

# INTEGRATED STRUCTURE/CONTROL DESIGN OF COMPOSITE PLATE WITH PIEZOELECTRIC ACTUATORS

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## ABSTRACT

A compromise minimization of structural weight and quadratic performance index is conducted for vibration suppression of the composite plate with segmented piezoelectric actuators. The optimal placement of the piezoelectric actuators, ply orientation and thickness distribution of the laminated plate are determined by optimization technique. Analysis for the laminated composite plate with piezoelectric actuators is formulated and conducted by Rayleigh-Ritz method. The control objective is the suppression of structural vibrations by minimizing the control system performance index associated with the linear quadratic regulator optimal control problem. The integrated structure/control design is formulated as a constrained multiobjective optimization problem. The solution approach is referred to as the sequential linear programming method. Numerical example demonstrates the capability in determining the optimal ply orientation, thickness distribution of the layer and optimal locations of the piezoelectric actuators for vibration suppression.

## Introduction

Due to a recent efforts to reduce the structural weight in aerospace industry, the structures tend to have more flexibility and poor passive damping, which causes certain unfavorable phenomena. Thus, the design of the vibration control system for large flexible structures with various types of actuators is becoming increasingly important. The application of piezoelectric materials for actuation of flexible structures gives new dimensions to the control problem[1,2]. There has been a considerable amount of research activity employing piezoelectric sensor/actuators to control vibration in flexible structure. Since the piezoelectric materials exhibit elastic deformation in proportion to the magnitudes of an applied electric field and transfer forces to the structure, piezoelectric materials which are bonded at the proper location of the mother structure can be used as actuators.

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Analytic models for induced strain actuator coupled with beams and plates have been developed by many authors[3-7]. Experimental study on the control of large truss structure using piezoelectric material is conducted by Preumont, et al.[8]. Hanagud, Obal and Calise[9] developed the optimal vibration control algorithm about cantilever beam utilizing piezoelectric material as sensor and actuator. They derived the dynamic equation of linear elastic beam with the sensor/actuator using finite element method, and observed the control technique by rate feedback and modal feedback control. Recently, the static and dynamic aeroelastic behaviors of wing structure with piezoelectric actuators have been studied[10,11]. Hajela and Glowasky[12] conducted parameter study to suppress the supersonic panel flutter with piezoelectric actuators. They used optimization technique to find the best configuration of panels for both structural weight reduction and flutter speed maximization.

Because of the interaction between the control system and structure system, the simultaneous design for a flexible structure has recently received much attention. There have been several investigation according to the simultaneous optimal design. By tailoring the thickness of flexible beam and truss-beam, Belvin and Park[13] introduced the integrated structure/control design technique to increase dynamic performance and reduce the control effort. Devasia, et al.[14] applied the optimization technique to design a simple beam structure for vibration suppression. They found the location and size of piezoelectric actuators to maximize modal damping of the closed loop control system.

The objective of this paper is to present a simultaneous optimization to minimize the structural weight and the control system performance index for vibration control of the composite plate with constraints on the natural frequencies and the closed loop damping ratios. It describes investigations pertaining to the optimal placement of piezoelectric actuators on the composite structure for minimization of the quadratic performance index and structural weight. Analysis on laminated composite plate with piezoelectric actuators is conducted by Rayleigh-Ritz method. The integrated structure/control design is formulated as a constrained multiobjective optimization problem. The design variables are the ply orientation and thickness distribution of the composite layer and the locations of the piezoelectric actuators. The solution approach

adopted in this paper is referred to as the sequential linear programming method. Numerical results of the optimized system in the control performance are compared to those of initial design model.

### Modeling of Adaptive Structures

Forces and moments acting on the laminated composite plate containing segmented piezoelectric actuators are derived by the classical laminate plate theory. The stress-strain relations of a thin piezoelectric layer are

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix}_p = \begin{bmatrix} Q_{p11} & Q_{p12} & 0 \\ Q_{p12} & Q_{p22} & 0 \\ 0 & 0 & Q_{p66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} - \begin{Bmatrix} d_{31} \\ d_{32} \\ 0 \end{Bmatrix} \epsilon_3$$

or

$$\{\sigma\}_p = [Q_p] \{ \{\epsilon\} - \{\Lambda\} \} \quad (1)$$

$[Q_p]$  is stiffness matrix of piezoelectric layer,  $d_{ij}$  is the piezoelectric strain coefficient and  $\epsilon_3$  is the applied electric field. This equation is very similar to stress-strain equation with thermal effect considering the fact that piezoelectric strain  $\Lambda$  has the same form with thermal strain  $\{\alpha\}\Delta T$ . Inplane forces and moments of laminated plate including the loads of the piezoelectric actuators are obtained by integrating stresses over the ply thickness and expressed as follows[15,16]

$$\{N\} = \int \{\sigma\} dz = [A] \{\epsilon\} + [B] \{\kappa\} - \{N_\Lambda\} \quad (2)$$

$$\{M\} = \int \{\sigma\} z dz = [D] \{\epsilon\} + [D] \{\kappa\} - \{M_\Lambda\} \quad (3)$$

where  $\{\epsilon\}$ ,  $\{\kappa\}$  are the midplane strain and curvature, respectively.  $[A]$ ,  $[D]$ , and  $[B]$  are extension, bending, and extension/bending coupling stiffness matrices. These matrices are influenced by not only ply orientation of the laminate but also the locations of the piezoelectric actuators. Inplane forces and moments due to actuator strain are given by

$$\{N_\Lambda\} = \int_{z_p} [Q_p] \{\Lambda\} dz \quad (4.a)$$

$$\{M_\Lambda\} = \int_{z_p} [Q_p] \{\Lambda\} z dz \quad (4.b)$$

where  $z_p$  is the coordinate through thickness in the piezoelectric material,  $[Q_p]$  is the stiffness matrix of piezoelectric materials.

It is impossible to obtain the closed form solutions to the laminated plate due to the complexity of equation, arbitrary boundary conditions and external forces. In this study, Rayleigh-Ritz method which is rather faster without loss of accuracy than finite element method is adopted for the structural analysis. Strain energy of plate with piezoelectric material is expressed

$$U = \frac{1}{2} \int \int_{A, A_p} [\epsilon \ \kappa] \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} dx dy - \int \int_{A_p} [N_\Lambda \ M_\Lambda] \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} dx dy \quad (5)$$

and the kinetic energy is

$$T = \frac{1}{2} \int \int_{A, A_p} \rho (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dx dy \quad (6)$$

where  $A, A_p$  represent area of composite plate and piezoelectric actuators, respectively and  $u, v, w$  are displacements in  $x, y, z$  direction. In Eqs. (5) and (6), strain energy and kinetic energy include effects of not only the composite laminate but also the piezoelectric materials. Therefore, we must carefully apply different integral region as  $A$  or  $A_p$  and different variable values in accordance with the material.

To apply Rayleigh-Ritz method, we introduce displacement functions in generalized coordinate system to represent displacements  $u, v, w$

$$u(x, y, t) = \sum_{i=1}^l X_i(x, y) q_i(t) \quad (7.a)$$

$$v(x, y, t) = \sum_{j=l+1}^{l+m} Y_j(x, y) q_j(t) \quad (7.b)$$

$$w(x, y, t) = \sum_{k=l+m+1}^{l+m+n} Z_k(x, y) q_k(t) \quad (7.c)$$

where  $X_i(x, y)$ ,  $Y_j(x, y)$ ,  $Z_k(x, y)$  are shape functions which satisfy boundary conditions of the structures. Using these displacement expressions, equations for strain energy and kinetic energy are written in matrix form

$$U = \frac{1}{2} \{q\}^T [K] \{q\} - [Q_\Lambda] \{q\} \quad (8)$$

$$T = \frac{1}{2} \{\dot{q}\}^T [M] \{\dot{q}\} \quad (9)$$

For the vibration control of plate with piezoelectric actuators, a symmetric laminated plate model which has the surface bonded piezoelectric actuators on opposite side of plate at the same location is considered. It is also assumed that the same magnitude but opposite direction of the electric field is applied to the actuator so as to create a pure bending moment as shown in Fig.1(a). With these assumptions, the displacements in  $x$  and  $y$  directions can be neglected. In order to express the displacement in the analysis, we used the free-free beam vibration modes in the  $x$ -axis direction (chordwise direction) and cantilever beam vibration modes in the  $y$ -axis direction (spanwise direction) as follows.

$$\begin{aligned} Z_k(x, y) &= \Phi_i(x) \Psi_j(y) \quad (10) \\ \Phi_i(x) &: 1, \quad i = 1 \\ &: \sqrt{3} \left(1 - 2 \frac{x}{c}\right), \quad i = 2 \\ &: \left(\cos \alpha_i \frac{x}{c} + \cosh \alpha_i \frac{x}{c}\right) \\ &\quad - \beta_i \left(\sin \alpha_i \frac{x}{c} + \sinh \alpha_i \frac{x}{c}\right), \\ &\quad i = 3, 4, 5, \dots \\ \Psi_j(y) &: \left(-\cos \bar{\alpha}_j \frac{y}{l} + \cosh \bar{\alpha}_j \frac{y}{l}\right) \end{aligned}$$

$$\begin{aligned} & +\bar{\beta}_j(-\sin \bar{\alpha}_j \frac{y}{l} + \sinh \bar{\alpha}_j \frac{y}{l}), \\ & j = 1, 2, 3... \end{aligned}$$

Making use of the Lagrange's equation, the system equations of motion can be written in matrix form;

$$[M] \{\ddot{q}\} + [K] \{q\} = \{Q_\Lambda\} \quad (11)$$

The stiffness and mass matrices are affected by the structural characteristics of both the composite material and the piezoelectric material, and the control force  $\{Q_\Lambda\}$  is a function of placement and applied voltage of the piezoelectric material. Therefore, when the actuators placement vary, not only  $[M]$ ,  $[K]$  but also  $\{Q_\Lambda\}$  vary, and should be recalculated. Since the vibration modes and frequencies are dependent on the ply orientation and piezoelectric actuator placement, it is necessary to devise some strategy for vibration suppression.

For vibration suppression using the derived equations of motion, we design a control system with four sets of segmented piezoelectric actuators which are arbitrarily bonded on the composite plate as shown in Fig.1(b). In Eq.(11), if we express  $\{Q_\Lambda\}$  in terms of the applied voltage  $\{u\}$  and the others  $[F_p]$ , it becomes as follows

$$\begin{aligned} \{Q_\Lambda\} &= 2 \sum_{i=1}^{np} \frac{1}{t_p} \left\{ \left( \frac{T}{2} + t_p \right)^2 - \left( \frac{T}{2} \right)^2 \right\} \\ & \int \int_{A_{p_i}} \left\{ [Q_p] \begin{Bmatrix} d_{31} \\ d_{32} \\ 0 \end{Bmatrix} \right\}^T \{Z''\} dx dy V(i) \\ &= [F_p] \{u\} \end{aligned} \quad (12)$$

where

$$\{Z''\}^T = - \left[ \frac{\partial^2 Z(x, y)}{\partial x^2}, \frac{\partial^2 Z(x, y)}{\partial y^2}, 2 \frac{\partial^2 Z(x, y)}{\partial x \partial y} \right] \quad (13.a)$$

$$\{u\}^T = [V(1), V(2), V(3), V(4)] \quad (13.b)$$

$T, t_p$  are the thickness of the laminate and piezoelectric layer, respectively.  $np$  is the number of bonded piezoelectric materials,  $[F_p]$  is the control force per unit voltage, and  $V(i)$  is the applied voltage at the  $i$ -th piezoelectric actuator.

Substituting Eq.(12) into Eq.(11) and introducing the state vector  $\{x\}^T = [q \dot{q}]$ , Eq.(11) can be written as the following state equation

$$\begin{aligned} \{\dot{x}\} &= \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & [0] \end{bmatrix} \{x\} \\ &+ \begin{bmatrix} [0] \\ [M]^{-1}[F_p] \end{bmatrix} \{u\} \\ &= [F] \{x\} + [G] \{u\} \end{aligned} \quad (14)$$

### Control System Design

When designing the active controller, the objective is to suppress the vibration with the smallest control effort. The linear quadratic regulator(LQR) theory

with the assumption that all states are available for feedback is employed for the design of vibration control system in this study. The LQR theory determines the optimal control input  $\{u\}$  to minimize the performance index expressed as follows[17]

$$J = \int_0^\infty [\{x\}^T [Q] \{x\} + \{u\}^T [R] \{u\}] dt \quad (15)$$

subject to

$$\{\dot{x}\} = [F] \{x\} + [G] \{u\}$$

$[Q]$  and  $[R]$  are positive semidefinite and definite matrices. In order that the state is penalized heavily,  $[Q]$  and  $[R]$  are taken as  $100[I]$  and  $[I]$ , respectively.

The corresponding optimal control input is given by

$$\{u\} = -[K_G] \{x\} = -[R]^{-1} [G]^T [P] \{x\} \quad (16)$$

where  $[K_G]$  is optimal feedback gain matrix, which can be obtained from the Riccati matrix  $[P]$ . The Riccati matrix  $[P]$  is determined by the solution of the following Steady State Riccati Equation;

$$[P][F] + [F]^T [P] + [Q] - [P][G][R]^{-1} [G]^T [P] = 0 \quad (17)$$

Using Eqs. (14) and (16), the closed loop state equation can be written as

$$\{\dot{x}\} = [F_c] \{x\} = [[F] - [G][K_G]] \{x\} \quad (18)$$

It is well known[18] that the minimum performance index is

$$J_{min} = \{x_0\}^T [P] \{x_0\} \quad (19)$$

for a given specified initial condition  $\{x_0\}$ . We take all the initial components of state vector to be 1.0 in this study.

Using Eq.(15),(16), the control system design is carried out for the basic model. Fig.1(b) shows the plate model containing four sets of the segmented piezoelectric actuators bonded on the arbitrary locations. The actuator is sized 1.5 inch  $\times$  3.0 inch. The laminate has six layers,  $[90/\pm 45]_s$  and each layer has uniform thickness in chordwise and spanwise direction. The thicknesses of  $90^\circ$ ,  $\pm 45^\circ$  layers are 0.03 inch, 0.015 inch, respectively. Total 16 vibration modes, which are 4 modes for each  $x$  and  $y$  directions are considered for the analysis. It seems to be reasonable from the fact that including more modes did not affect so much the response accuracy based on the numerical results.

With the material properties of composite and piezoelectric material from Table 1, the open loop and closed loop responses are calculated as shown in Fig.2 and Fig.3, respectively. To get those figures, the initial disturbances are chosen arbitrary. Fig.3 shows the tip displacement and twist to the initial disturbance at the mid-chord ( $x = 3, y = 12$ ). Fig.4 shows the control input voltages applied to the piezoelectric actuators for the vibration suppression. As shown in Figs.3 and 4, the vibration control effect of piezoelectric material is fairly good. Since the purpose of current study is not only to find the optimal placement of the piezoelectric actuators but also to reduce the structural weight, integrated structure/control design is performed using optimization technique.

## Integrated Structure/Control Design

Using the equations presented in the preceding sections, integrated optimal design is conducted for the simultaneous optimization of structure and control. In the multiobjective optimization problems, there is no unique solution where all the single objective functions are optimum. Therefore, for most structure/control interaction problems, a trade-off between a structural objective and a control objective is needed. Usually, the concept of Pareto-optimum solution is used to find the solution of a multiobjective optimization problem.

The improved compromise multiobjective optimization by using reduction factor of objectives is applied in this study. In improved compromise multiobjective optimization method, the objective function is expressed as the following mathematical model[19].

$$\begin{aligned} & \text{Minimize } F(\vec{X}) \\ & = \left[ \sum_{p=1}^2 W_p^r \left| \frac{RF f_p(\vec{X})}{(f_{MAX})_p - (1 - RF) f_p(\vec{X})} \right|^r \right]^{\frac{1}{r}} \end{aligned} \quad (20)$$

subject to

$$g_j(\vec{X}) \leq 0, \quad j = 1, 2, \dots, m \quad (21)$$

$$\vec{X}_l \leq \vec{X} \leq \vec{X}_u \quad (22)$$

where  $\vec{X}$  is the design variables,  $g_j$  is the inequality constraints,  $m$  is the number of constraints,  $\vec{X}_l, \vec{X}_u$  are the lower, upper bounds of the design variables, respectively.  $RF$  is the reduction factor and  $W_p$  is the weighting factor for the  $p$ -th objective function  $f_p$ .  $f_{MAX}$  is taken as the initial objective value which is evaluated by given initial design variables.  $r$  is the geometric distance, which is set to 2 in this study.  $f_1$  and  $f_2$  are the structural and control objective function, respectively.

Structural objective function  $f_1$  is the weight of composite plate and expressed as follows,

$$f_1(\vec{X}) = \sum_{i=1}^n \int_x \int_y \bar{\rho} t_i(x, y) dx dy \quad (23)$$

where  $\bar{\rho}$  is density of composite plate,  $t_i(x, y)$  is thickness distribution of the  $i$ -th ply. The control objective function  $f_2$  is adopted from Eq.(19),

$$f_2(\vec{X}) = \{x_0\}^T [P] \{x_0\} \quad (24)$$

As the structure design variables, we adopt the ply angle  $\theta$  of  $[\theta/\pm 45]_s$  laminated plate and the thickness coefficients  $C_1, C_2, \dots, C_{10}$  of the following cubic equation which represents the thickness distribution of  $\theta$  layer

$$\begin{aligned} t_\theta(x, y) = & C_1 + C_2x + C_3y + C_4x^2 + C_5xy + C_6y^2 \\ & + C_7x^3 + C_8x^2y + C_9xy^2 + C_{10}y^3 \end{aligned} \quad (25)$$

The  $x$  and  $y$  coordinates of the four piezoelectric actuators  $pzx(i)$ ,  $pzy(i)$  as shown in Fig.1(b) are taken as the control design variables. The total 19 structure/control design variables are thus considered, and the total 21 constraints are imposed as the followings.

- 9 constraints on minimum ply thickness (minimum skin gage) ;

$$\begin{aligned} & t_\theta(x_i, y_j) \geq 0, \\ & x_i = 0, 3, 6, y_j = 0, 6, 12 \end{aligned} \quad (26)$$

The first 3 natural frequencies and damping ratios of the closed loop are considered as the constraints.

- 3 constraints on open loop natural frequencies ;

$$\omega_i \geq \omega_{0i}, \quad i = 1, 2, 3 \quad (27)$$

- 3 constraints on closed loop damping ratios ;

$$\xi_i = -\frac{\bar{\sigma}_i}{\sqrt{\bar{\sigma}_i^2 + \bar{\omega}_i^2}} \geq \xi_{0i}, \quad i = 1, 2, 3 \quad (28)$$

where  $\bar{\sigma}_i, \bar{\omega}_i$  are real and imaginary part of the closed loop eigenvalues.

- 6 constraints on placements of four piezoelectric actuators without overlapping ;

$$\begin{aligned} & px(i) - px(j) \geq 1.5 \\ & \text{or } pzy(i) - pzy(j) \geq 3.0, \\ & i, j = 1, 2, 3, 4 \end{aligned} \quad (29)$$

There are also side constraints on design variables for the actuators to be placed in the plate boundary.

Numerical solution to the nonlinear optimization problem is obtained by using the sequential linear programming method in ADS[20]. First order derivatives necessary in the optimization process for iterative direction search are internally calculated by finite difference.

## Numerical Results

As an example, the laminated plate which is composed of six symmetric layers shown in Fig.1(b) is investigated. Initial dimension of model and locations of piezoelectric actuators are shown in Fig.1(b). The initial ply angle and thickness are set to  $\theta = 90^\circ$  and 0.03 inch which is the uniform thickness distribution. For the initial values of design variables, structural weight and performance index are 0.525 lb and  $1.24 \times 10^5$ , respectively. Weighting factors  $W_p$  and reduction factor  $RF$  in the objective function are set to be 0.5, 0.8, respectively. Constraints on initial natural frequencies and closed loop damping ratios are reasonably set to be  $\omega_{0i} = 15, 90, 130Hz$ , and  $\xi_{0i} = 0.01, 0.01, 0.01$ .

Typical optimization results are listed in Table 2. Ply angle converges to  $73.5^\circ$  from the initial value of  $90^\circ$ . Natural frequencies  $\omega_i$  and closed loop damping ratios  $\xi_i$  at the initial design variables are  $\omega_i = 40.3, 104.1, 252.9Hz$  and  $\xi_i = 0.067, 0.032, 0.034$ , respectively. At the final designs obtained by the optimization algorithm, those values converged  $\omega_i = 23.8, 90.0, 130.0Hz$  and  $\xi_i = 0.136, 0.097, 0.102$ , where the constraints on  $\omega_2, \omega_3$  are active.

Fig.5 illustrates the iteration histories of structure weight, control performance index, and the integrated structure/control objective function. Optimized structural weight and control performance index are respectively 0.343 lb,  $3.74 \times 10^4$ , which show

the reduction of 35% and 70%, respectively. Objective function converges to the final value of 0.3232 from 0.7071 in 9 iterations.

Fig.6 shows placement history of the piezoelectric actuators during the optimization process. The actuator number 2 and 4 first approach together, and move to the inboard region. The actuator number 1 moves to the leading edge, the actuator number 3 approaches to the actuator number 1. Optimized thickness distribution of the  $\theta$  layer is shown in Fig.7. It is the thickest at the root, becomes thinner toward tip/trailing edge. The minimum skin gage constraint is active at the tip/trailing edge ( $x = 6.0, y = 12.0$ ).

In order to compare the suppression performance of optimized geometry to that of initial basic model, the response and the control input are plotted. The open loop and closed loop time history of the displacement and twist at tip to an initial disturbance are presented in Figs.8 and 9, respectively. The corresponding control input voltages applied to piezoelectric actuators for the vibration suppression are shown in Fig.10. Comparing to the responses of initial basic model in Figs.2 and 3, we can see the faster response time, substantial reduction in torsional mode, and much decrease in control input voltages applied to the piezoelectric actuators. In addition, there is substantial saving in the value of the performance index. The control performance index which represents the energy for the control has far less value of  $3.74 \times 10^4$  than that of basic model  $1.24 \times 10^5$ .

### Conclusions

For the efficient vibration suppression and weight reduction of the laminated composite plate containing piezoelectric actuators, an integrated structure/control design is performed considering the structure and control design variables. Linear Quadratic Regulator theory under the assumption that all state vectors are available for feedback is considered to find the optimum placements of piezoelectric actuators. From the numerical simulations, it is possible to determine both the structure design variables such as ply orientation and thickness distribution, and control design variables such as placements of piezoelectric actuators. There was the reduction of 35% in weight and 70% in control performance index for the example model used in this paper.

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Table 2. Optimization Results

	Initial Design		Optimization Results	
Ply Angle	90°		73.5°	
Thickness	3.0	0.0	1.1883	-0.3468
Coeff. ( $\times 10^{-2}$ )	0.0	0.0	-0.3659	0.3266
$C_1, C_2$	0.0	0.0	-0.2504	-0.2298
$C_3, C_4$	0.0	0.0	0.3380	-0.2349
.....	0.0	0.0	-0.1269	-0.2453
pzx(i)	0.5	0.5	0.20	0.82
	3.5	3.5	2.21	2.52
pzy(i)	1.0	7.0	1.29	5.49
	1.0	7.0	1.44	5.29
Weight	0.525		0.343	
Prf. Index	$1.24 \times 10^5$		$3.74 \times 10^4$	
Natural Freq.(Hz)	40.3		23.8	
	104.6		90.0	
	252.9		130.0	
Damping Ratio	0.067		0.136	
	0.032		0.097	
	0.034		0.102	

Table 1. Material properties

Composite Materials
$E_1 = 14.21 \times 10^6 \text{ psi}$
$E_2 = 1.146 \times 10^6 \text{ psi}$
$G_{12} = 0.8122 \times 10^6 \text{ psi}$
$\bar{\rho} = 0.05491 \text{ lb/in}^3$
$\nu_{12} = 0.28$
Piezoelectric Materials
$E_p = 9.137 \times 10^6 \text{ psi}$
$\rho_p = 0.28 \text{ lb/in}^3$
$\nu_p = 0.3$
$d_{31}, d_{32} = 6.5 \times 10^{-9} \text{ in/V}$

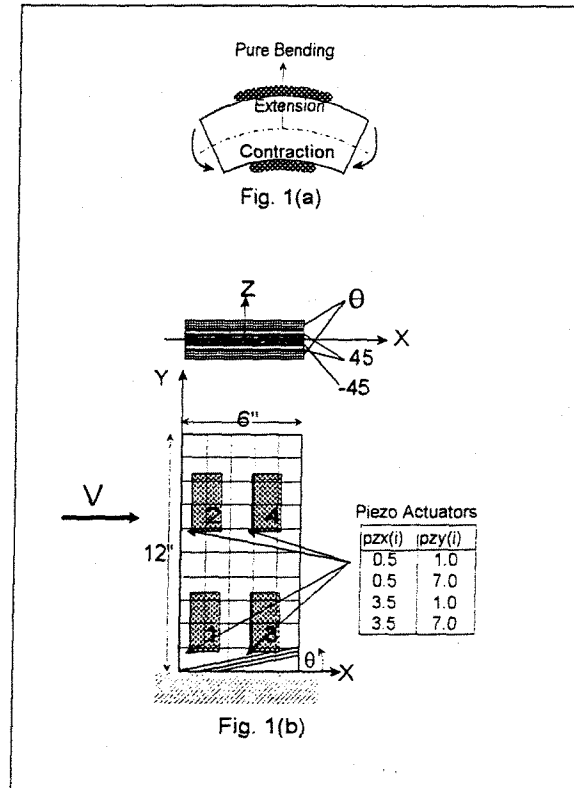


Fig.1(a). Pure bending due to piezoelectric actuators, (b). Laminated plate model with piezoelectric actuators.

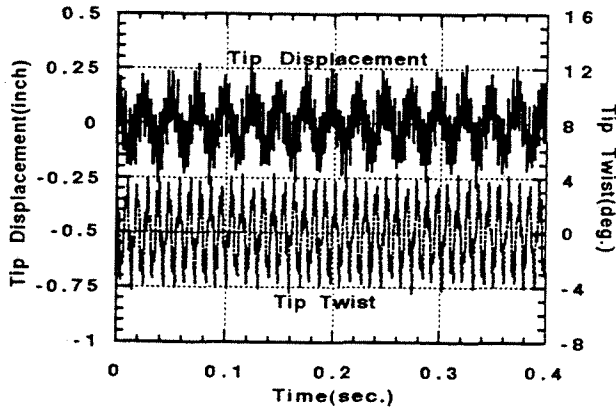


Fig.2. Open loop responses of the initial model.

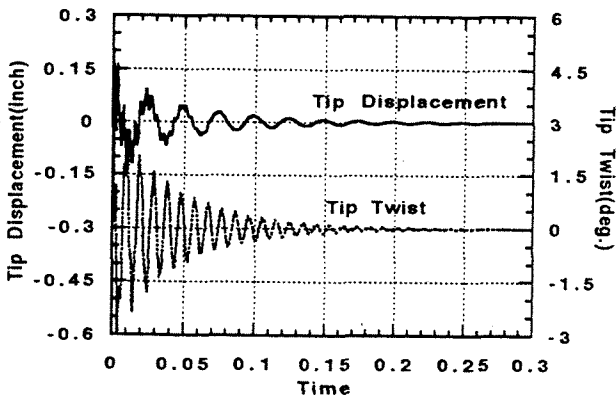


Fig.3. Closed loop responses of the initial model.

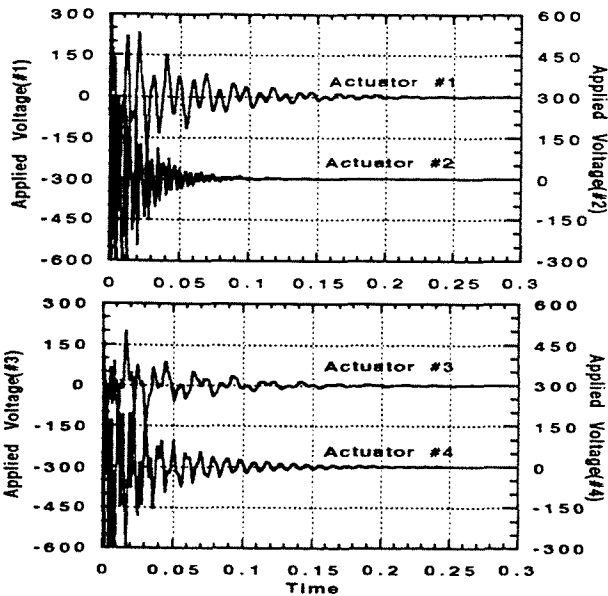


Fig.4. Control input voltages for piezoelectric actuators.

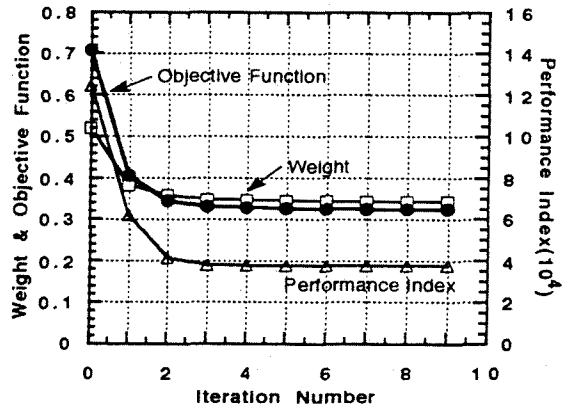


Fig.5. Iteration histories of the objective function, weight, and performance index.

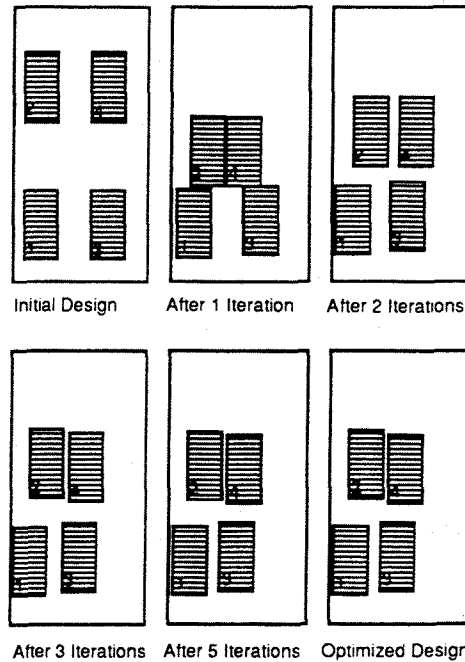


Fig.6. The changes in placements of the piezoelectric actuators during the optimization.

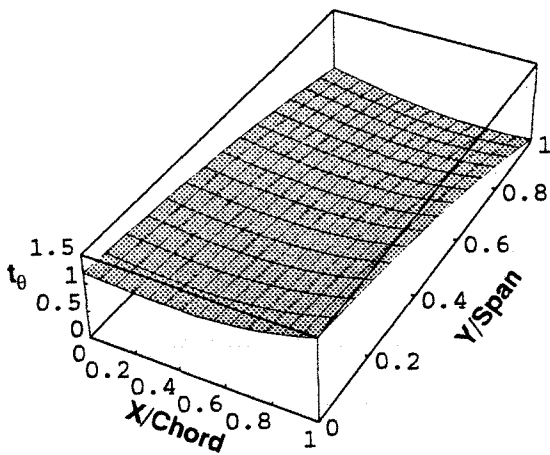


Fig.7. The optimized thickness distribution of the  $\theta$  layer.

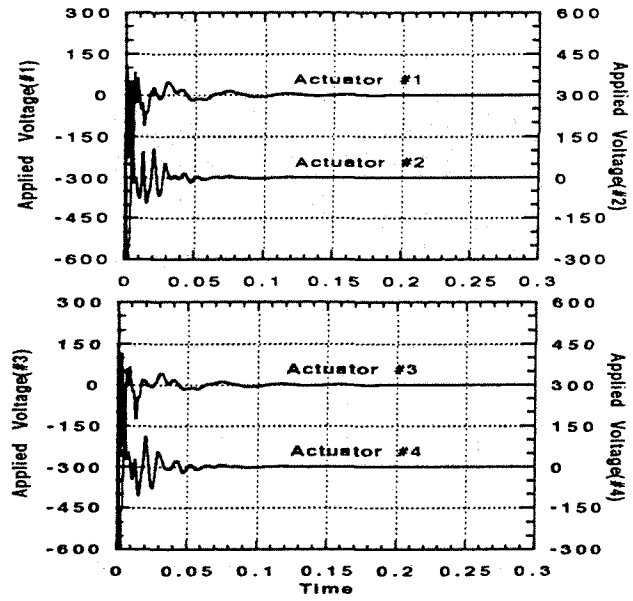


Fig.10. Control input voltages for piezoelectric actuators.

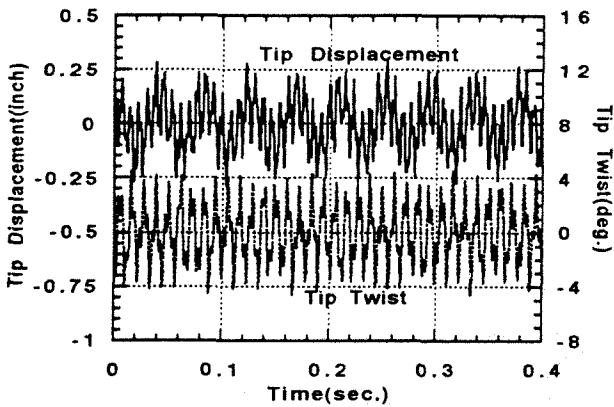


Fig.8. Open loop responses of the optimized model.

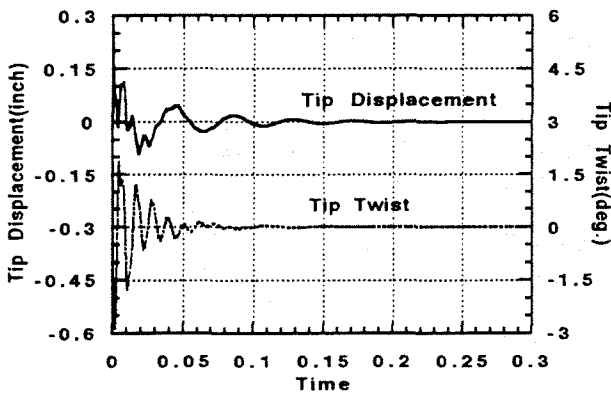


Fig.9. Closed loop responses of the optimized model.