π is the argument of perigee; Ω is the angular velocity of the Earth's rotation about its axis; Q is the drag of the atmosphere; ρ_* is the atmospheric density; S is the cross-sectional area of the satellite; C_x is the drag coefficient. In equations (1)-(2) the point denotes differentiation with respect to time t.

In deriving of expressions (4), (5) (7) it was assumed that the atmosphere is completely carried away by the rotating Earth, the influence of atmospheric drag on translation motion of the satellite can be ignored, the effect of the atmosphere on the satellite attitude motion is reduced to the aerodynamic drag force applied to the center of pressure and directed against the velocity of the satellite's center of mass with respect to the free air stream.

<u>Influence of the atmosphere rotation</u>

We consider first the effect of the Earth's atmosphere rotation on the satellite attitude motion in a circular orbit ($e = 0$). Then with $\tau = \omega_0 t$ as an independent variable, equations (1) has the form:

$$
\begin{aligned}
& A\dot{p} + (C-B)qr - 3(C-B)a_{32}a_{33} = 0, \\
& B\dot{q} + (A-C)rp - 3(A-C)a_{33}a_{31} + \\
& \bar{\kappa}_1\left(\frac{V_X}{V}a_{13} + \frac{V_Y}{V}a_{23}\right) = 0, \\
& C\dot{r} + (B-A)pq - 3(B-A)a_{31}a_{32} - \\
& \bar{\kappa}_1\left(\frac{V_X}{V}a_{12} + \frac{V_Y}{V}a_{22}\right) = 0.
\end{aligned} \tag{8}
$$

Here

$$
\begin{aligned}
\frac{V_X}{V} &= \frac{1-\varepsilon\cos i}{\sqrt{(1-\varepsilon\cos i)^2 + \varepsilon^2\sin^2 i\cos^2 u}}, \\
\frac{V_Y}{V} &= \frac{\varepsilon\sin i\cos u}{\sqrt{(1-\varepsilon\cos i)^2 + \varepsilon^2\sin^2 i\cos^2 u}},
\end{aligned} \tag{9}
$$

$$
\bar{\kappa}_1 = -\frac{Q a_g}{\omega_0^2}, \quad \varepsilon = \frac{\Omega}{\omega_0}.
$$

It can be seen from the equations (8), (9) that the rotation of the atmosphere is reduced to the existance of a forced solution that depends on the parameters $\bar{\kappa}_1$, ε, i and has the form (ε is a small parameter):

$$
\begin{aligned}
\alpha &= \alpha_0 + \varepsilon \alpha_1 + \cdots \quad , \\
\beta &= \beta_0 + \varepsilon \beta_1 + \cdots \quad , \\
\gamma &= \gamma_0 + \varepsilon \gamma_1 + \cdots \quad .
\end{aligned} \tag{10}
$$

By substituting the solution (10) into (8) we obtain (up to ε^2):

$$
\begin{aligned}
\alpha &= 0, \\
\beta &= A_\beta \cos u, \\
\gamma &= A_\gamma \sin u,
\end{aligned} \tag{11}
$$

where

$$A_\beta = \frac{x(y-1)\varepsilon}{(x-1)(y-1)-1}\sin i,$$

$$A_\gamma = \frac{x\varepsilon}{(x-1)(y-1)-1}\sin i,$$

$$x = \frac{\bar{\kappa}_1}{A-B+C}, \quad y = \frac{3(B-C)}{A-B+C}.$$

If aerodynamic torque is zero ($Qa_g = 0$) or the satellite moves along an equatorial orbit ($i = 0$), the amplitudes of the forced solution (11) are equal to zero. In a polar orbit the amplitudes of the solution (11) achieve the maximal values. If the gravitational torque is negligibly small in comparison with the aerodynamic torque then

$$\alpha = 0, \quad \beta = \varepsilon \sin i \cos u, \quad \gamma = -\varepsilon \sin i \sin u. \tag{12}$$

In the altitude range from 300 to 1000 km the parameter ε weakly depends on the altitude and at $i = 65°$ $\varepsilon \sin i \cong 0{,}06$. This value of $\varepsilon \sin i$ corresponds to $A_\beta = A_\gamma \cong 3{,}5°$.

Equilibria and their stability

Let us consider the attitude motion equations of the satellite at $e = 0$, $\Omega = 0$. Then

$$
\begin{aligned}
&A\dot{p} + (C-B)qr - 3(C-B)a_{32}a_{33} = \\
&\bar{h}_2 a_{13} - \bar{h}_3 a_{12}, \\
&B\dot{q} + (A-C)rp - 3(A-C)a_{33}a_{31} = \\
&\bar{h}_3 a_{11} - \bar{h}_1 a_{13}, \\
&C\dot{r} + (B-A)pq - 3(B-A)a_{31}a_{32} = \\
&\bar{h}_1 a_{12} - \bar{h}_2 a_{11}.
\end{aligned} \tag{13}
$$

Here,

$$
\begin{aligned}
\bar{h}_1 &= -Qa_g / \omega_0^2, \\
\bar{h}_2 &= -Qb_g / \omega_0^2, \\
\bar{h}_3 &= -Qc_g / \omega_0^2.
\end{aligned}
$$

Equilibrium orientations of the satellite in the orbital coordinate system correspond to the stationary solutions α = const, β = const, γ = const of (13) and are determined by the following system of equations:

$$
\begin{aligned}
(C-B)(a_{22}a_{23} - 3a_{32}a_{33}) &= \bar{h}_2 a_{13} - \bar{h}_3 a_{12}, \\
(A-C)(a_{23}a_{21} - 3a_{33}a_{31}) &= \bar{h}_3 a_{11} - \bar{h}_1 a_{13}, \\
(B-A)(a_{21}a_{22} - 3a_{31}a_{32}) &= \bar{h}_1 a_{12} - \bar{h}_2 a_{11},
\end{aligned} \tag{14}
$$

System (13) posses the integral of energy

$$
\begin{aligned}
&\frac{1}{2}\left(A\bar{p}^2 + B\bar{q}^2 + C\bar{r}^2\right) + \\
&\frac{3}{2}\left[(A-C)a_{31}^2 + (B-C)a_{32}^2\right] + \\
&\frac{1}{2}\left[(B-A)a_{21}^2 + (B-C)a_{23}^2\right] - \\
&\left(\bar{h}_1 a_{11} + \bar{h}_2 a_{12} + \bar{h}_3 a_{13}\right) = h_0 \,.
\end{aligned} \tag{15}
$$

Here

$$\bar{p} = p - a_{21}, \ \bar{q} = q - a_{22}, \ \bar{r} = r - a_{23} \,.$$

Using left-hand part of the integral of energy (15) as Lyapunov function, it is possible to investigate stability of any stationary solution of system (14).

It is possible to show that the stationary solution

$$\alpha = 0, \quad \beta = 0, \quad \gamma = 0 \tag{16}$$

exists if $\bar{h}_2 = \bar{h}_3 = 0$, i. e. the center of pressure is situated on axis Ox of the satellite. The sufficient conditions for the stability of the solution (16) are the following:

$$\begin{aligned} &3(A-C)+\bar{h}_1 > 0, \\ &(B-A)+\bar{h}_1 > 0, \\ &B-C > 0. \end{aligned} \tag{17}$$

For aerodynamically stable satellite (the center of pressure should lie behind the center of mass) $\bar{h}_1 > 0$. It follows from (17) that the stability of the satellite in pitch and yaw can be provided with the aerodynamic torque if $\bar{h}_1$ is big enough. The stability in roll depends on the gravitational torque only.

Aerodynamic stabilization system

The aerodynamic stabilization system of the satellite is based on the properties of the stable equilibrium solution (16). The successful operation of the stabilization system depends on a damping device. As a rule, in passive stabilization systems are used the following types of dampers: viscous-fluid or eddy-current magnetically anchored spherical damper, magnetic hysteresis rods, gyrodamper.

The attitude motion of satellites under the influence of gravitational and aerodynamic torques was investigated in many papers [2-9]. But the practical implementation of the aerodynamic stabilization system has been effected in the russuian satellites "Space Arrow" (Cosmos-149, Cosmos-320) [10-13]. These satellites were placed in low orbits to investigate physical processes in the Earth's atmosphere and to determine atmospheric parameters. This stabilization system is of special interest, since it represents the first (and unique up to the present) use in space technology of the aerodynamic principle of satellite control in pitch and yaw.

To obtain aerodynamic stabilization the satellite carried an aerodynamic stabilizer in the form of a trincated conical surface. It was placed off from the satellite body at a distance of about 4m by four tubes (Fig. 2). The aerodynamic stabilizer of this construction provides aerodynamic restoring torque with a comparatively small increase of the aerodynamic drag force. Attitude control of the satellite in roll and damping of transient oscillations in pitch, yaw and roll angles is accomplished with the aid of gyrodamper (two single-degree-of-freedom gyroscopes). The angular momentum vectors of the gyroscope rotors in the equilibrium orientation are situated in the local horizontal plane, symmetrically relative to the normal to the orbit plane. The functioning of the satellites Cosmos-149 and Cosmos-320 with aerodynamic stabilization system in the orbit was quite successful.

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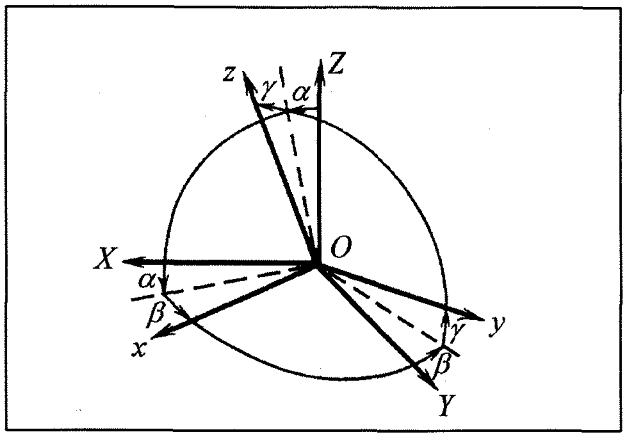

Fig.1. Orientation angles pitch (α), yaw (β), roll (γ)

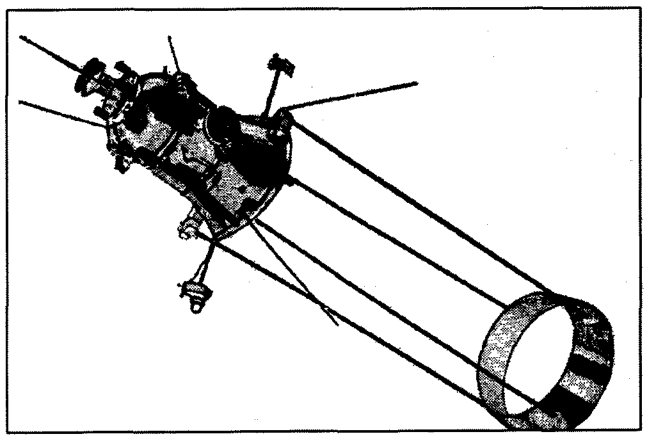

Fig.2. The Cosmos-149 Satellite