

# INVESTIGATION OF THE VORTEX WAKE EVOLUTION AND FLIGHT SAFETY

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## Abstract

Investigated in the present report are the problems of decreasing separation distances between aircraft during their takeoffs and landings. This work reflects in a condensed form the efforts of a large collective of scientists and presents a general view on the problem under study, whose complexity lies mainly in the fundamental impossibility to create a single method and universal tool of investigation. The authors use a fragmentary approach with asymptotically non-strict splitting of the problem into subregions and a multiple mosaic selection of separate subproblems, in the strict sense improperly posed for solving main problem.

## Nomenclature

- $V_\infty$  flight speed (for landing and takeoff cases  $V_\infty \approx 70$  m/s);
- $\rho_\infty$  ambient air density;
- $g$  acceleration due to gravity;
- $G$  aircraft weight;
- $b$  wing span;
- $S$  wing area;
- $AR$  wing aspect ratio ( $AR=b^2/S$ );
- $C_L$  lift coefficient;
- $C$  wing chord ( $C_r$  - root chord,  $C_t$  - tip chord,  $C_{av} = 2\Gamma_0/(C_L V_\infty)$  - mean aerodynamic chord);
- $\Gamma$  velocity circulation for each of the vortices:  
 $\Gamma_0 = G/(\rho_\infty \cdot V_\infty \cdot b_v)$ ;
- $b_v$  initial vortex span (for elliptical span circulation distribution  $b_v = b \cdot \pi/4$ );
- $Ro$  Rossby number characterizing the degree of flow three-dimensionality in core,  $V_{x,max}/V_{\tau,max}$ ;
- $r$  vortex radius ( $r_0$  initial vortex radius);
- $h$  flight height;
- $W_0$  initial vortex descent rate  $\Gamma_0/(2\pi b_v)$ ;
- $q^2$  relative energy of turbulent fluctuations;
- $L_f$  turbulence scale;
- $\varepsilon$  turbulence level [ $m^2/c^3$ ] (turbulent dissipation rate  $dq^2/dt$ ): for isotropic turbulence  $\varepsilon = 0.25 \cdot q^3/L_f$ ;
- $Ri$  Richardson number - the ratio of buoyancy forces (stratification effects) to inertial forces (shear effects):  $Ri = N_0^2 \cdot \left(\frac{dU}{dz}\right)^{-2}$ ;
- $U(z)$  mean value of wind velocity;
- $N_0$  Brunt-Väisälä frequency:  $N_0^2 = -g/\rho \cdot \frac{d\rho}{dz}$ ;

- $\nu$  kinematic coefficient of viscosity;
- $\hat{a}$  condensation drop size;
- $\hat{n}$  condensation particle density

## Introduction

The scarcity of free land areas on the Earth's surface and a limited number of existing airports around the world have disagreed with the required 5-6 percent annual increase in passenger and cargo air traffic. Economical requirements pose the problem of increasing the capacity of current airports. This problem can be solved by efforts in three directions. First, by artificial aging, attenuation and provoking early breakdown of vortex wakes behind wake-generating aircraft and, as a result, shifting them downward by one row in the safe separation matrix. Second, by considering of atmospheric factors in evaluating the safe separation matrix. Third, by increasing the survivability of encountering aircraft and shifting them to the left by one column in the matrix. The air traffic control system can use the detection and tracing of vortex wakes on takeoff and landing paths as well as ordering pairs and groups of aircraft performing takeoffs and/or landings.

The approach to the solution of the problem under study is focused on the creation of a vortex-wake model consistent with the wake-generating aircraft, conditions of vortex wake existence and decay, and providing the assessment of hazard for the encountering aircraft. Such a model should allow its subsequent evolution, refinement and complement. For its development the present authors have used data of numerous flight and model (first of all, in tow tanks) experiments, results of mathematical modeling in the framework of engineering approaches and algorithms for solving boundary-value problems for the Euler and Navier-Stokes equations.

The application of the model in solving the problem of the "second" (encountering) aircraft allows one to assess situations unfavorable for the aircraft from the standpoint of flight dynamics and airframe strength, and evaluate the possible increase in survivability of the aircraft affected by aerodynamic turbulence of the type considered.

In this work, the formulations of the problem are considered, the potentialities of the research tools are assessed, the ways and phases of model refinement are outlined. Some concrete measures are discussed concerning to decreasing safe separation distances between aircraft. These investigations may significantly

influence the selection of concepts of passenger and cargo ultra heavy aircraft.

**I. The region of the wake-generation aircraft.**

In this region it is impossible to use analytical approaches because they do not able to adequately represent configurations under study. Mathematical modeling in the framework of engineering approaches and boundary-value problems does not currently permit to account for the total range of physical effects essential for the given problem. Scale-model experiments are constrained by the framework of similarity criteria, while full-scale experiments are expensive and do not always provide comprehensive information.

Non-dimensional parameters characterizing the phenomenon in the given region are the following:  $\bar{\Gamma} = \Gamma / (b_v \cdot V_\infty)$ ,  $\bar{r} = r_0 / b_v$ ,  $Ro$ . From all standpoints, the most difficult case is takeoff and landing trajectories: a minimum velocity  $V_\infty$  corresponds to a maximum circulation of the wake vortices, and, in addition, aircraft follow one another along a common straight path, takeoff and landing being distinguished by differences in positions of wing high-lift devices, aircraft weights and engine regimes.

Main difficulties in solving the problem in the given region are associated with the need of modeling flow about the complete aircraft configuration with wing high-lift devices deflected and engines operating. Unsolved problems of boundary-layer separation and empirical character of near-wall turbulence models used impose a limit on the rigor of the mathematical modeling in the case considered.

The main problem for this region is the construction of a vortex structure adequate to the aircraft. In this case, it is necessary to model the rolling-up of the many-core vortex system in the region extending downstream at a distance up to 10 wing spans.

As a baseline configuration of the wake-generating aircraft, the B-747 aircraft is used with the following main geometric parameters (given in the parentheses are the values corresponding to a landing regime) and flight data (1):

Mass, kg	b, m	b <sub>v</sub> , m	S, m <sup>2</sup>
340000	59.64	46.8	511 (796)
AR	C <sub>r</sub> , C <sub>t</sub> , m	C <sub>av</sub> , m; C <sub>L</sub>	r, m
6.96 (4.47)	16.56, 4.06	11; 0.44 (17;1.4)	~ 1

h, m	Mach number M	V <sub>∞</sub> , m/s	Γ <sub>0</sub> , m <sup>2</sup> /s	W <sub>0</sub> , m/s
0-300	0.206	70	833	2.83
9000	0.82	250	605	2.06

One should note a large value of the longitudinal component of the velocity at the vortex axis (2) (comparable to V<sub>t, max</sub>) at sufficiently large long

distances from the wake-generating aircraft at low velocities.

**I.1. Mathematical modeling of the aircraft vortex structure in the near field in the framework of a discrete vortex scheme.**

The problem is considered on the assumption of attached flow and low flight velocities:

$$\Delta\Phi = 0;$$

the flow velocity at an infinitely distant point coincides with the undisturbed one:

$$\text{grad}(\Phi)|_\infty = V_\infty;$$

the tangency condition is satisfied on the surface  $\partial\Omega$  :

$$Vn|_{\partial\Omega} = 0;$$

the Kutta-Zhukovsky condition is satisfied at sharp trailing edges; there is no static pressure difference across the vortex sheet.

Distributed on the panels of the wing surface are layers of sources and vortices, the intensity of the vortex layer varies according to a linear law along the panel's chord; the intensity of the source layer is constant within a panel. The vortex sheet behind the wing is modeled with curvilinear vortex cords of constant intensity shedding from the trailing and side edges. At a significant distance from the wing they end in semi-infinite segments aligned with the velocity of the undisturbed flow.

The main difficulties in solving the given problem (3) are associated with the development of the mechanism of vortex linking; reasonable setting of their initial position including the zone of the side edges which significant influences the size of the turbulent core; topology of vortices behind a multi-element wing ("vortex scissors" problem).

Numerical investigations (see Figure 1) being performed in the framework of this model allow one

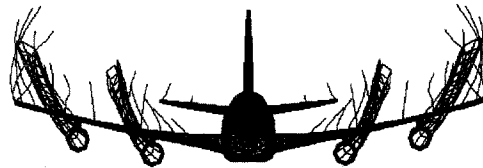
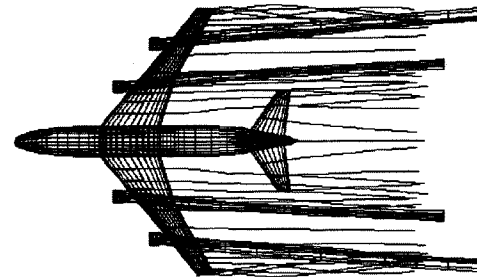


FIGURE 1 - Computation of flow around the B-747 configuration during landing with high-lift devices deployed and engines operating (cold jet approximation).

to study the role of the high-lift devices, influence of jet blowing at the wing ends for accelerating tip-vortex decay, effects of the engines (their type, number, location, operating regime including thrust reverse). An additional thrust of the engines results in an alleviating effect on the subsequent vortex system through the direct influence on the wing vortex sheet during its rolling-up.

**1.2. Mathematical modeling of the aircraft vortex structure in the near field in the framework of a boundary-value problem for the Euler equations.**

The problem is formulated on the assumption of attached inviscid flow about a wing-fuselage configuration:

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial \bar{E}}{\partial x} + \frac{\partial \bar{F}}{\partial y} + \frac{\partial \bar{G}}{\partial z} = 0,$$

where

$$\begin{aligned} \bar{q} &= [\rho, \rho u, \rho v, \rho w, e]^T, \\ \bar{E} &= [\rho u, \rho u^2 + P, \rho uv, \rho uw, u(e + P)]^T, \\ \bar{F} &= [\rho v, \rho uv, \rho v^2 + P, \rho vw, v(e + P)]^T, \\ \bar{G} &= [\rho w, \rho uw, \rho vw, \rho w^2 + P, w(e + P)]^T. \end{aligned}$$

Pressure P is determined from the equation of state for an ideal gas with the adiabatic index  $\kappa$ :

$$P = (\kappa - 1) \left[ e - \rho / 2 \cdot (u^2 + v^2 + w^2) \right].$$

On the body surface with the normal  $\bar{n} = (n_x, n_y, n_z)$  the tangency condition is imposed:  $un_x + vn_y + wn_z = 0$ ; in the oncoming flow:  $\rho \rightarrow \rho_\infty, e \rightarrow e_\infty, \bar{V} \rightarrow \bar{V}_\infty$ , entropy is equal to zero. On the downstream boundary:  $p \rightarrow p_\infty$ . The non-reflection condition on the upstream and downstream boundaries are of the form:  $R_{ext/int} = V_n / \pm 2\kappa / (\kappa - 1)$ . The Kutta-Zhukovsky condition in the numerical scheme is realized automatically.

For discretization of the equations the finite volume method is used:

$$\frac{\partial}{\partial t} \iiint_{\Omega} \bar{q} dx dy dz + \iint_{\partial \Omega} (\bar{E} n_x + \bar{F} n_y + \bar{G} n_z) ds = 0,$$

with an explicit ( $Cu < 2$ ) four-stage Runge-Kutta time-stepping scheme.

For numerically solving the problem the Jameson scheme (4), second-order accurate in space and with artificial dissipation, is used.

The presence of significant artificial viscosity results in the generation of vorticity in the region above the wing up to the trailing edge and non-physical "wash-out" of vortex structures. Additionally, a specific problem appeared of vortex visualization and identification due inadequacy of intuitive assessments such as the criterion  $P_{min}$ , closure or helicity of streamlines and trajectories, constructions of surfaces  $\omega = \text{const}$  (see Figure 2). All these means of visualizations are inapplicable in the three-dimensional

cases, for unsteady flows and in the case of large non-physical viscosity which by itself can be the source of centripetal acceleration at constant pressure.

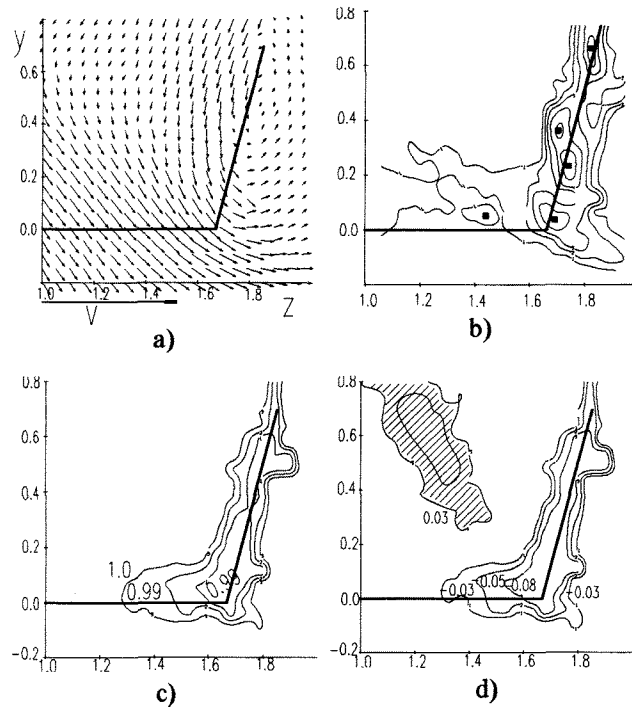


FIGURE 2 - Flow around a wing with a winglet,  $M_\infty = 0.6, \alpha = 8^\circ$ , section  $x/c = 2$ : a) velocity field ( $|\bar{V}| = 1$  shown by the arrow), b) vorticity contours,  $\omega = \text{const}$ , c) isobars,  $P - P_\infty = \text{const}$ , d) isotachs,  $U - U_\infty = \text{const}$ .

An analysis of the evolution of two vortices of the same intensity and sign in the presence of the symmetry plane near the ground shows a deceleration of their rate of descent and velocity of transverse movement, the latter phenomenon being associated not with the dissipation mechanism, as it commonly supposed, but with a noncircular form of the core.

The experience of the present authors (5,6) shows that the given mathematical model can be useful only in the framework of a fragmentary approach: in the region behind a wing or a wing-fuselage configuration extending up to 3-5 wing spans downstream; in a region of the far field behind the configuration in investigating an isolated vortex structure - up to 20 - 30 s in time, what is obviously insufficient both in the first and second cases.

**II. Far wake region: the problem of the dynamics of a vortex pair.**

In this region asymptotic approaches are widely used (usually in an inviscid approximation - in this case the core structure must be coordinated with the deformation of the vortex column's centroid. Their shortcoming is the impossibility of accounting for the influence of the atmosphere. Mathematical modeling accounting for viscosity rests on lack of free turbulence models, which have resulted in a wide

spreading of approaches in the framework of the two-dimensional Navier-Stokes equations for laminar flows. Methods of full-scale and model experiments are the most reliable tools (however, it should be remembered that atmospheric turbulence influences tracer movement in the vortex wake field).

Non-dimensional parameters, characterizing the phenomenon under study in the given region, are  $\bar{h} = h/b_v$ ,  $\varepsilon^* = (\varepsilon \cdot b_v)^{1/3} / W_0$ ,  $\bar{L}_r = L_r/b_v$ ,  $Ri$  ( $Ri = 0$  corresponds to wind shear without stratification,  $Ri = \infty$  corresponds to stratification with no shear). Thus, the vortex structure, with which the encountering aircraft interacts and which determines a safe separation distance  $L_s$ , is specified at least by the following set of parameters:  $L_s/b_v = f(\bar{\Gamma}, \bar{r}, Ro, \bar{h}, \varepsilon^*, \bar{L}_r, Ri)$ .

A peculiarity of the problem lies in large spatial and temporal scales of the phenomenon: the vortex wake behind an aircraft can be potentially dangerous at distances up to 12 km and for times up to 3 min, which imposes very strict requirements for methods of investigations.

The vortex wake is sufficiently stable with respect to the influence of small-scale atmospheric turbulence (by a certain extent, due to relaminarization in the core), but fortunately is vulnerable to large-scale instability. The main form of wake decay depending on the intensity and scale of atmospheric turbulence are sinuous instability (see Figure 3) having three-dimensional character due to realization of modes with various wave lengths and oscillation planes, and vortex breakdown (see Figure 4). Sinuous instability is generated by the mutual induction of the vortex pair and excited by atmospheric (or artificial) turbulence. Scorer (7) was the first who has revealed it, its linear analysis was performed by Crow (8). Subsequently this model was supported by flight experiments (9), its generalization for the case when the core structure is considered was performed by S.E. Widnall and her co-workers (10). In particular, it was shown that the presence of the longitudinal velocity in the core decelerates self-induced vortex motion due to necessity of the turn of axial flow (jet effect) and somewhat delays the development of sinuous instability.

Sinuous instability is the main form of wake decay in a calm atmosphere ( $\varepsilon^* < 0.01$ ). Large-scale atmospheric turbulence,  $L_r/b_v \geq 1$ , accelerates its development decreasing the lengths of main modes and life time of the wake. In the case of strong turbulence ( $\varepsilon^* > 0.4$ ) the main form of decay becomes vortex breakdown, its premature onset being provoked by small-scale ( $L_r/b_v \ll 1$ ) turbulence. For relatively weak ( $\varepsilon^* \approx 0.1$ ) small-scale turbulence the life time is shorter by a factor of 3-4 as compared to large-scale turbulence.

Increased atmospheric stability and the absence of the wind decelerate wake decay. Temperature inversion of the atmosphere, (for example, in night-time) and increased turbulence of the atmosphere contribute to wake decay.



FIGURE 3 - Initial phase of vortex-wake breakdown in the form of three-dimensional "sinuous" instability<sup>(11)</sup>.

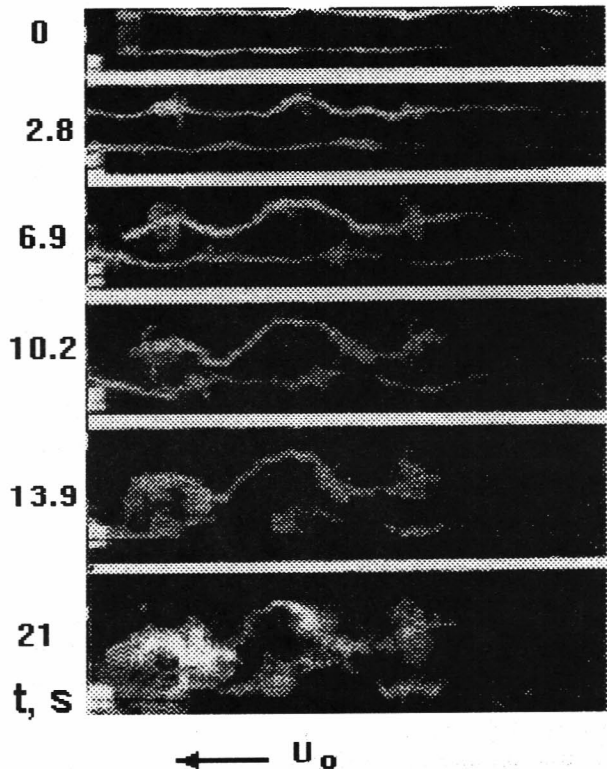


FIGURE 4 - Experiments in a tow tank (12),  $\varepsilon^* \approx 0.4$  (for anisotropic turbulence  $\varepsilon = 0.18 \cdot q^3 / L_r$ ).

The aircraft is a source of not only a turbulent vortex pair but also of additional turbulence due to

generation of multi-core vortex and separated-flow structures as a result of deflection of flaps and spoilers, installation of splines and fences, undercarriage lowering, etc., which decreases the magnitude of the rolling moment experienced by the encountering aircraft, especially at short distances from the wake-generating aircraft.

**II.1. Asymptotic methods for investigation on forms of instability and decay of the vortex wake.**

Asymptotic methods for investigating of local instability of vortex cores are based, as a rule, on the underlying work by Crow (8). The vortex decay mechanism described there has its foundation in the supposition about realization of an unsteady, exponentially growing (in time) solution for vortex pair evolution. It is found, that steady-state solutions also exist (in a body-fixed coordinate system) which exponentially decay (along the longitudinal coordinate) towards  $-\infty$ , and exponentially grow in the  $+\infty$  direction, with vortex filaments having the form of unwinding spiral. An example of such a solution is shown in Figure 5. This solution qualitatively describes the experimentally observed picture shown in Figure 3.

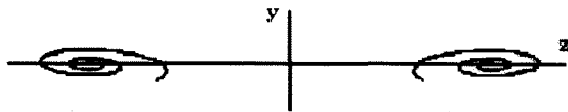


FIGURE 5 - Sinuous instability: a projection of vortex core locations on a cross-section plane.

**II.2. Zonal method for computing the wake behind an aircraft.**

For studying the wake behind an aircraft with the help of a numerical method with minimum use of empirical relationships, a zonal approach was used basing on an asymptotic analysis of the problem. This approach rests upon the existence of several scales in the given problem. The following zones were singled out:

1. zone in the vicinity of the aircraft and near wake;
2. zone of the intermediate wake;
3. far-wake zone;
4. turbulent-core zone.

For each zone corresponding equations are solved, on their boundaries the numerical matching of the solutions are performed. The purpose of the package is the computation of the wake vortex structure and intensity for an aircraft of real configuration and the evaluation of the time of wake breakdown.

The package consists of four modules: module for computing the near wake behind an aircraft of real configuration with a panel method; module for computing the intermediate wake based on an approximation of two-dimensional unsteady analogy;

module for computing the far wake; module for computing the turbulent core. It is essentially that in the zones of the intermediate and far wake the second approximation of unsteady analogy is used which permit one to simulate its instability. To compute the turbulent core, the anisotropic algebraic turbulence model is used.

Figure 6 shows the results computing the circumferential velocity in the wake behind an elliptically loaded wing compared with the Betz (13) semi-empirical model used in the compilation wake model, section III.1.

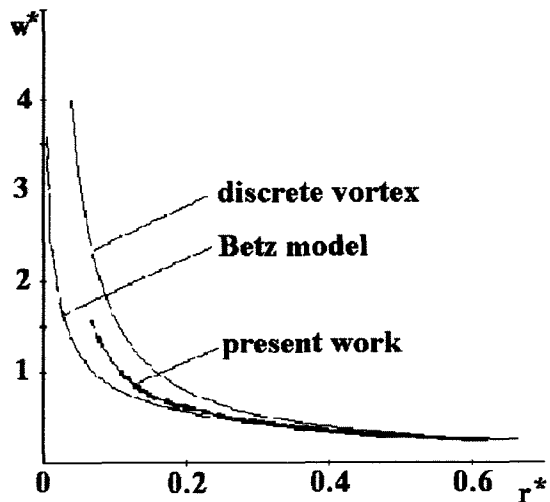


FIGURE 6 - Correction of the Betz model;  $w^*$ ,  $r^*$  are correspondingly the nondimensional tangential velocity and radius.

**II.3 Mathematical simulation within a framework of a boundary-value problem for the two-dimensional  $\Psi$ ,  $\Omega$  Navier-Stokes equations.**

In the Boussinesq approximation  $\left| \frac{\rho(z) - \rho_0}{\rho_0} \right| \ll 1$  a laminar vortex pair is considered ( $Ro=0$ ); in a stratified atmosphere ( $Ri = \infty$ );

$$\psi: v = \frac{\partial \psi}{\partial z}, w = -\frac{\partial \psi}{\partial y}; \zeta = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z};$$

$$\frac{\partial \zeta}{\partial t} + v \frac{\partial \zeta}{\partial y} + w \frac{\partial \zeta}{\partial z} = \frac{1}{Re} \left( \frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \zeta}{\partial z^2} \right) + \frac{1}{Fr} \frac{1}{T_0} \frac{\partial T}{\partial y};$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{1}{Re} \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right);$$

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\zeta;$$

where  $Re = \frac{\Gamma_0}{\nu}$ ,  $Fr = \frac{\Gamma_0^2}{gb_v^3}$ .

Conditions at the inlet boundary are  $\zeta = 0, T = T_0$ ; at the side and outlet boundaries -  $\frac{\partial \zeta}{\partial n}, \frac{\partial T}{\partial n} = 0$ .

Used in the numerical method is a finite-difference scheme with a third-order accurate upwind approximation for advective terms at the internal nodes of the five-point difference pattern (first-order at

the boundary on the two-point pattern) and a second-order accurate centered approximation for dissipative terms on the three-point pattern. The inertial reference system descends with a velocity of  $W_0$ . Computations are performed within the  $6 \times 6$  domain on a uniform  $200 \times 200$  mesh with square cells, time step is  $\tau_0=0.01$ , initial vortex radius is  $\bar{r}_0=0.3$ , temperature is  $T_0=280K$ , atmospheric temperature lapse is  $\frac{\partial \Delta T}{\partial z} = 2^\circ / 100 \text{ m}$  (see

Figures 7 and 8).

Computed results allow one to study the effect of Reynolds number and atmospheric stratification (which accelerates vortex-wake breakdown) on the descent rate of the vortex pair that increases (cf. with Ref.<sup>(14)</sup>) as the secondary vorticity is accumulated within the wake of a vortex cluster and the velocity is induced favorable to approaching the vortices each other.

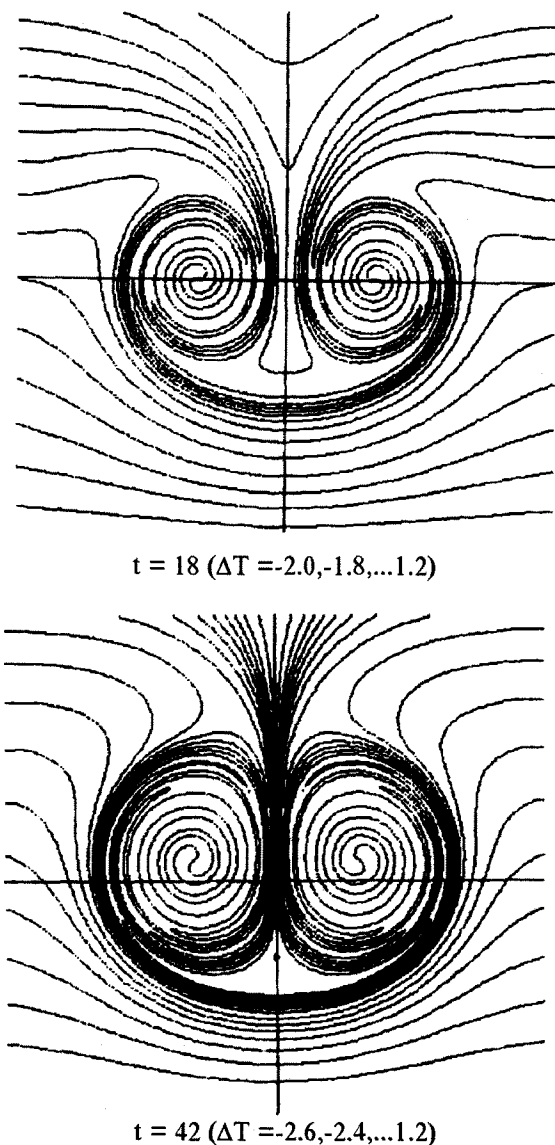


FIGURE 7 - Contours of  $\Delta T = \text{const}$ , ( $Re=10^6$ ).

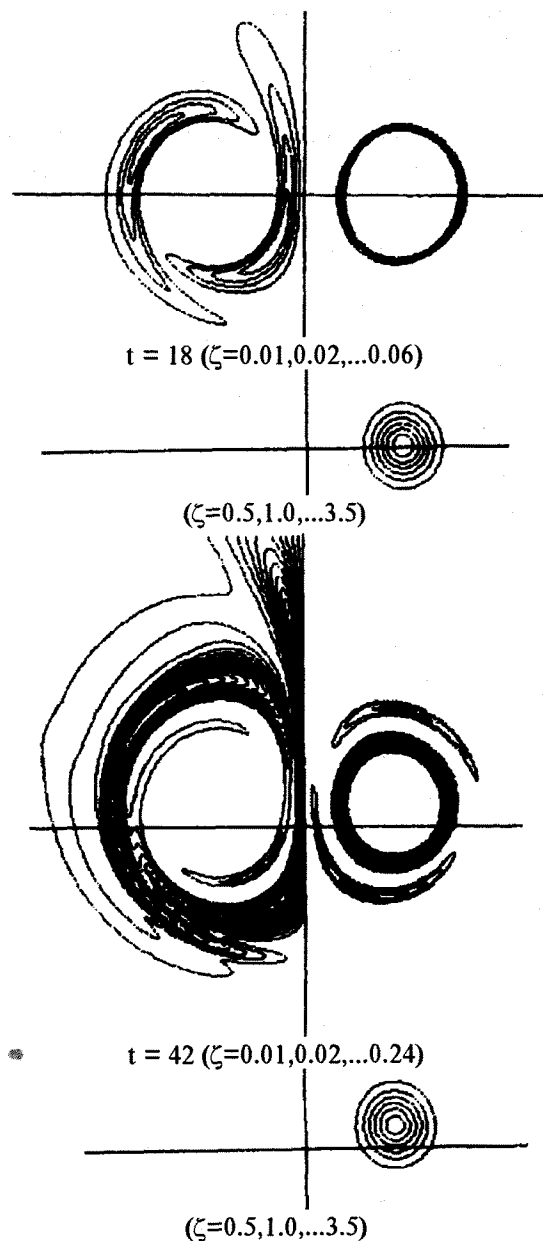


FIGURE 8 - Isovorticity contours  $\zeta = \text{const}$  ( $Re=10^6$ ).

### III. Following-aircraft problem: vortex diffraction by an obstacle.

Successfully used in this area are analytical assessments (in so doing a mathematical model can be reduced to the trivial one while retaining the physical nature of the phenomenon) and simulation methods in the framework of boundary-value problems and engineering methodology. Full-scale experiments are difficult to use here; leaning to full measure upon model experiment is nearly impossible due to the vortex diffraction by measuring instruments, the lack of coincidence of tracer trajectories with those of gas particles in their unsteady motion and the impossibility of properly modeling a vortex structure which interacts with the model. A useful guide in solving the problems of this kind can be provided by

the statistics of aircraft accidents and knowledge of accident scenarios (15).

We consider the interaction between an aircraft and a columnar vortex featuring canonical forms of instability in any combination: flute-like (longitudinal flutes on the column), bending-like (curve of the axis of the columnar vortex, for example, in developing a sinuous instability of a vortex pair where the centroid wavelength is much more than an aircraft size), neck-like (longitudinal variation in vortex cross-sectional area in consequence, for instance, of vortex bursting) or a ring vortex (formed in the last stage of vortex breakdown) with various elongation ratios.

The in-ground effect leads to diverging the vortices from each other and their rebound near the ground. The visible pattern of the vortex structure therewith is dependent on the wavelength of a sounding radiation (see Figure 9).

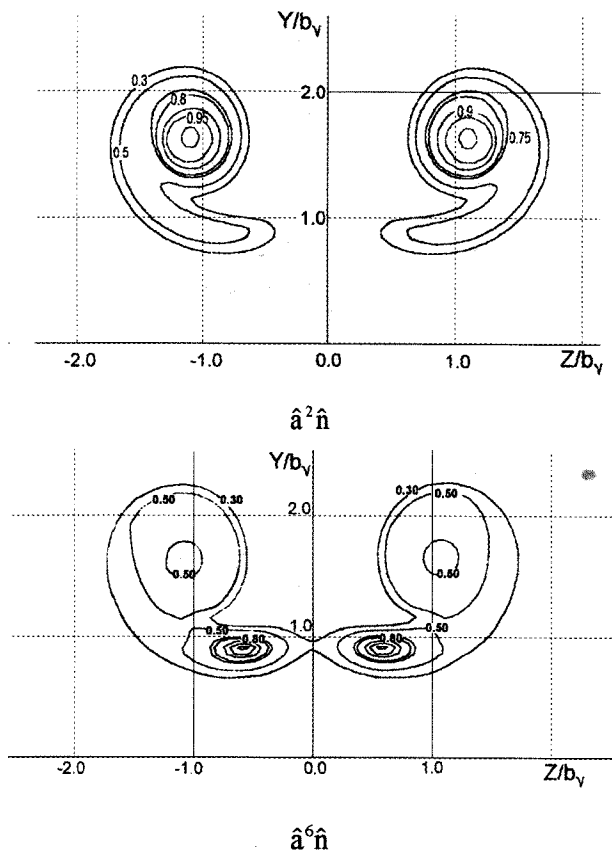


FIGURE 9 - Volume density of scattering short-  $\hat{a}^2 \hat{n}$  and long-period ( $\hat{a}^6 \hat{n}$ ) sounding radiation (jet-vortex wake, B-747, glide path,  $h = b_v$ ,  $x \approx 500$  m).

The vortex dissipation coefficient increases by 3 to 4 times as compared to its conventional value depending on the turbulence level near the ground surface and its roughness (16).

### III.1 Block-compiled wake model.

Many factors, both deterministic and random, influence the behavior of a vortex wake behind an

aircraft. On the basis of the analysis of available experimental data and existing theoretical wake models, one can set off the following factors that, if not included into a wake model, can result in the explicit inconsistency with experimental data:

1. geometrical shape of the aircraft;
2. effect of growing a turbulent core of the vortex on an external vortex flow;
3. effect of atmospheric turbulence;
4. effect atmospheric stratification;
5. ground effect, including the near-ground wind;
6. effect of wing high-lift devices that leads to generating a multi-core wake structure.

In addition, the assessment of the wake lifetime is necessary.

The algorithm for calculating a velocity field and vortex locations within the aircraft wake with accounting for the above-indicated factors is based on experimental data.

The well-proved Betz model (13) is proposed to be used for describing the external inviscid flow field. The use of the Betz model begins with the calculation of the flow over the aircraft according to a linear theory for obtaining the spanwise distribution law of the circulation  $\Gamma(z)$ . Then, using Betz formulas (13) which duplicate the circulation and momentum conservation laws for every separate portion of the vortex sheet in the course of its rolling up, one recalculates the circulation and obtains its distribution  $\Gamma(r)$  over the inviscid portion of the vortex sheet rolled up.

The size of the turbulent core for the aircraft is calculated from the formula (17):

$$r_c [m] = 0.35 \sqrt{1 + 0.327 t [\text{sec}]}$$

The core circulation is calculated from the formula (18) with accounting for losses due to turbulent diffusion in the atmosphere:

$$\Gamma_c(t) = 0.4 \Gamma_0 \exp(-0.8qt/b_v)$$

For the effect of atmospheric stratification and turbulence on the vortex pair motion away from the ground board to be accounted for, we use the Greene model (19):

$$\frac{d^2 H}{dT^2} + K_2 \frac{dH}{dT} + K_3 H = 0,$$

$$\text{where } H = \frac{h}{b_v}, T = \frac{t \cdot W_0}{b_v}, \frac{dH}{dT} = \frac{W}{W_0} = \frac{\Gamma}{\Gamma_0}.$$

Here  $\Gamma_0$  is the initial circulation,  $W$  is the vortex descent rate,  $t$  is the time. The coefficients  $K_{2,3}$  are determined by the following formulas:

$$K_2 = 0.82 \frac{q}{W_0}, K_3 = 0.45 \frac{N_0^2 b_v^2}{W_0}$$

Initial conditions for the equation are:  $H=0$ ,  $dH/dT=1$  at  $T=0$ . This model is correct in the case of absence in-ground effect - for altitudes  $h > 2 b_v$ .

Vortex breakdown is usually associated with the mechanism of Crow instability (8) of two vortex filaments. According to this theory, at some simplifying assumptions regarding atmospheric



turbulence the following estimate for the time-to-link<sup>(19)</sup> is obtained:

$$e^{T_L} (e^{-10} + e^{-\frac{1.6}{Q}}) = 1,$$

where  $Q = q/W_0$ ,  $T_L = t_L \cdot W_0/b_v$  - nondimensional time for vortex linking. This formula holds for the cases of both strong and weak turbulence.

Figure 10 shows tangential velocity profiles as functions of radial distance for the wake produced by the B-747 aircraft when flying along the glide path. The non-monotone character of radial variations in tangential velocity is resulted from the presence of the high-lift devices extended and the formation of a two-core flow structure.

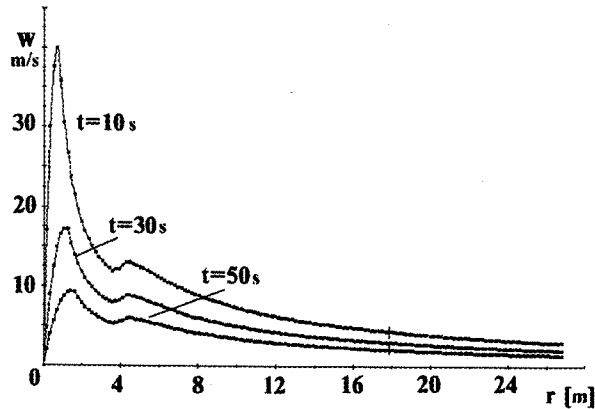


FIGURE 10 - Tangential velocity profiles of the vortex behind B-747 aircraft.

The above-outlined wake model allows the behavior and structure of a wake to be described on the basis of empirical relationships. However such an approach does not relieve one of having to rationally describe the phenomenon of wake development.

### III.2. Flight-dynamics aspects: basic assumptions.

The along-track penetration of a following aircraft into the vortex wake generated by a leading aircraft is the most severe situation from the flight-dynamics standpoint. Usually, when approaching the vortex at encountering angles of more than  $6^\circ$ , the maximum vortex-induced upset effect does not exceed the available roll control power of the trailing aircraft.

The model of a frozen vortex wake that does not account for the reciprocal effect of the aircraft on the vortex structure into which it penetrates is the assumption universally adopted when solving the problems of such kind. But the more rigorous formulation requires estimating the reciprocal reaction of the vortex (solution to the problem of vortex diffraction by an obstacle).

An assumption is also made on the atmospheric turbulence being independent of the organized vortex structure, which enables summing up velocity disturbances in the core region induced by the vortex and stochastic component featuring the Kolmogorov<sup>(20)</sup> distribution. In actuality, behind the

aircraft the vortices become curved gradually under the action of atmospheric turbulence and are far from being statistically homogeneous and isotropic in nature.

Besides, atmospheric turbulence is assumed to be stationary in the coordinate system descending together with vortices. This assumption is valid within the scales of about  $\sim 8.6 \cdot b_v$ , since, as usual, prior to the coherent structure being destroyed, the vortex wake has time to descent only by several vortex separation distances  $b_v$ .

In describing wind effects when statistically modeling the automatic landing, the ICAO model is used: the longitudinal wind component has the expected value shifted upwind, the tail wind speed is less than 5 m/s,

$$U_x(10) = (-2.7 + 3.75\xi_x), \quad U_z(10) = 3.75\xi_z,$$

where  $\xi_x, \xi_z$  are the random variables with a normal distribution, zero expected value and unit variance. The standard deviation of the atmospheric turbulence is

$$\sigma_w = 0.2\sqrt{U_x^2(10) + U_z^2(10)}.$$

The shear wind in the atmospheric boundary layer was specified according to the logarithmic velocity profile:

$$U_x(h) = U_x(10) \cdot (1 + \log h) / 2,$$

$$U_z(h) = U_z(10) \cdot (1 + \log h) / 2.$$

The solution (in a linear approximation) to the unsteady problem of determining aerodynamic forces and moments acting on the aircraft encountering the vortex wake is a constituent of this mathematical model. The choice of an aerodynamic model for the wake-penetrating aircraft was made according to the disturbance scale  $L_f$ : the single-point model is used when  $L_f \gg b$ ; a decrease in  $L_f$  leads to the necessity of passing to the four- or N-point models. In so doing, for saving computation time spent for the calculation of aircraft aerodynamic characteristics, a matrix approach is implemented according to which matrices of aerodynamic influence are calculated in advance and stored in the main computer memory.

The results of the investigation are also dependent on the model adopted of pilot's actions. The automatic control algorithm allows reducing the available roll control power as compared to an uncontrolled motion (wheel-free motion or at rapid action) by a factor of 1.5-2 (see Figure 11).

The results obtained allow the conclusion to be drawn that the equipment available makes it possible to opportunely detect the presence of a vortex wake. The structural flexibility and pilot's location away from the aircraft's center of gravity make it impossible to be guided by subjective sensation of the pilot when assessing the situation. The identification of the aircraft response to atmospheric disturbances is also dependent on the piloting technique used (smooth or sharp), which defines the duration of the subsequent structural response.



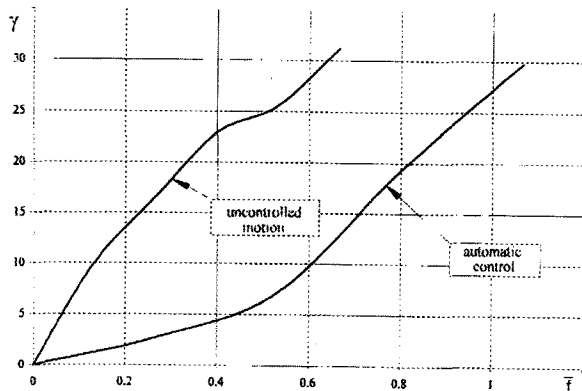


FIGURE 11 - Estimated maximum vortex-induced rolling moments when encountering the B-747 vortex wake by the Tu-154 aircraft at a separation distance between them of 3 nmi ( $\gamma$  is the roll angle,  $\bar{F}$  is the ratio of the over-the-trajectory maximum induced rolling moment to the available roll control power).

The assessment of unfavorable situations in landing approach has shown that low altitudes of 10-30 m are the most dangerous. Maximum rolling moments occur at a weak tail wind (in this case the vortices are advected to the runway), at a cross wind of  $U_z(10) = 1-2$  m/s that holds the vortex over the runway (in this case the extreme values of practically all parameters are realized). An increase in horizontal wind due to increasing  $\sigma_w$  leads to decreasing circulation and  $V_{\tau, \max}$ . The most difficult and poorly studied is the area of low altitudes (the evolution of vortices formed at low altitudes is not clearly understood).

### III.3. Structural-strength and aeroelastic aspects.

The cross-track penetration into a vortex pair is the most severe situation from the structural-strength standpoint, where the structural flexibility results in the fact that the greatest loads occur when encountering the second vortex. Accounting for natural oscillation frequencies (for a given vortex span and strength and vertical distance of aircraft passage near the vortex) can change the extreme wake intersection angle by 10-15°. The vortex separation distance influences in the same non-monotone way (see Figure 12).

The mathematical model used for determining aerodynamic forces and moments is based on a quasi-steady approach in the framework of a strip theory and an assumption of "frozen" velocity field, and the model for determining elastic and inertial forces is based on the beam representation of the structure.

Illustrated in Figure 13 by way of example is the penetration of the light aircraft with  $V_{\infty} = 78$  m/s through the centers of a vortex pairs with a vortex separation distance of  $b_v = 40$  m, a vortex radius of  $r_0 = 2$  m, and a maximum tangential velocity of  $V_{\tau, \max} = 10$  m/s. The large altitude overshoot of the aircraft can be

considered as a measure of the accuracy of the hypothesis that the flow velocity field is "frozen".

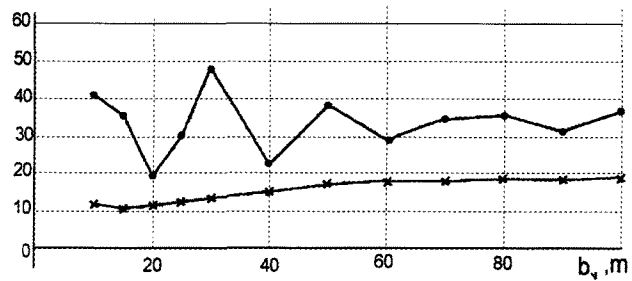


FIGURE 12 - Maximum acceleration (in  $m/s^2$ ) at the physical (●) and mathematical (×) c.g. of the IL-114 aircraft ( $G = 20,000$  kg) as a function of distance between vortices.

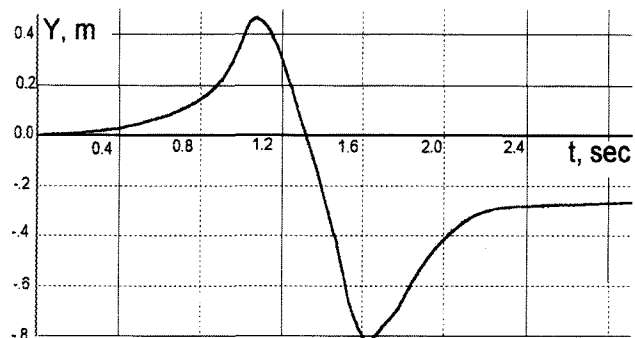


FIGURE 13 - Vertical excursions of the c.g. of the Il-103 aircraft ( $G=1,200$  kg) between +0.5 and -0.8 m when encountering a vortex wake at an angle of penetration of 90°.

Shown in Figures 14 and 15 for a medium-payload aircraft are the values of vertical accelerations at mathematical and physical (where the accelerometer is located) centers of gravity. Vertical acceleration at the physical c.g. are seen to be substantially different from those at the mathematical c.g. As to the g-loads experienced by the pilots, they do not reflect the structure of external disturbances.

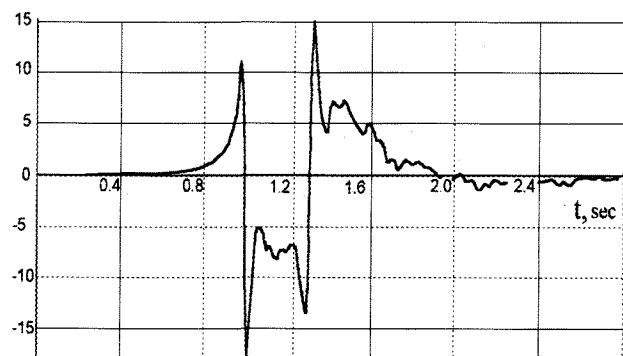


FIGURE 14 - Vertical accelerations (in  $m/s^2$ ) at mathematical c.g. of the Il-114 aircraft.

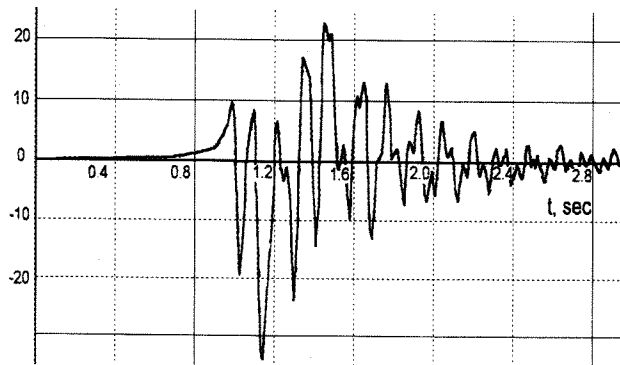


FIGURE 15 - Vertical accelerations (in  $m/s^2$ ) in the section passing through the Il-114's c.g.

### III.4. Aircraft aerodynamics aspects.

Studies aimed at developing the system for active alleviation of the vortex-wake effect on aircraft were performed in TsAGI's T-102 and T-103 wind tunnels. The results obtained have shown that the use of control surfaces (flaps, interceptors) allow changing the wing lift and producing additional rolling moments of  $\Delta C_l = 0.05-0.06$  that are comparable to those of the wing-tip ailerons while retaining a satisfactory level of aircraft aerodynamic characteristics in takeoff and landing flight regimes (see Figure 16).

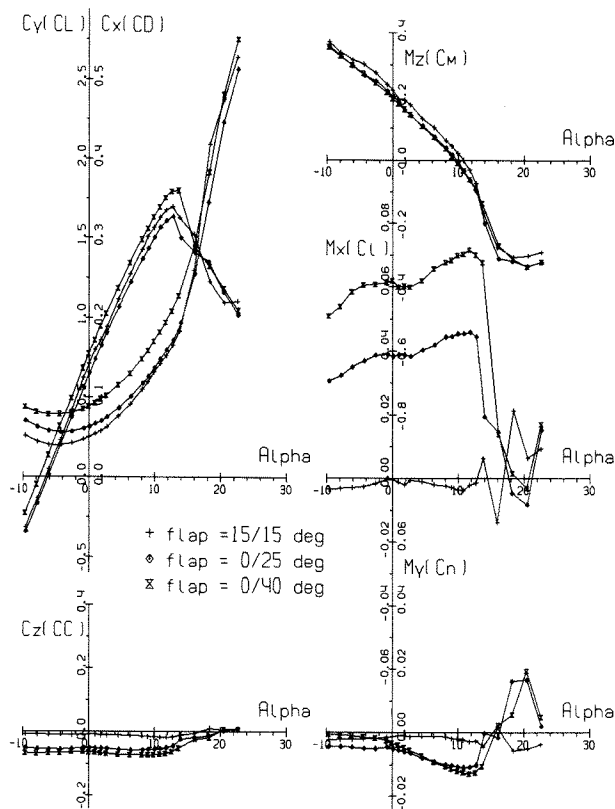


FIGURE 16 - Wing adaptation using trailing edge device for producing additional rolling moments (aircraft model:  $C_{flap} = 30\%$ ,  $M_{\infty} = 0.15$ ,  $Re = 1 \cdot 10^6$ ).

### Conclusions

The development of an adequate, self-consistent vortex-wake model that therewith leaves room for further refinement and growth is the major result of the studies performed.

The availability of the model allows the measures aimed at attenuating aircraft-generated vortex wakes to be considered. In so doing, the effect of engines is taken into account, including their type, number, location, mode of operation, and possible thrust reversing. Additional engine thrust has an alleviating influence on the trailing vortex system through directly interacting with the wing vortex sheet during its rolling up. The engine as a source of small-scale turbulence exerts a provocative effect on a vortex bursting. Jet blowing at the wing tips also accelerates vortex decay.

In investigating the wing-tip devices including the interceptors mounted on them, one should remember that they can be used not only for vortex alleviation through dissipating tip vortices (winglets turned down are especially effective) but also for enhancing aircraft stability when flying through rough air.

Aircraft certification for minimum safe-separation distances would allow solving the question as to the advisability of introducing into service new heavy transport aircraft.

Parameters of the atmosphere have an appreciable effect on wake breakdown<sup>(21)</sup>, hence they must be registered in conducting field observations. It should be taken into account that realized in actual practice are not a single and rapidly growing mode of instability but also all modes featuring different wavelengths and oscillation planes, which is responsible for a three-dimensional character of the vortex pair motion. Also, accounting for the effect of atmospheric factors and in-ground effect on the matrix of safe-separation distances allows the airport flight schedule to be tightened.

The wake-tracer issue is of interest not only from the scientific standpoint but also can become the near-future contribution to solving the problem under study. It should therewith be remembered about the possible inconsistency between the trajectories of tracer particles and vortex structures.

The improvement of aircraft survivability in encountering vortex wakes is also a factor favoring the reduction of safe-separation distances. This issue should be solved in a complex manner with considering the aircraft's general layout characterized by available aerodynamic moments, moments of inertia (mass distribution, including engines and fuel), roll controllability, use of wind-shear-detection and automatic control systems.

Mathematical models of the second (following) aircraft should be developed both for a two-dimensional vortex wake and with accounting for the transformation of the two-dimensional vortex structure into the three-dimensional one as well as the

time of aircraft-vortex wake interaction and aircraft control characteristics.

Air traffic organization with taking into account the topology of the airport environment, wind rose, geological formations (especially for multi-runway airports) and ground effect in combination with the above-indicated measures would allow increasing the airport capacity by 20-25%.

#### Acknowledgments.

The research reviewed here has been supported by the International Science and Technology Center under grant No 201. The authors wish to acknowledge all performers of the Project especially Profs. V.A. Yaroshevsky, O.A. Kuznetsov, A.L. Stasenko as well as Drs. A.V. Voevodin, A.M. Gaifullin, A.A. Pavlenko, Yu.S. Mikhailov, Yu.N. Sviridenko for useful discussions of the work results.

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