ON THE SIGNIFICANCE OF PROBABILISTIC PARAMETERS FOR THE ASSESSMENT OF MSD IN THE CASE OF AGING AIRCRAFT

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Abstract

The Multiple Site Damage (MSD) problem is treated by means of a Monte-Carlo Simulation. The severity of a damage scenario as well as the probability of occurrence of MSD-like scenarios are assessed by this method. Different parameters are taken into account and are checked with regard to their influence on the likelihood of MSD scenarios and the inspection intervals which have to be taken into account.
The model itself is presented briefly and preliminary results are shown.

1. Introduction

In consideration of potential consequences of widespread fatigue damage it is important to assess the likelihood of its occurrence on a given airplane and the ability of the current maintenance system to discover the damage in a reliable manner. Therefore, the Airworthiness Assurance Working Group (AAWG) proposes an airplane evaluation process in the case of 11 aging aircraft types [1]. In order to make it possible to perform such an evaluation process, different tasks have to be treated. One of these aspects is presented in this paper.

Multiple Site Damage (MSD) is a key effect which may influence the structural integrity of aging aircraft. In order to assess this problem, three main questions have to be taken into account:

- how to calculate the actual fatigue and crack growth problem, if two or more fatigue critical sites are apparent
- what are the initial crack (flaw) patterns like, which may be regarded as "critical" and at which time are they to be taken into account
- what is the influence of discrete source damages, if multiple site damage is already present in a structure.

While the third point is not addressed in this paper, the first point is only briefly discussed. The main objective of this paper is the second point. Therefore, this paper presents a "Monte-Carlo"-Simulation of the entire MSD problem (fatigue and crack growth), which is based on the following idea:

- Starting with a randomized fatigue distribution, a deterministic calculation of the subsequent fatigue accumulation process and crack growth is performed. Obviously, it is important that the chosen computational method requires little computer time. This is achieved either using the compounding method or by means of simple numerical methods. Different parameters, e.g. the probability of detection, will also influence the MSD problem and may therefore be included in the simulation process.

- This procedure is performed many times, using different initial damage configurations and possibly different other parameters.

- It is possible to treat the results of all different configurations statistically, which reveal a significant performance of the MSD problem with regard to the scatter of the fatigue data and mainly the probability of detection. Other parameters only exhibit smaller effects.

Obviously, synergetic effects can be shown, which rule MSD, especially if loaded holes are taken into account. Some examples of theoretical calculations are presented, completed by the comparison of the Theory with some experimental results.

2. The Monte-Carlo Simulation Model

Multiple Site Damage (MSD) is an effect which has been realized only recently, it therefore has been accepted by some authors to be a strange phenomenon of a completely new quality. In the eyes of the authors of this paper this is not the case. Hence, the MSD case is treated in this paper as an ordinary crack initiation and crack propagation problem for each individual fatigue critical location. It only requires a special treatment, if cracks are likely to influence each other mutually. It will turn out that this approach results in the problem to find a strict definition of MSD.

2.1 Calculation of the Damage Scenario

The actual model of crack initiation is demonstrated by means of the sketch in figure 1. It actually shows the most simple two–rivet–row lap joint without doubler. If the rivets are not countersunk, those rivet rows of each sheet are fatigue critical, where a high by-pass stress level is combined with a high load transfer by the rivets. This row is indicated for the upper sheet by small cracks.

In the proposed model it is assumed that for a given stress level fatigue test results are known, which reflect the mean value of the fatigue life as well as the scatter of the fatigue life (e.g. in terms of a standard deviation) for a single rivet pitch. The fatigue life should be defined up to a relatively small crack size (e.g. 1 mm). It is obvious that, due to different reasons it is not easy to achieve these results, but they are those results which normally are needed for a Wöhler–curve, and in the ideal case would reflect all manufacturing and design effects on fatigue of the item.

If the mean value of the fatigue life and the scatter factor for a single rivet pitch are known, it is possible to use a random process in order to derive a damage scenario for a complete frame–bay or even a complete lap joint. Provided the fatigue life is distributed according to a log–normal distribution, this may be done by means of an ordinary random processor which provides a smooth distribution of random numbers in the interval [0,1] (see e.g. Press et al. [2]), equivalent to 0% – 100%, and by inversion of the log–normal distribution by an approximate method (an analytical inversion is not feasible). This situation is indicated in figure 2.

After the damages of all fatigue critical locations have been determined, it is assumed that the fatigue damage at the most damaged location is high enough to start crack growth, while all other locations still have to accumulate more fatigue damage until the crack starts. The fatigue damage rate of each location "i" is calculated by equation (1)

$$dD_i = \frac{N_i}{N_{max}} \qquad (1)$$

Special attention has to be paid to the interaction of cracks and holes or one–sided cracks with regard to the fatigue problem at the uncracked hole. This has either been done by using an ellipse – instead of a hole with one crack – in order to calculate the α_k – value or for the interaction by using a compounding method like procedure.

2.2 Crack Propagation Calculation

Generally the crack propagation has been calculated by means of the SIF (stress intensity factor) K_I and the Forman equation (2) [3].

$$\frac{da}{dN} = \frac{c_f \, \Delta K^{n_f}}{(1 - R)\, K_{1c} - \Delta K} \qquad (2)$$

The Forman parameters for the aluminum 2024 T3 alloy have been used as follows within this paper :

- $c_f = 2.01 \times 10^{-8}$
- $n_f = 2.70$
- $K_{1c} = 2256 \text{ MPa} / \text{mm}^{-1/2}$.

This leaves to assess the appropriate SIFs for different crack scenarios, which may or may not include a mutual interaction of the cracks or other boundaries. The model does not include up to now any special treatment of short cracks, although it starts crack propagation calculation at considerably short crack sizes.

Three different methods of SIF detection are provided in the model:

- the compounding method [4]
- a numerical method based on complex stress functions [5]
- the finite element method.

Since one of the main requirements for the method used within a Monte–Carlo Simulation is its low computer time consumption, the compounding method is the most promising. All results,which are quoted in this paper are based on this method. Only some facts, e.g. regarding the redistribution of pin–loads after cracking, are based on one of the others.

The basic idea of the compounding method is very simple. A set of solutions for the determination of the SIF is known. Many of these solution comprise the interaction of one crack with some kind of boundary, where a boundary may be a hole, a straight line, an other crack , etc.. The mutual interaction of cracks with a number of boundaries is now achieved by a certain procedure of summation or product of the influences of all boundaries. Within this model the general method of Rooke et al. [4] has been adopted, where the SIF is calculated by

$$K_r = K_0 + \sum_{n \neq 0} (K_n' - K_0) + K_e \qquad (3)$$

with K_r as the actual SIF including all interactions, K_0 indicating the SIF without interaction with other boundaries; K_n' is the SIF according to the interaction with one single boundary "n" and K_e is the most problematic parameter, since it comprises the influence of all boundaries together. This last term is not very easy to obtain. Based on some assumptions of Rooke et al. [6] this term is not essential for normal rivet row pitches.

Special attention has to be paid to the case of cracks emanating from holes or intersecting other boundaries. This procedure is very clearly presented in the ESDU data sheet [7]. For pin–loaded fasteners some further assumptions have to be made which are also given in [4]. The required stress intensity factors for the simple basic configurations mainly have been taken from Rooke and Cartwright [8], Tada, Paris and Irwin [9] and some single publications. One of the most valuable has been the report on two unequal cracks emanating from a hole by Rooke and Tweed [10]. It is not the intention of this paper to repeat the theory of the compounding method. If any information on this subject is needed, please refer to the mentioned papers.

Since the compounding method is based on already known solutions, which may to a large extent be stored in data arrays and only have to be interpolated, this method requires only very little computational time. On the other hand there are also certain shortcomings, e.g. the fact that only a constant remote stress may be treated by this method. But it is shown in the subsequent sections that this is not such an important feature.

2.3 The Residual Strength Problem

Up to now the residual strength problem has been used in this model mainly for the detection of the link–up of relatively small cracks. As stated in section 3.2 the case of the link–up of one large lead crack with additional small MSD cracks is not essential in the application discussed here. The interaction of MSD cracks with one large accidental crack has not been treated, since it is the feeling of the authors that this is a very special problem – likely to occur only in relatively small areas of the aircraft–, which should not disturb the understanding of the initial problem of the initiation of MSD scenarios.

Therefore, only the link–up problem of small cracks remains, which may be treated in different ways. One of the very simple models for this problem seems to be the model proposed by Swift [11]. It actually checks a criterion which is very similar to a "net section yielding" criterion.

The model is visualized in figure 3. The main idea is to use the radius of the plastic zone in front of each crack tip

$$r_p = \frac{K_I^2}{\pi\ \sigma_y^2} \qquad (4)$$

and to apply the contact of both plastic zones as criterion for the link–up of the cracks. This is in line with the argument of Irwin [12] concerning the effective length of the crack. From equation (4) it is obvious that this criterion depends very much on the yield stress σ_y. Within the present model the stress intensity factor K_I has been taken from the compounding method instead of the equations given by Swift in ref. (11).

This model is extremely simple and is therefore easy to be used in the Monte–Carlo Simulation. After all the results of Swifts method are not too bad, if flat specimens are considered. This has been shown by Moukawsher [13]. Actually, the results are even better, if the plastic radius has been derived by means of the compounding method. On the other hand there are some doubts, whether this criterion still holds, if specimens are used, which also exhibit bending.

Anyway, the model does not have a major influence on the calculated inspection interval, since the number of loading cycles with highly interacting cracks is low for each possible configuration. The real residual strength calculation, which requires the limit load, is only needed in this model, when a complete large panel is assessed. This is not the subject of this paper.

2.4 Comparison with Existing Experimental Data

In order to show that the general method for the calculation of fatigue and crack growth works properly, the results of one larger coupon test has been calculated by means of the model.

The results of the test and the calculation are shown in figure 4. The test specimen consisted of the aluminum alloy 2024 T3. One single crack emanated from the central hole. The crack growth curves have been calculated using standard crack growth and fatigue data.

Two major points may be derived from figure 4. First, the crack propagation is calculated in a sufficient manner (actually, other specimens have been calculated much more exactly also in the MSD case). Second, and this is the more important point, the number of cycles for the initiation of cracks on the uncracked side of a hole, where the crack approached from the other side, is very good.

3. Numerical Experiments

3.1 The New Concept for the Definition of the Inspection Interval

As already indicated in section 2, the definition of the inspection interval is altered completely by the present model. The normal way to define the inspection interval in case of a single crack is shown in figure 5. The inspection interval is a fixed value defined as the difference of load cycles between critical crack length and detectable crack length. Both values do not depend on anything else but the crack length of the single crack itself.

Now, for the MSD case the problem is completely opposite. Cracks may initiate at different locations at different times. Therefore, very different configurations may occur, which result in a very different crack propagation behaviour. This means that the inspection interval is not fixed at all, and it even becomes apparent that both of the values are linked together. This is mainly discussed in section 4.

What actually does remain is that the results of the Monte–Carlo Simulation may only be treated statistically. This is to say that the inspection interval is also a statistical value. MSD has not to be defined for an interpretation of the results, it is just a part of the possible configurations that may occur.

Actually, MSD–like configurations are those which result in a relatively short inspection interval. On the other hand it will follow that the "worst case of equal cracks at all rivet holes" is not only very unlikely to occur; it also is shown that it may only occur after a very long "crack free life". This "crack free life" is called "threshold" in the following.

3.2 Important Features of Multiple Site Damage

3.2.1 Performance

If more than one single crack appears at adjacent rivet holes two major features are responsible for the fact that multiple site damage is a very serious phenomenon [14]:

- in MSD–like scenarios crack sizes tend to be of very similar size in a certain region of nearly equal remote stress.

- after reaching a considerable crack size, the cracks start to influence their crack growth mutually.

The first of these facts may only be explained by the fact that for loaded rivet holes the crack growth rate of small cracks is higher than for larger cracks, or for adjacent holes

$$\frac{da_{small}}{da_{large}} > 1 \qquad (5)$$

This kind of mechanism is also reflected in the MSDS parameter of de Koning [15]. If larger cracks additionally are spread over a considerable region, a redistribution of the load transfer at the rivets will also occur, and result in a higher loading of the outer, normally less loaded rivet holes. Some finite element calculations, which have been performed in order to assess the effect of small cracks on the load–transfer redistribution, have shown that this is not a crucial point, as long as the cracks do not exceed a length of some millimeters. Only a small amount of the load–transfer is distributed between the adjacent rivets, both parallel and perpendicular to the rivet row direction.

In longitudinal lap joints (and this is the main subject of the following sections) a nearly quadratic stress distribution is found within one frame–bay. The maximum of this distribution is located in the center of the frame–bay. This stress distribution results in the fact that approximately 8 to 10 rivets are loaded in such a manner that the fatigue life is very similar at these rivets, i.e. MSD is likely to occur at these rivets, if it occurs at all [5]. Therefore, it is sufficient in the first step to look at approximately 9 rivet–spacings in a Monte–Carlo Simulation. Larger models would be necessary, if accidental damages are taken into account. A large effect of the damage scenario of one frame–bay on the other is only essential shortly before the crack scenario is critical in one of the frame–bays. The few load cycles which will occur in this state are not essential for the overall result of the following studies and are therefore neglected in the following calculations.

3.2.2 What does the result of a Monte–Carlo Simulation look like?

A typical result of a Monte–Carlo Simulation is shown in figure 6. The inspection interval (without safety factor) has been collected for 125 different configurations of a simple two–rivet row lap joint. The results are plotted versus the probability of occurrence. Furthermore, a log–normal distribution has been calculated from these results. Obviously, the log–normal distribution fits the results very well.

The main point that may be derived from this diagram is the fact that the inspection interval is not a constant value at all. The question is, why the results for the interval are so different? The answer is given in figures 7 and 8. They show the crack propagation in the "worst" and the "best" case, i.e. it is the configuration of the far left and far right result in figure 6.

In figure 7 and 8 the x–position of the rivet–holes in the rivet row as well as the position of the crack tip are plotted on the abscissa. On the ordinate the cycles are plotted. Dashed lines indicate that the rivet–hole remains intact and only damage accumulation takes place, while a solid line indicates that crack propagation has started.

It gets apparent that the "worst case" in figure 7 is a configuration where cracks start at almost all rivets at the same time; this is nearly the classical MSD case. In the "best" case of figure 8, one single crack starts to grow up to quite a number of cycles before an other crack starts to grow; this is almost the classical damage tolerance configuration with one fatigue critical location.

When looking at a Monte–Carlo Simulation the question occurs, how many different damage configurations have to be taken into account. Figure 9 shows for one example the calculated interval (for three small probabilities of occurrence) as they are calculated for different numbers of configurations. Obviously, the values vary in the beginning, but already after some hundred configurations the differences are not large anymore, compared to the uncertainties of the calculation model itself. Therefore, 250 different configurations have been used in the following.

3.3 Significance of Parameters

It may be assumed that different parameters are likely to influence the inspection interval in the case of more than one fatigue critical location. In this study the following parameters have been tested by means of the Monte–Carlo Simulation described above:

Parameter	Average	Scatter	Symbol
fatigue life	X	X	FAT
detection	X	X	POD
position		X	POS
crack growth		X	CGR
friction		X	LTR

where detection stands for the probability of detection, position for the position of the rivet in the rivet row, crack growth for a scatter in the crack growth data of the material and friction for the scatter in the load–transfer due to friction between the sheets of the lap joint. While the fatigue life has been assumed to follow a log–normal distribution, all other values have been approximated by a normal–distribution.

All of the subsequent examples have been calculated for a three–rivet row example with 20 mm pitch, 1.6 mm sheet thickness, 4 mm rivets and a remote stress of 84 MPa. All of the figures 10 to 15 include one reference calculation with a fatigue life of log N = 5.057, a standard deviation of 0.058 and a detectable crack length of 5.0 mm. Nine fasteners have been used as basic configuration.

The first parameter discussed in figure 10 is the scatter of the fatigue data. It shows that an increase of the scatter increases the interval considerably. The interpretation of this result is simple: the probability that at adjacent fasteners cracks occur at nearly the same load cycle decreases with increasing scatter; the result is discussed in figures 7 and 8.

The next parameter, discussed in figures 11 and 12 is the average value of the fatigue life. It is obvious that this parameter influences the threshold considerably, but the influence on the interval is only small. This small influence results from the fact that the scatter is defined on a logarithmic scale and is therefore larger for the higher average values, if it is compared on a normal decimal scale.

Figure 13 presents the influence of the average value of the probability of detection. It is quite obvious that the inspection interval increases with a refined inspection method. Therefore, this result is not at all astonishing.

If now a scatter in the probability of detection is discussed, it turns out that obviously positive and negative influences balance each other. Hence, there is nearly no change in the curves shown in figure 14.

All of the further parameters given in the table have the same small effect on the inspection interval as the scatter of the probability of detection has. Since it is not appropriate to repeat three more figures without an essential new information, only the combination of all of these influences is shown in figure 15. The following scatter values have been used exceeding the basic model mentioned above :

- σ (POD) 1.0 mm
- σ (POS) 1.0 mm
- σ (LTR) 50 N
- σ (CGR) $c_f = 0.522 \times 10^{-8}$

These scatter values seem to be of reasonable magnitude. It therefore seems to be justified to say that the main influences on the inspection interval and the threshold are the mean value and scatter of the fatigue life and the average value of the probability of detection.

4. Example

It does not seem to be easy to find test results, which may serve as a representative example for the Monte–Carlo Simulation presented above. Also the following example may not be considered to be perfect, but it provides at least a strong hint that the general approach of this paper is right.

The main obstacle for representative full scale tests is the fact that a well designed lap joint is not likely to exhibit MSD during a reasonable number of flight cycles. Since no company will pay for a full scale test with hundreds of thousands of flight cycles, other measures have to be taken. During a full scale test at Deutsche Aerospace Airbus in Hamburg one lap joint has been manufactured in a way, which was likely to enforce Multiple Site Damage. This was done by means of rivet holes which have been countersink too deeply.

A deep countersunk rivet hole, which results in a non–protruding rivet head, will show a relatively bad fatigue life behaviour and a reduced scatter of the fatigue life. It is therefore much more likely that MSD will occur in such a lap joint after a relatively short initiation period.

From coupon test specimens of the same lap joint design, the drop in the fatigue life can easily be derived for the appropriate stress level:

- in the case of a normal rivet hole with protruding rivet head:
  - $N_{10\%}$ = 1,171,039 cycles
  - $N_{50\%}$ = 535,268 cycles
  - $N_{90\%}$ = 233,653 cycles

- in the case of a deep countersunk rivet hole with non–protruding rivet head:
  - $N_{10\%}$ = 132,695 cycles
  - $N_{50\%}$ = 114,062 cycles
  - $N_{90\%}$ = 94,250 cycles.

It must be stated that the coupon tests have not been performed in a way which delivered the number of cycles up to a small crack length. Actually, the number of cycles up to a failure of the complete small coupon has been tested. Therefore, the interval between crack initiation and failure of the complete coupon test specimen have also been assessed by means of a Monte–Carlo Simulation. This leads to a decrease of 31,332 cycles for the $N_{50\%}$ value of the deep countersunk specimen.

The general design of the lap joint specimens is shown in figure 16.

The results of the Monte–Carlo Simulation are shown in figure 17. A load–transfer of 37% at the critical rivet row and a detectable crack length of 5 mm have been assumed. The remote stress level (this includes a "coupon → full scale correction term") of 84 MPa were used as remote stress level. Figure 17 shows two distributions: first, the case of the normal protruding head riveting and second the case of the non–protruding head riveting. On the abscissa the "life up to the detectable crack length" is plotted versus the inspection interval. Both values do not include any safety factor. Please note that the results for the calculated interval must be considered as conservative, since the high "coupon–full scale correction term" has also been used for the crack propagation calculation.

A simulation using 250 different damage configurations has been performed in both cases. Symbols indicate the position of the results for each single calculation. Additionally, the distribution of the results has been treated by statistical means, indicating which percentage of points is located within a certain region.

The following points may be derived from figure 17:

- there is an obvious relation between the number of cycles up to detectable crack length and the interval in both cases (see section 3.1).

- the results of both of the types of lap joints are extremely different. While the badly manufactured lap joint exhibits an early initiation of cracks, combined with a high probability of an MSD–like crack scenario (i.e. the inspection interval is small), the properly manufactured lap joint shows a much lower probability that a MSD–like scenario occurs. And if this scenario is likely to occur, it will occur much later in the service life, which is not likely to be reached by the aircraft anyway.

The data given in figure 17 still are completely theoretical. If they are compared with data from the full scale fatigue test mentioned above, the problem occurs that no crack initiation has been found in the properly manufactured lap joints of the test specimen within 120,000 flight cycles. Therefore, no validation of the Monte–Carlo Simulation of this problem is given, except from the fact that also the Monte–Carlo Simulation provides no damage in this case.

If only the deep countersunk lap joint is considered, this is the region indicated by a dotted line in figure 17, and is plotted in greater detail in figure 18. Additionally to the information of figure 17, results from the above mentioned full scale test are indicated in figure 18. A solid line indicates the region of intervals and fatigue life up to the initiation of a 5 mm crack which has been covered by the test itself, i.e. all points on the upper right–hand side of the solid line were not in the range of the test.

Within the deep countersunk lap joint of the full scale test crack initiation has been found in different frame bays up to different degrees and often with cracks at adjacent rivets. On the other hand, only one of the crack scenarios has nearly reached a critical state after 120,000 flights, while all the others still were not at all critical. The development of the cracks has been monitored by means of eddy current techniques from the beginning. Hence, a good database of initiation up to detectable crack length and interval data exists.

The single finding, where one frame–bay nearly became critical is indicated by a filled dot in figure 18, while for different further frame–bays an arrow indicates the initiation life up to a 5 mm crack and the interval, which was reached at the time when the full scale test has been stopped. The arrow also indicates the region where the real point would have been situated, if the test would not have been stopped at 120,000 flight cycles.

A comparison of the theoretical results as well as the experimental results shows that the one critical scenario was not very probable. But, all measured points from the full scale test indicate that the results of the Monte–Carlo Simulation are very likely to occur. This means that, although the model itself certainly has its shortcomings, it is already able to assess the likelihood of widespread fatigue damage in a considerable way. Furthermore, it supports the thesis that threshold and interval are not fixed values, if more than one fatigue critical location is taken into account.

5. Conclusion

A completely new approach for the assessment of Multiple Site Damage (MSD) has been presented. While, in the case of a single crack the inspection interval is one fixed value, which is related to one fixed threshold value, this is not the case for MSD. Obviously both, the interval and threshold depend on each other and are not fixed anymore.

This thesis is supported by means of a Monte–Carlo Simulation model, which has been tested by comparison with one existing result from a full scale fatigue test. The results are already very good, although the model still is in a preliminary state and needs some refinement.

The main parameters which influencing both the threshold and the interval are : the average crack initiation life, the scatter of the crack initiation life and the detectable crack length. This shows that the design (and the applied stress) as well as the quality of manufacturing, which both influence the fatigue life, are essential for the likelihood of MSD–like scenarios. It therefore is not necessary to anticipate MSD, if both the design and the production of the item are good, and no deteriorating influences such as corrosion occur.

Furthermore, some typical phenomena of MSD are discussed and a way is presented to limit the computational effort needed for the Monte–Carlo Simulation by reducing the number of fatigue critical locations.

The work on this method will be continued in order to refine and extend the model.

6. References

[1] –,"Structural Fatigue Evaluation for Aging Airplanes", Airworthiness Assurance Working Group (AAWG) Final Report, October 1993

[2] Press, W.H.; Flannery, B.P.; Teukolsky, S.A.; Vetterling, W.T.: "Numerical Recipes – The Art of Scientific Computing", Cambridge Univesity Press, 1988

[3] Forman, R.G.; Kearney, V.E.; Engle, R.M., "Numerical Analysis of Crack Propagation in a Cyclic–Loaded Structure", *ASME Trans. J.Basic Eng. 89D*, 1967, p. 459

[4] Rooke, D.P., "Stress Intensity Factors for Cracks at Fastener Holes in a Row of Fastener Holes", Royal Aircraft Establishment, Techn. Report 81144, 1981

[5] Horst, P.; Schmidt, H.–J.; Bauch, J.; "Grundlegende Untersuchung zum Einfluß der statistischen Ermüdungsverteilung bei "Multiple Site Damage"", in: 'Proceedings of the "Deutscher Luft– und Raumfahrt–Kongress 1992"', Bremen, 29th September to 2nd October 1992

[6] Rooke, D.P., "Stress Intensity Factors for Cracked Holes in the Presence of Other Boundaries", in: "Fracture Mechanics in Engineering Practice", ed. P. Stanley, London Applied Science Publishers, 1977, pp. 149–163

[7] "The Compounding Method of Estimating Stress Intensity Factors for Cracks in Complex Configurations Using Solutions from Simple Configurations", ESDU Item Number 78036, 1978

[8] Rooke, D.P.; Cartwright, D.J., "Compendium of Stress Intensity Factors", HSMO, 1976

[9] Tada, H.; Paris, P.; Irwin, G., "The Stress Analysis Handbook", Del Research Corp., 1973

[10] Rooke, D.P.; Tweed, J., "Opening–Mode Stress Intensity Factors for Two Unequal Cracks at a Hole", RAE Technical Report 79105, 1979

[11] Swift, T., "Damage Tolerance Capacity", in: "Fatigue of Aircraft Materials–Proceedings of the Specialists' Conference, dedicated to the 65th birthday of J. Schijve', eds.: Beukers, A.; de Jong, Th.; Sinke, J.; Vlot, A.; Vogelesang, L.B., Delft University Press, 1992, pp. 351–387

[12] Irwin, G.R., "Fracture", in: "Handbuch der Physik Vol. VI", ed. Flügge, Springer, 1958, pp. 551–590

[13] Moukawsher, E.J., "Fatigue and Residual Strength of Panels with Multiple Site Damage", MSc–Thesis, Purdue University, 1993

[14] Schmidt, H.–J.; Horst, P.,"Industry Research & Development Needs Regarding Inspection and Maintenance Technologies for Aging Aircraft", in: "Proceedings of the 5th Int. Conf. on Structural Airworthiness of New and Aging Aircraft", Hamburg, Germany, 1993

[15] de Koning, A.U.,"A Parameter Characterizing Structural Sensitivity for Multiple Site Damage (MSD)", NLR CR 92094 L, 1992

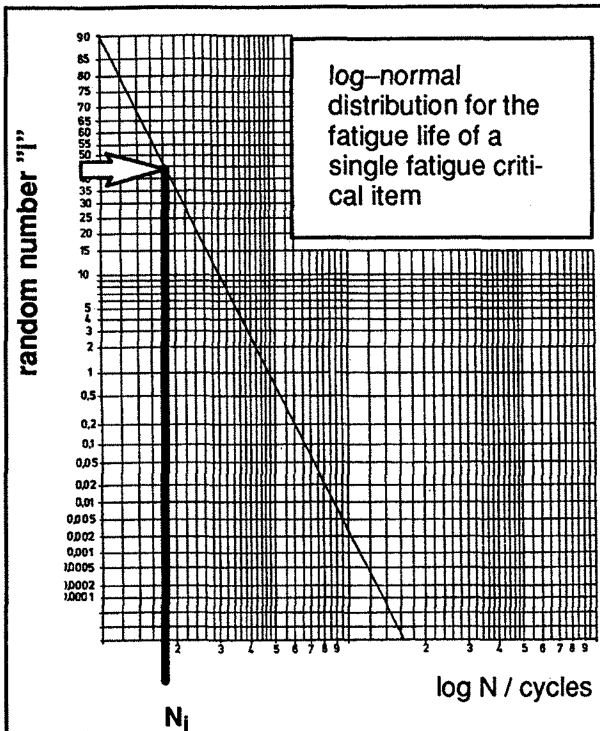

Figure 2 : Determination of the Fatigue Damage by a Random Process

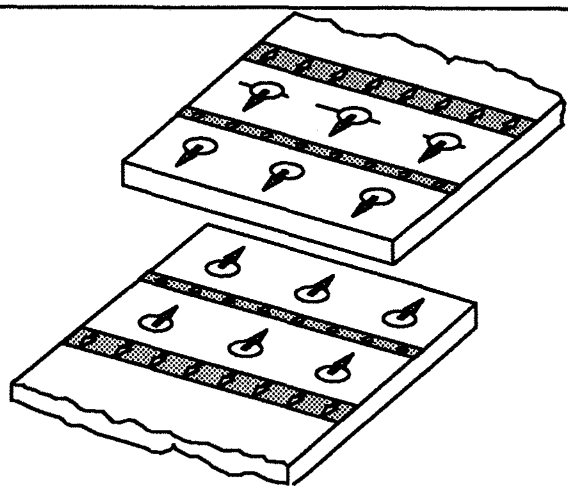

Figure 1: Typical Lap Joint Design (Generic)

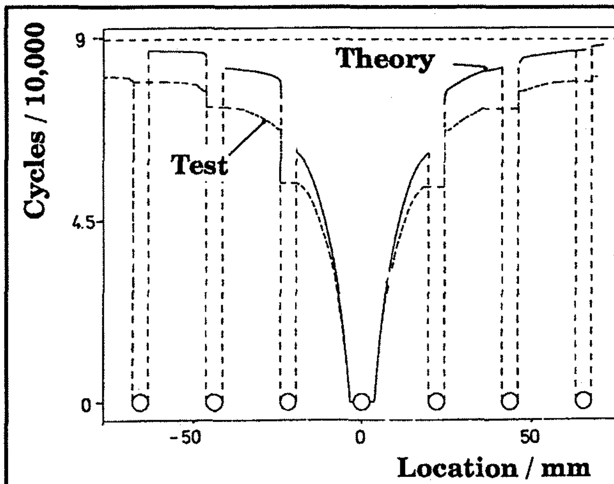

Figure 4: Results of a Crack Growth Test

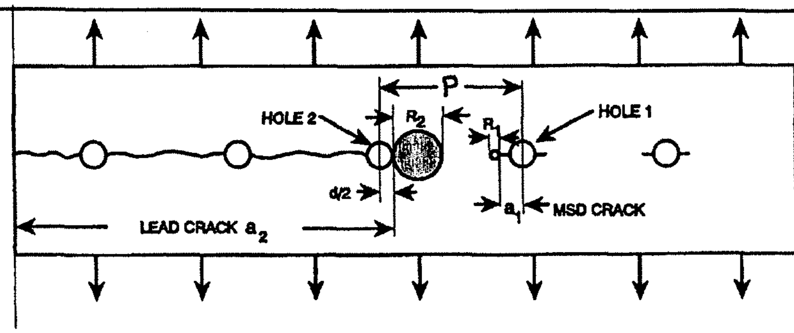

Figure 3: Swifts Model for the Assessment of Residual Strength (from (11))

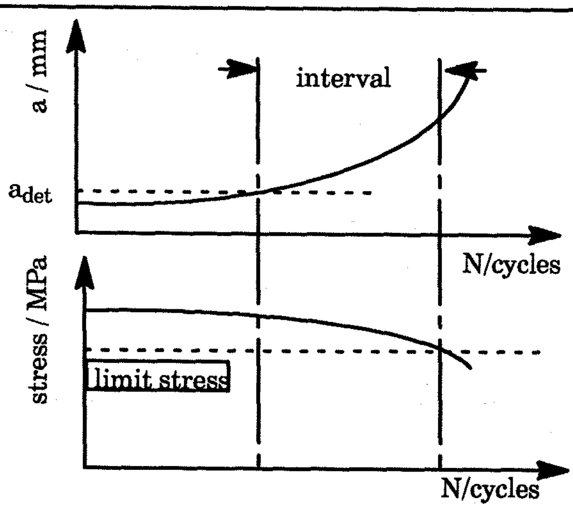

Figure 5: Determination of the Inspection Interval in the Case of a Single Fatigue Critical Location

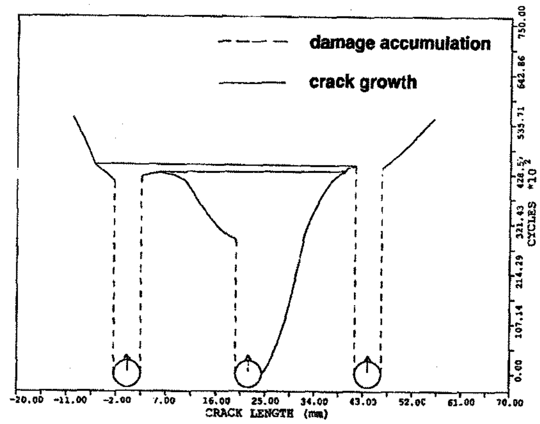

Figure 8: "Best Case" of the Monte–Carlo Simulation

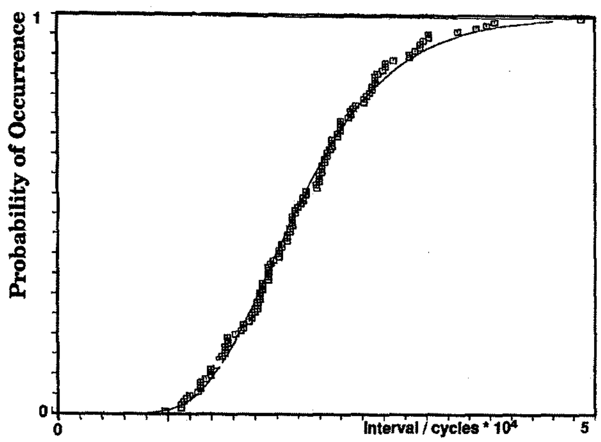

Figure 6: Typical Probability of Occurrence of Different Intervals due to a Monte–Carlo Simulation

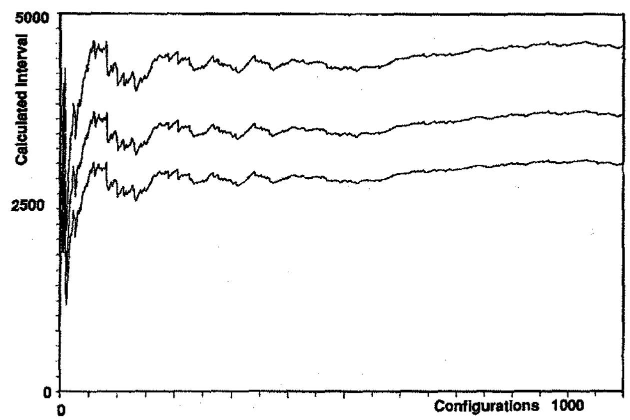

Figure 9: Convergence of the Data of a Monte–Carlo Simulation

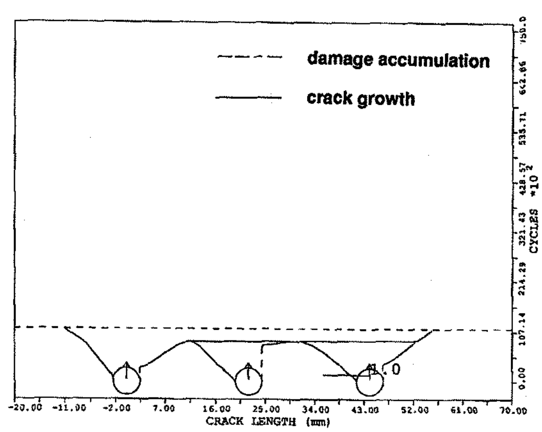

Figure 7: "Worst Case" of the Monte–Carlo Simulation

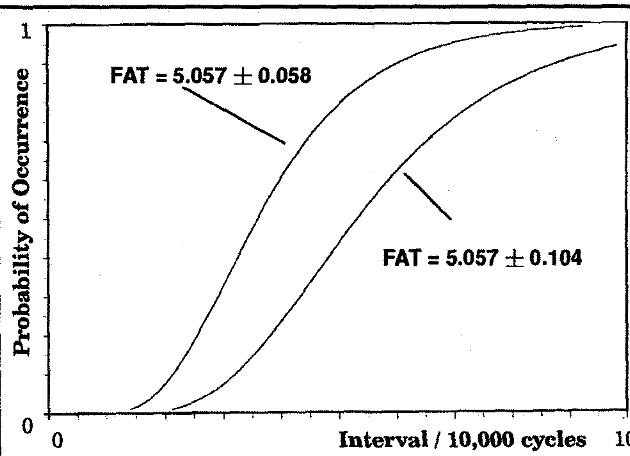

Figure 10: Effect of Scatter of Fatigue Data

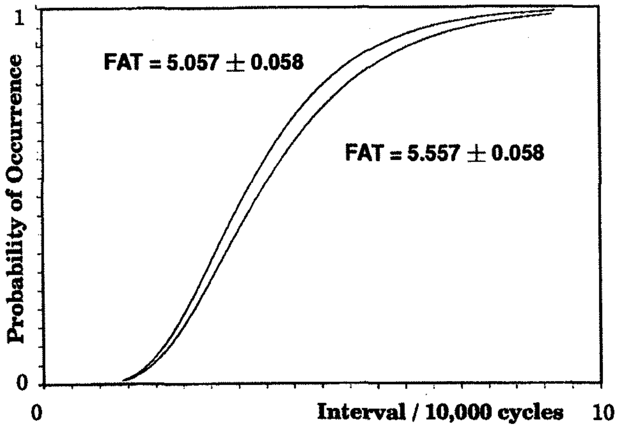

Fig. 11 : Effect of Different Average Values of the Fatigue Life (POD = 5.0)

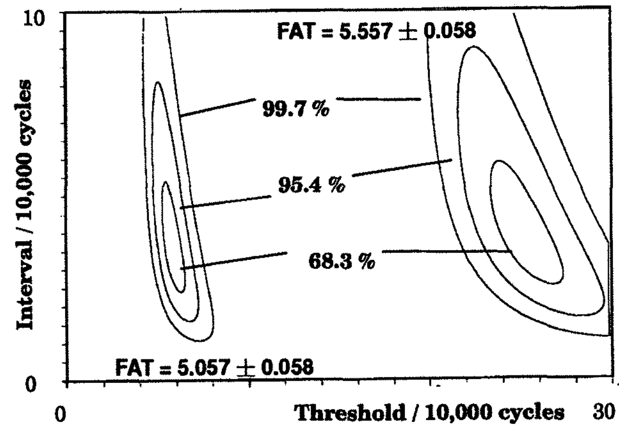

Fig. 12: Different Average Values of the Fatigue Life (POD = 5.0)

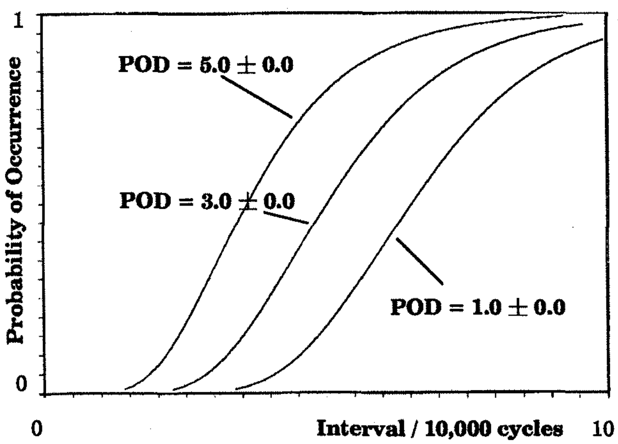

Fig. 13: Effect of Different Detectable Crack Lengths

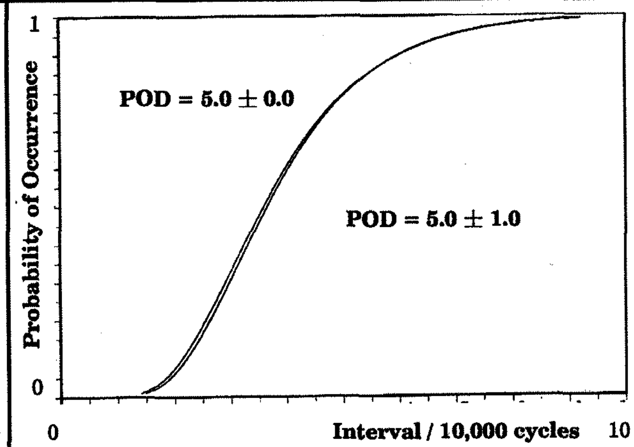

Fig. 14: Effect of Scatter in the Probability of Detection

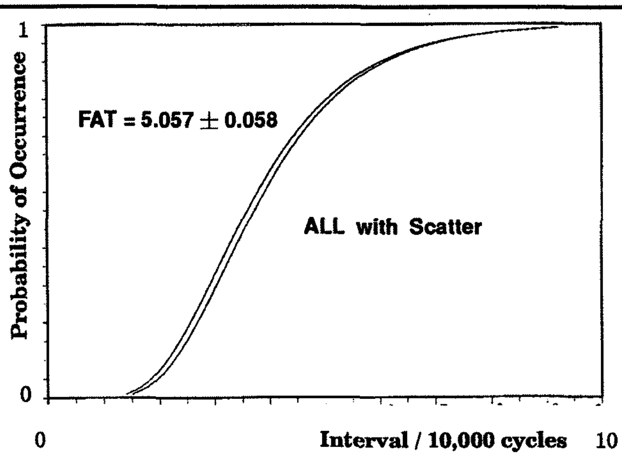

Fig. 15: Effect of Scatter in FAT, POD, POS, CGR, LTR

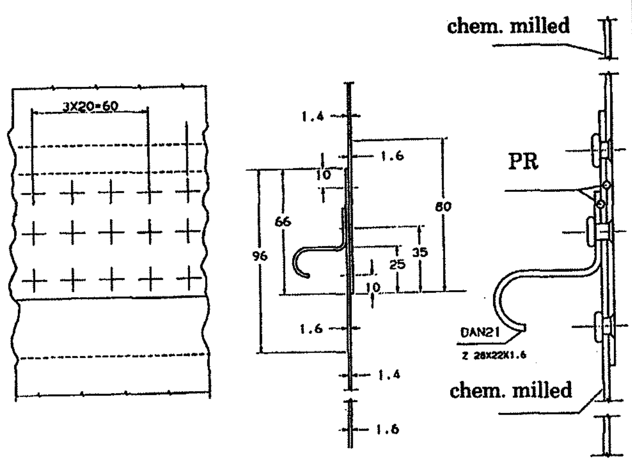

Fig. 16: Lap Joint Design (2024 T3)

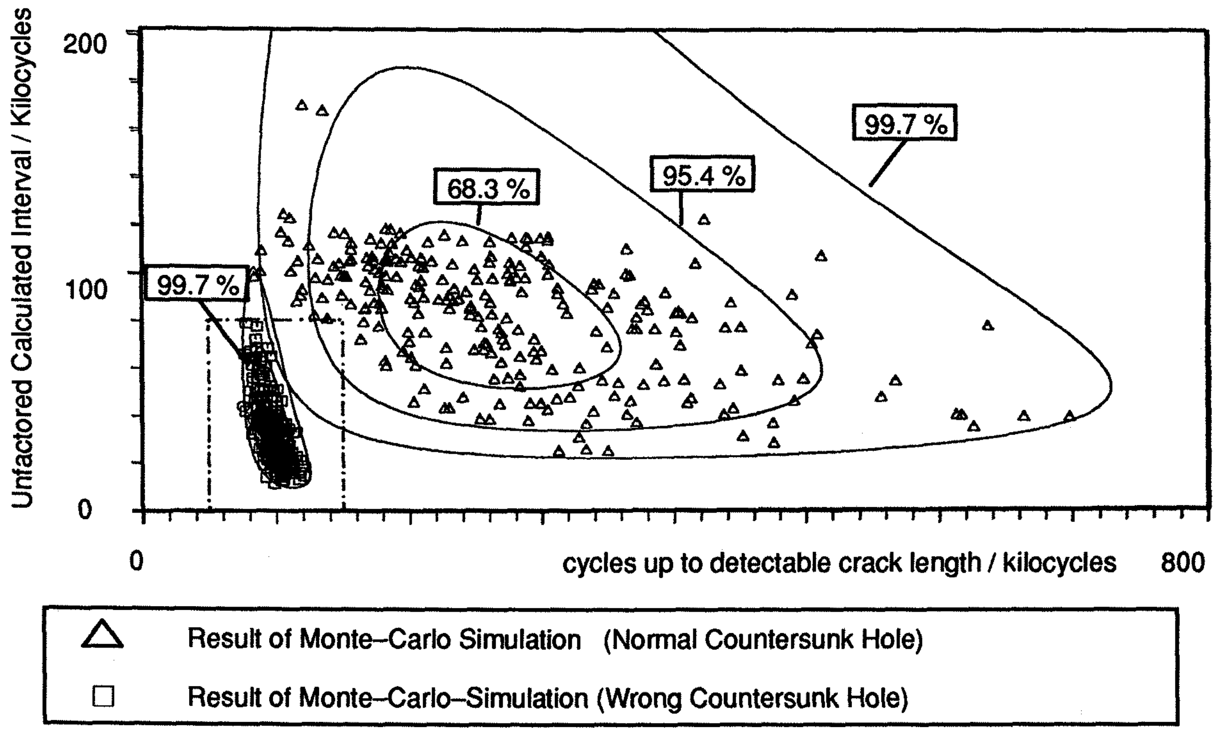

Figure : **Comparison of Monte–Carlo–Simulations in the Case of Normal and too Deeply Countersunk Holes**

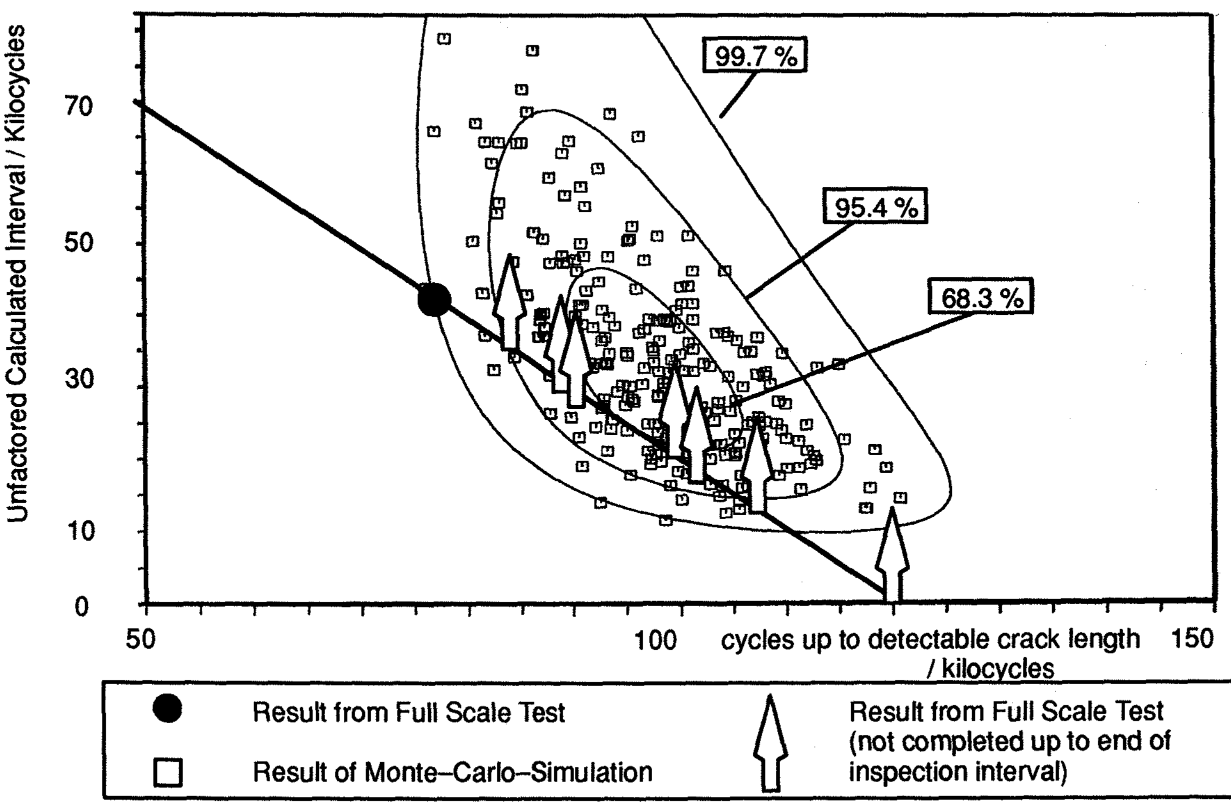

Figure : **Comparison of Monte–Carlo–Simulation and Full Scale Test Results of a Lap Joint with Countersunk Rivet Holes too Deep**