COUPLED FLIGHT MECHANICS AND AEROELASTICITY- SOME EFFECTS OF AIRCRAFT MANEUVERS ON AEROELASTIC DIVERGENCE AND FLUTTER

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Abstract

This paper shows that the aeroelastician's tools of energy methods and Lagrange's equations are simple and convenient methods of developing the scalar equations of motion for the flight mechanics of flexible aircraft. Illustrations include (a) the longitudinal motion of a slender, flexible beam, (b) the general (longitudinal/lateral-directional) motion of a thin, flexible, plate-like surface and (c) the divergence and flutter of a bending - torsion airfoil attached to a rigid fuselage in the longitudinal plane. The presence of imposed rigid body rotations is seen to reduce the effective generalized stiffnesses of the natural vibration modes, thereby having a second order effect on aeroelastic stability and response. It also is pointed out (perhaps rediscovered) that it is not necessary to develop the tedious expressions for the kinetic energy and its many partial and time-derivatives when applying Lagrange's equations.

1. Introduction

This paper combines the points of view of aeroelasticity and flight mechanics in treating the motions of flexible aircraft that can be: (a) in steady flight or (b) accelerating along and/or rotating about inertial coordinate axes. The customary examinations of divergence and flutter[1] either: (a) assume that an aircraft is in trimmed, balanced flight or (b) drop the coupling between the accelerations of the rigid body and the structural deflections[2]. Even in examining the effects of flexibility on stability and control, a frequent assumption[3] is that the aircraft axis system is an inertial one, allowing Lagrange's equations for flexibility effects to be used directly in a body-fixed axis system.

However there are some examples of aircraft that have divergence and/or flutter speeds in maneuvers that differ from those speeds in steady, level flight. Usually the first culprits blamed in such cases are the altered aerodynamic forces.

Of course another possibility is that the structural dynamic properties are altered by the aircraft accelerations themselves - that is the natural vibrations and mode shapes in the accelerating system are significantly different than their counterparts in steady, level flight.

The FLEXSTAB program[4] was a careful examination of the differential equations of motion for a flexible aircraft from first principles. However, even there it was necessary to neglect the interactions of the rotation rates with the "perturbation displacements". Intuitively one would expect those effects to be proportional to the ratio of the rigid-body accelerations (say pitch rate or acceleration along a significant structural axis) to the lowest vibration frequencies of the aircraft in unaccelerated flight.

A related work[5] by Bekir et al uses Likins'[6] idea of a hybrid coordinate system to describe the motion of a flexible aircraft. In an earlier work Rodden[7] showed the care that is necessary to account for inertia-relief effects in the flexibility matrix in order to obtain results that are independent of the the restraining system to measure (or calculate) structural flexibility.

To the author's knowledge, the first applications of Lagrange's equations to the flight mechanics of flexible aircraft was by Waszak and Schmidt[8]. This paper extends the work of reference [8] and also reference [9] to apply to the coupled rigid and flexible motions of a thin, flexible beam. It also also shows how the same approaches can be used for the (seemingly daunting) case of a flexible, plate-like surface in general (longitudinal and lateral/directional motion). Along the way we (re ?) discover that there are great simplifications that are available in the use of Lagrange's equations. The simplifications are always available, but are most useful in the cases of nonlinear problems with complicated geometries.

For the cases considered, the results are not large, but they are "configuration-dependent" and are potentially larger for other cases. In Rodden's terminology [10], at this point they are "secondary considerations".

2. The Slender, Flexible Beam in Accelerating, Longitudinal Motion

As a useful introductory example, consider the static and dynamic response of a slender, flexible beam. The degrees of freedom are the translations along the inertial X and Z directions (attached to a "flat" earth), the rotation θ of the "body-fixed" x, z axis system with respect to the X axis and the bending of the beam with respect to the body-fixed system. The beam is "attached" to the x, z coordinate system in the sense that the elastic potential energy is given by the form that follows below, and the bending displacement of the beam is small enough so that it involves motion only normal to the x axis.

2.1 Coordinates

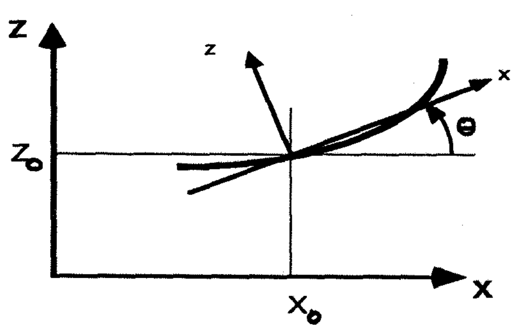

Figure 1. The Idealized Thin, Flexible Beam

In the idealized examples presented here, we use an orthogonal inertial coordinate system, fixed to a "flat earth", with Z pointed "up".

The coordinates of a point on the body are:

$$z = z(x,t)$$

$$X = X_0 + x\cos\theta - z\sin\theta$$

$$Z = Z_0 + x\sin\theta + z\cos\theta$$

2.2 Kinetic Energy

We choose the first three generalized coordinates as:

$$q_1 = X_0(t) \quad q_2 = Z_0(t) \quad q_3 = \theta(t)$$

and assume that the bending displacement is given by the series of "assumed modes" with the generalized coordinates $q_4, q_5, q_6, \dots q_n$:

$$z(x,t) = \sum_{i=4}^{n} f_i(x) q_i(t)$$

Define:

$$S_3 = \sin q_3 \quad C_3 = \cos q_3$$

$$\hat{m} = \int dm \quad \hat{S} = \int x dm \quad \hat{I} = \int x^2 dm$$

$$F_i = \int f_i \, dm \quad M_{ij} = \int f_i f_j \, dm \quad G_i = \int x f_i \, dm$$

The kinetic energy becomes:

$$T = \tfrac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2\right)\hat{m} + \left(\dot{q}_2 C_3 - \dot{q}_1 S_3\right)\left(\dot{q}_3 \hat{S} + \sum_{i=4}^{n} \dot{q}_i F_i\right)$$

$$-\dot{q}_3\left(\dot{q}_1 C_3 + \dot{q}_2 S_3\right)\sum_{i=4}^{n} q_i F_i + \tfrac{1}{2}\dot{q}_3^2\left(\hat{I} + \sum_{i=4}^{n}\sum_{j=4}^{n} q_i q_j M_{ij}\right)$$

$$+\dot{q}_3 \sum_{i=4}^{n} q_i G_i + \tfrac{1}{2}\sum_{i=4}^{n}\sum_{j=4}^{n} \dot{q}_i \dot{q}_j M_{ij}$$

2.3 Elastic Potential Energy

The elastic potential energy of the beam in bending is assumed to be the form

$$U_e = \tfrac{1}{2}\int EI_b (z'')^2 \, dx \equiv \tfrac{1}{2}\sum_{i=4}^{n}\sum_{j=4}^{n} K_{ij} q_i q_j$$

This requires some restraints on the orientation and bending displacement of the beam with respect to the "body-fixed" axis system.

2.4 Gravitational Potential Energy

The beam begins to accumulate gravitational potential energy U_g at a position Z_g by moving vertically against gravity.

$$U_g = g\int\left(Z - Z_g\right)dm$$

$$= g\left[\left(q_2 - Z_g\right)\hat{m} + S_3\hat{S} + C_3\sum_{i=4}^{n} q_i F_i\right]$$

2.5 Lagrange's Equations of Motion

Lagrange's equation for the i^{th} degree of freedom is:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U_e}{\partial q_i} + \frac{\partial U_g}{\partial q_i} = Q_i$$

The differential equations for $i = 1, 2, 3$ are:

$$\hat{m}\ddot{q}_1 - S^*\ddot{q}_3 - S_3\sum_{j=4}^{n}\ddot{q}_j F_i - C^*\dot{q}_3^2$$

$$-2C_3\dot{q}_3\sum_{j=4}^{n}\dot{q}_j F_j = Q_1$$

$$\hat{m}\ddot{q}_2 + C^*\ddot{q}_3 + C_3\sum_{j=4}^{n}\ddot{q}_j F_j - S^*\dot{q}_3^2$$

$$-2S_3\dot{q}_3\sum_{j=4}^{n}\dot{q}_j F_j + g\hat{m} = Q_2$$

$$-S^*\ddot{q}_1 + C^*\ddot{q}_2 + \hat{I}\ddot{q}_3 + \sum_{j=4}^{n}\dot{q}_j G_j + gC^* \approx Q_3$$

where:

$$S^* = S_3\hat{S} + C_3\sum_{j=4}^{n} q_j F_j$$

$$C^* = C_3\hat{S} - S_3\sum_{j=4}^{n} q_j F_j$$

The differential equations for $i \geq 4$ are:

$$\left(-\ddot{q}_1 S_3 + \ddot{q}_2 C_3\right)F_i + \sum_{j=4}^{n}\ddot{q}_j M_{ij} - \dot{q}_3 G_i$$

$$-\dot{q}_3^2\sum_{j=4}^{n} q_j M_{ij} + \sum_{j=4}^{n} q_j K_{ij} + gC_3 F_i = Q_i$$

We have used the symmetry of the "generalized mass" and the "generalized stiffness" matrices. We have dropped products of the "flexible generalized coordinates" $q_j q_k$ for $j,k \geq 4$ in the differential equations, but have **not** assumed q_1, q_2, q_3 to be small. If we use the actual natural modes of vibration as our "assumed modes", some of the weighted mass integrals turn out to be zero.

These equations are in an **unusual form** for flight mechanics or stability and control in that q_1, q_2 represent the motion along the **inertial coordinates**, X, Z. The usual rotations of coordinates are needed to replace q_1, q_2 with a set of coordinates q_ξ, q_η in a body-fixed system, wind-axis system or any other conveniently rotated coordinate system.

2.6 The "Flexible Approximation":

The fully coupled solution for arbitrary motions would require the determination of the (generalized) aerodynamic and propulsive forces on the RHS of Lagrange's equations and then the simultaneous solution of those equations. However, analogous to the "phugoid approximation" and the "short period approximation", consider the form that the "flexible" equations would take in the special case of a "steady maneuver" - the linear accelerations and the angular velocity assumed to be constant (but not necessarily zero):

$$\ddot{q}_1, \ddot{q}_2, \dot{q}_3 = \text{Constants}$$

$$\sum_{j=4}^{n}\ddot{q}_j M_{ij} + \sum_{j=4}^{n} q_j\left(K_{ij} - \dot{q}_3^2 M_{ij}\right)$$

$$= Q_i + \left(\ddot{q}_1 S_3 - \ddot{q}_2 C_3\right)F_i + \dot{q}_3 G_i - gC_3 F_i$$

The RHS contains the usual generalized forces, constants and specified functions of time. So, considering the eigenvalue problem, an important effect of "pitch rate" $\dot{\theta} = \dot{q}_3$ in this approximation is to reduce the effective generalized stiffnesses to:

$$\boxed{K_{ij} - \dot{q}_3^2 M_{ij}}$$

Of course aircraft pitch rates are usually small with respect to any of the natural frequencies of vibration, so the effect normally is what Rodden[10] has referred to as a second order effect on aeroelasticity.

3. The Slender, Flexible Surface in Accelerating, General Motion

A more challenging case is the acceleration of a thin flexible surface in general (longitudinal and lateral-directional) motion.

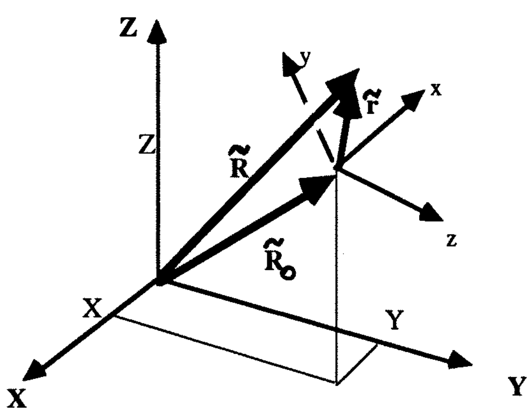

Figure 2. The Translating, Rotating Coordinate System

3.1 Coordinates

Again, we use an orthogonal inertial coordinate system, fixed to a "flat earth", with Z pointed "up". This not only influences the sense of our gravitational forces, but also the physical interpretation we apply to the successive Euler angles of rotation.

We obtain the orientation of the body-fixed axis system by thinking of the three successive translations as our first three generalized coordinates:

$$X_0 = q_1$$
$$Y_0 = q_2$$
$$Z_0 = q_3$$

to obtain coordinate systems (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3). Then the next three generalized coordinates are three successive Euler rotations - q_4 about $z_3 = z_4$ to obtain coordinate system (x_4, y_4, z_4), q_5 about $y_4 = y_5$ to obtain coordinate system (x_5, y_5, z_5) and q_6 about $x_5 = x_6$ to obtain the final body-fixed coordinate system:

$$x_6 = x$$
$$y_6 = y.$$
$$z_6 = z$$

The inertial X, Y, Z coordinates of a point in the body-fixed x, y, z axis system then are:

$$\{\tilde{R}\} = \{\tilde{R}_0\} + [T_4][T_5][T_6]\{\tilde{r}\}$$

where:

$$\{\tilde{R}\} = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \quad \{\tilde{R}_0\} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \quad \{\tilde{r}\} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

$$[T_4] = \begin{bmatrix} C_4 & -S_4 & 0 \\ S_4 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T_5] = \begin{bmatrix} C_5 & 0 & S_5 \\ 0 & 1 & 0 \\ -S_5 & 0 & C_5 \end{bmatrix}$$

$$[T_6] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_6 & -S_6 \\ 0 & S_6 & C_6 \end{bmatrix}$$

We have used the abbreviations:

$$S_i = \sin q_i \quad C_i = \cos q_i$$

3.2 Flexible Displacements

We assume that the surface is thin, lies essentially in the x, y plane and that the flexible displacements are given by the assumed modes $f_j(x, y)$ and generalizeed coordinates $q_j(t)$:

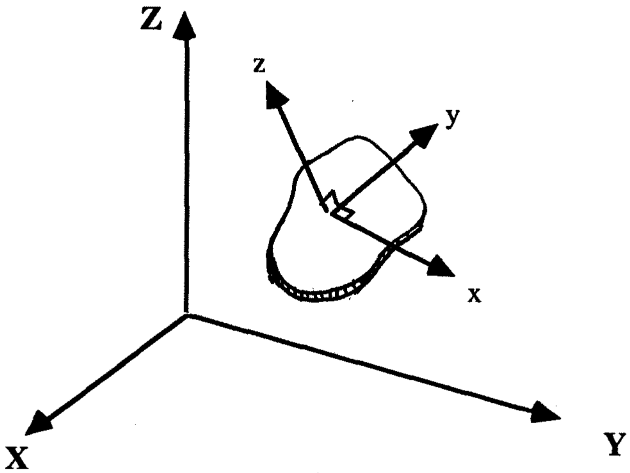

Figure 3. Thin, Plate-like Surface with Body-fixed Coordinate System

$$z(x,y,t)=\sum_{j=7}^{n} f_j(x,y)q_j(t)$$

3.3 Kinetic Energy and Lagrange's equations:

An expression for the kinetic energy would require that we obtain expressions for all of the inertial velocities, square them and integrate over the body.

$$T=\frac{1}{2}\int_{Body}\left\lfloor\frac{d\tilde{R}}{dt}\right\rfloor\left\{\frac{d\tilde{R}}{dt}\right\}dm$$

Then obtaining the necessary expressions for Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U_e}{\partial q_i}+\frac{\partial U_g}{\partial q_i}=Q_i$$

would require tedious expressions for all of the partial derivatives with respect to the generalized coordinates and their time derivatives. An attractive alternative is to return to the derivation of Lagrange's equations and note that we can write:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=\sum_{j=1}^{n}\ddot{q}_j\int_{Body}\lfloor\tilde{R}_i\rfloor\{\tilde{R}_j\}dm+\sum_{j=1}^{n}\sum_{k=1}^{n}\dot{q}_j\dot{q}_k\int_{Body}\lfloor\tilde{R}_i\rfloor\{\tilde{R}_{jk}\}dm$$

where:

$$\{\tilde{R}_i\}=\frac{\partial\{\tilde{R}\}}{\partial q_i}\qquad\{\tilde{R}_{ij}\}=\frac{\partial^2\{\tilde{R}\}}{\partial q_i\partial q_j}$$

Perhaps this is a rediscovery of an old principle, but it means that <u>we do not need the complete expressions for the kinetic energy and its derivatives when we apply Lagrange's equations</u>. <u>We only need to obtain the partial derivatives of the inertial coordinates with respect to the generalized coordinates</u>. This is always correct, but is particularly helpful in nonlinear problems with complicated geometries.

Breaking the summations into discrete parts, plugging in the flexible deflections and eliminating terms that are inherently zero allows us to arrive at what we call the "acceleration vector" terms in the kinetic energy and gravitational potential energy:

$$\{\tilde{a}\}=\begin{Bmatrix}\ddot{q}_1\\ \ddot{q}_2\\ \ddot{q}_2\end{Bmatrix}+\left[\hat{T}_L+\hat{T}_R\right]\begin{Bmatrix}x\\ y\\ z\end{Bmatrix}+\left[\hat{T}+\hat{T}_F\right]\begin{Bmatrix}0\\ 0\\ \ddot{z}\end{Bmatrix}+g\begin{Bmatrix}0\\ 0\\ 1\end{Bmatrix}$$

where:

$$\left[\hat{T}\right]=\left[T_4(q_4)\right]\left[T_5(q_5)\right]\left[T_6(q_6)\right]$$

$$\left[\hat{T}_i\right]=\frac{\partial}{\partial q_i}\left[\hat{T}\right]\qquad\left[\hat{T}_{ij}\right]=\frac{\partial^2}{\partial q_i\partial q_j}\left[\hat{T}\right]$$

$$\left[\hat{T}_L\right]=\left[\hat{T}_{Linear}\right]=\sum_{j=4}^{6}\left[\hat{T}_j\right]\ddot{q}_j$$

$$\left[\hat{T}_R\right]=\left[\hat{T}_{Rotate}\right]=\sum_{j=4}^{6}\sum_{k=4}^{6}\left[\hat{T}_{jk}\right]\dot{q}_j\dot{q}_k$$

$$\left[\hat{T}_F\right]=\left[\hat{T}_{Flexible}\right]=2\sum_{j=4}^{6}\left[\hat{T}_j\right]\dot{q}_j$$

The kinetic and gravitational energy terms in the first three equations of motion (inertial translations) turn out to be:

$$q_1: \quad \int_{Body} \lfloor 1 \quad 0 \quad 0 \rfloor \{\tilde{a}\} dm$$

$$q_2: \quad \int_{Body} \lfloor 0 \quad 1 \quad 0 \rfloor \{\tilde{a}\} dm$$

$$q_3: \quad \int_{Body} \lfloor 0 \quad 0 \quad 1 \rfloor \{\tilde{a}\} dm$$

and in the next three equations of motion (Euler angles of rotation):

$$q_4: \quad \int_{Body} \lfloor x \quad y \quad z \rfloor \left[\hat{T}_4\right]^T \{\tilde{a}\} dm$$

$$q_5: \quad \int_{Body} \lfloor x \quad y \quad z \rfloor \left[\hat{T}_5\right]^T \{\tilde{a}\} dm$$

$$q_6: \quad \int_{Body} \lfloor x \quad y \quad z \rfloor \left[\hat{T}_6\right]^T \{\tilde{a}\} dm$$

and for the flexible equations:

$$q_i\big|_{i \geq 7}: \quad \int_{Body} \lfloor 0 \quad 0 \quad f_i \rfloor \left[\hat{T}\right]^T \{\tilde{a}\} dm$$

3.4 <u>The "Flexible Approximation"</u>

Again, the kinetic and gravitational energy terms contribute the most complicated parts to the completely coupled equations of motion for the displacements along the inertial coordinates, the Euler angles of rotation and the flexible deflections. We would have to add the potential energies from stiffness and the generalized forces from aerodynamics and propulsion.

Since the first three generalized coordinates are the displacements along the inertial coordinates, we could introduce appropriate rotations of coordinates to replace them with motions along the body axes, wind axes or any other convenient coordinate system. Similarly, the second three generalized coordinates are the Euler angles of rotation in their successive coordinate systems. We would need appropriate rotations of coordinates to replace them with other convenient angles, such as the instantaneous rotations about the body axes.

To get additional insight into the flexible equations, abbreviate the matrices:

$$[A] = \left[\hat{T}\right]^T$$

$$[B] = \left[\hat{T}\right]^T \left[\hat{T}_L + \hat{T}_R\right]$$

$$[C] = \left[\hat{T}\right]^T \left[\hat{T} + \hat{T}_F\right]$$

The kinetic energy terms in the flexible equations become:

$$\left(A_{31}\ddot{q}_1 + A_{32}\ddot{q}_2 + A_{33}\ddot{q}_3\right)F_i + B_{31}G_i + B_{32}H_i + \sum_{j=7}^{n} M_{ij}\left(B_{33}q_j + C_{33}\ddot{q}_j\right)$$

where:

$$H_i = \int_{Body} y f_i dm$$

Now consider the case of a "steady state maneuver" where $\ddot{q}_1, \ddot{q}_2, \ddot{q}_3; \dot{q}_4, \dot{q}_5, \dot{q}_6$ are constants, but not necessarily zero. Then assuming simple harmonic motion for the flexible degrees of freedom produces:

$$\left(A_{31}\ddot{q}_1 + A_{32}\ddot{q}_2 + A_{33}\ddot{q}_3\right)F_i + B_{31}G_i + B_{32}H_i + \left(B_{33} - \omega^2 C_{33}\right)\sum_{j=7}^{n} M_{ij} q_j$$

In expanding the matrices $[B]$ and $[C]$ we see:

$$B_{33} = -\dot{q}_4^2\left(S_5^2 + S_6^2 C_5^2\right) - 2\dot{q}_4\dot{q}_5 C_5 S_6 C_6 + 2\dot{q}_4\dot{q}_6 S_5 - \dot{q}_5^2 C_6^2 - \dot{q}_6^2$$

$$C_{33} = 0$$

Just as in the case of the longitudinal motion of a bending beam, the above equation reveals that the <u>**effective forms of the generalized stiffnesses**</u> in the ultimate equations of motion are:

$$\boxed{K_{ij} - M_{ij}\begin{bmatrix} \dot{q}_6^2 - 2\dot{q}_6\dot{q}_4 S_5 + \dot{q}_5^2 C_6^2 \\ +2\dot{q}_5\dot{q}_4 S_6 C_6 C_5 + \dot{q}_4^2\left(S_5^2 + S_6^2 C_5^2\right) \end{bmatrix}}$$

If we use the right hand rule to visualize angular velocity and examine the components of the Euler angular velocity vectors, the bracketed expression is just the <u>**square of magnitude of the total angular velocity in the plane of the flexible surface**</u>.

4. Some Related Results

As an example, we present the results from reference [9]. Consider a two dimensional airfoil, with a mass distribution that is symmetric (fore and aft) about the midchord.

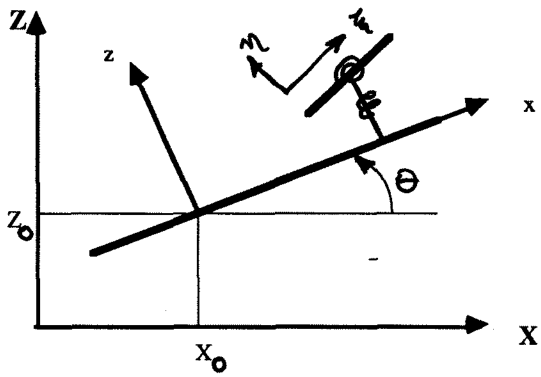

Figure 4. Idealized Airfoil on a Rigid Fuselage

The mass levels and bending and torsional stiffnesses were chosen to give "uncoupled" bending and torsion frequencies of 1 and 2 Hz, respectively, when the torsional axis was located at the midchord. These also gave divergence and flutter speeds that were in the range of validity of Theodorsen's theory for low speed unsteady aerodynamics. For the baseline case, the torsional axis was varied across the chord. For subsequent cases the pitch rate was imposed with values of 0, 2 and 4 rad/sec. As shown in the two previous sections, if the pitch rate has been allowed to approach 1 Hz, the effective bending stiffness could have been driven to zero.

In all that follows, "speed" refers to "equivalent airspeed" in meters/second; frequency is in Hertz and pitch rate $\dot{q}_3$ is in rad/sec. In the axis system chosen the trailing edge of the airfoil is at $\xi = 0$, and the leading edge is at $\xi = 1$.

4.1 Nominal Divergence and Flutter

Figure 5 presents the calculated divergence and flutter speeds vs the location of the torsional axis for $\dot{q}_3 = 0$. As expected the divergence speed is a minimum for slightly aft locations of the torsional axis and approaches infinity as the torsional axis approaches the center of pressure at the quarter chord. Flutter occurs for positions of the torsional axis between (about) 0.44 and 0.84.

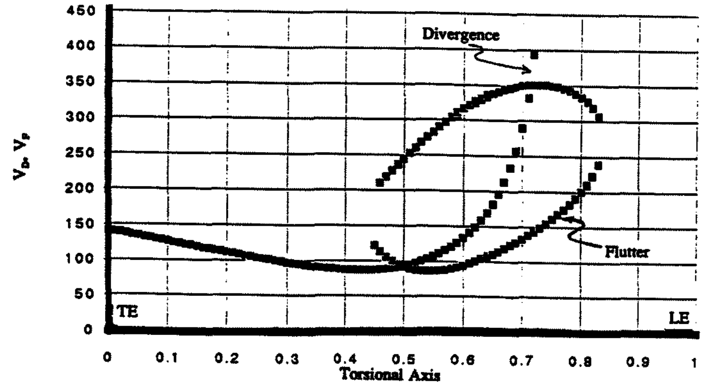

Figure 5. Divergence and Flutter for the Baseline

4.2 Pitch Rate Effects on Vibrations

Figure 6, shows the effects of pitch rate on the natural frequencies in bending and torsion vs location of the airfoil torsional axis.

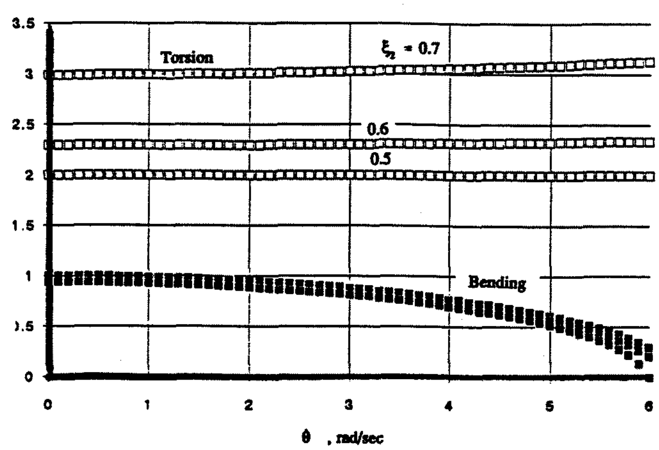

Figure 6. Pitch Rate Effects on Natural Frequencies

4.3 Pitch Rate Effects on Divergence

Figure 7 gives the effects of pitch rate on torsional divergence. In all cases, increasing pitch rate decreases the divergence speed slightly for aft torsional axes and is beneficial for torsional axes ahead of the midchord.. Recall that, in the axis system chosen, the trailing edge of the airfoil is at $\xi = 0$, and the leading edge is at $\xi = 1$.

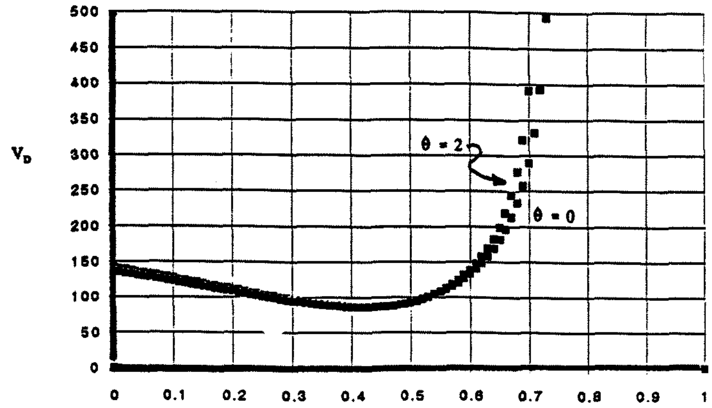

Figure 7. Pitch Rate Effects on Divergence

4.4 Pitch Rate Effects on Flutter

Figure 8 plots flutter speed vs torsional axis for the pitch rates of 0, 2 and 4 rad/sec. The effects are slightly beneficial for the flutter mode that appears at the lowest speeds and even more beneficial for the second flutter mode that appears at the higher speeds. This is certainly due to the insensitivity of the torsional mode to pitch rate and the fact that increasing pitch rate actually increases the separation between the bending and torsion frequencies.

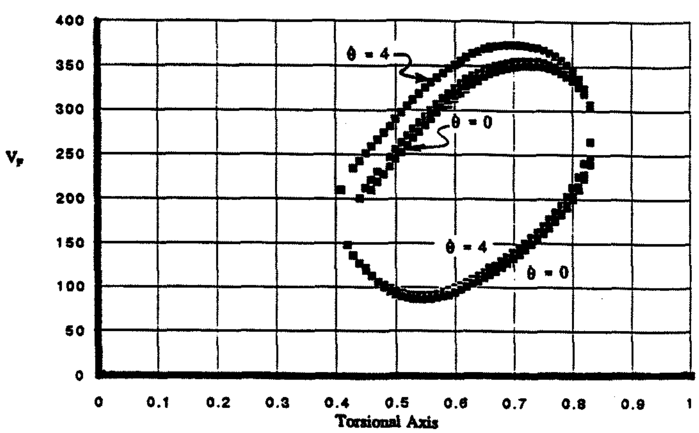

Figure 8. Pitch Rate Effects on Flutter

5. Conclusions

5.1 The equations of motion were developed, using energy methods and Lagrange's equations, for three cases: (a) a thin, flexible beam in longitudinal motion, (b) a thin, plate-like surface in general motion and (c) an idealized aircraft with a flexible wing, attached to a rigid fuselage in longitudinal motion. The analysis was limited to solutions of the vibration or aeroelastic eigenvalue problems for the flexible surfaces, with imposed rigid-body accelerations.

5.2 A significant simplification was (re ?) discovered in the application of Lagrange's equations. There now is no need to write out the tedious expressions for the kinetic energy and its various derivatives with respect to the generalized coordinates and their time derivatives.

5.3 Imposed pitch rates decreased the "effective" beam generalized stiffnesses in the beam bending problem.

5.4 Similarly, for the thin plate-like surface in general motion, the component of the total imposed angular velocity in the plane of the surface decreased the effective plate generalized stiffnesses.

5.5 For the idealized wing, increasing values of pitch rate had beneficial effects on torsional divergence for forward locations of the wing torsional axis and detrimental effects for aft locations. Increasing values of pitch rate increased the separation between the coupled bending and torsion frequencies, contributing to beneficial effects on flutter.

5.6 In advance of fully coupled solutions, the technique used here is an extension of the usual techniques used for flutter and divergence calculations and is very simple to incorporate in any aeroelastic analysis.

6. References

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