# A POSITION CONTROL OF AN ELECTRO-HYDROSTATIC ACTUATOR SYSTEM USING SLIDING MODE

# Cláudio Barroso\*, J. António Dente+

\*Escola Náutica Infante D. Henrique

Av. Eng. Boneville Franco - Paço d' Arcos - 2780 Oeiras - Portugal

+Instituto Superior Técnico ( Lab. Mecatrónica / CAUTL )

Av. Rovisco Pais - 1900 Lisboa Codex - Portugal

E-mail: DENTE@alf4.cc.fc.ul.pt Fax nº 351-1-8482987

Abstract - The paper presents a position controller for an electro-hydrostatic actuator with fixed displacement pump based on the sliding-mode technique. The main task for sliding-mode controller design is to determine an adequate commutation law, that assures a vanishing position error and makes the system insensive to parameters and load variations. In a recent paper a priniciple of synthesinig a commutation law was presented. However, the measure of certain state variables, like piston acceleration and its derivates, put some implementation problems. To overcome these difficulties a simplified commutation law was used. In this paper the same procedure is presented with a different commutation law, that assures smaller position tracking error.

### I. INTRODUCTION

Primary flight control actuators on modern aircraft are hydraulic actuators with mechanical and/or electrical signalling. The electrically powered actuators may be an advantageous alternative solution for primary flight control actuators on future civil aircraft. The progress in magnetic materials and the development of hybrid electronics, in the last decade, have basically changed the situation. Nowadays, the permanent-magnet synchronous motor with electronic commutation is used in high dynamics variable speed drive systems. However, as hydraulic force transmission offer significant advantages over mechanical coupling when high dynamics in high inertial loads are necessary (4), an electro-hydraulic drive as an hybrid system, may be a good solution for primary flight control actuators. In this application there are many aspects to verify as the efficiency, power weight ratio and dynamic beaviour. This paper only looks for control and dynamic aspects.

To command the system ones can use sliding mode control to achieve robustness and to simplify the command and control hardware. In a recent paper <sup>(3)</sup> a methodology to determine commutation law was proposed. However, implementation of this result is difficult, because no direct measures of some variables are not directly available. A simplified form of the commutation law was used and interesting results was obtained. In this paper the same methodology is used, but a different simplified commutation law is considered

to obtain a better performance concerning the position tracking error.

#### II. SYSTEM DESCRIPTION

A schematic representation of the electro-hydraulic actuator system that is being considered in this study is shown in Figure 1.

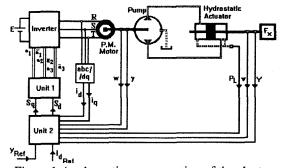


Figure.1-A schematic representation of the electrohydrostatic actuator system in study

A permanent magnet synchronous motor drives a fixed displacement pump, which output fluid is used to drive a hydrostatic actuator. A voltage source inverter, whose power devices, on-off operated, must be controlled to generate adequate motor stator voltages. The values of commutation functions  $S_d$  and  $S_q$  defines the state conduction of the power devices of the inverter.

One must note that figure 1 presents a laboratory electrohydraulic actuator. As a primary fly control actuator some modifications must be included for instance in the hydraulic layout.

## **III. THEORETICAL STUDY**

To assure a better understanding one underlines the main ideas of reference <sup>(2)</sup> to find a commutation law for a position tracking electro-hydraulic system.

The electric motor and hydraulic pump are represented by a simplified model using equations (1). This simplified model results from the application of electrical circuit theory and the conservation mass law. A particular dq reference frame was also used in order to

Copyright © 1994 by ICAS and AIAA. All rights reserved.

achieve an easy mathematical representation to the electric motor. In spite of the simplifications, the model gives good indications concerning the performances of the controlled system.

$$\frac{di_d}{dt} = \frac{1}{L_d} \left[ u_d + \omega L_q i_q - r i_d \right]$$

$$\frac{di_q}{dt} = \frac{1}{L_q} \left[ u_q - \omega L_d i_d - r i_q - \omega \phi_f \right]$$

$$\frac{d\omega}{dt} = \frac{1}{J} \left\{ \left[ \left( L_d - L_q \right) i_d + \phi_f \right] i_q - d_b P_L \right\}$$

$$\frac{dP_L}{dt} = \frac{1}{V} \left[ -L K_B P_L - C_p K_B v + K_B d_b \omega \right]$$

$$\frac{dv}{dt} = \frac{1}{M} \left[ C_p P_L - B v - F_x \right]$$

$$\frac{dy}{dt} = v$$
(1)

#### A. Command Voltages

To obtain the control action, that are the command voltages that assure a pre-defined performance to the system, a more suitable representation that the one given by equation (1) may be found. Using the same thecnique as the one presented in reference,  $^{(2)}$  it is possible to use a new state variables set according (2), where  $\gamma_c$  is the piston acceleration.

$$(i_d, i_q, \omega, P_L, v, y) \leftrightarrow (i_d, \ddot{\gamma}_c, \dot{\gamma}_c, \gamma_c, v, y)$$
 (2)

When a position regulator or a position tracking problem is considered it is useful to find the position error dynamics equation (3). This equation is found using (2) and the position error definition.

$$\begin{bmatrix}
\frac{de_{y}}{dt} = e_{y} \\
\frac{de_{y}}{dt} = e_{\gamma_{c}} \\
\frac{de_{\gamma_{c}}}{dt} = e_{\dot{\gamma}_{c}}
\end{bmatrix}$$

$$\frac{de_{\gamma_{c}}}{dt} = e_{\dot{\gamma}_{c}}$$

$$\frac{de_{\dot{\gamma}_{c}}}{dt} = e_{\dot{\gamma}_{c}}$$

$$\frac{de_{\dot{\gamma}_{c}}}{dt} = A_{\gamma_{c}}(e_{y}, e_{y}, e_{\gamma_{c}}, e_{\dot{\gamma}_{c}}, t) - B_{\gamma_{c}}u_{q}$$
(3)

As usual when  $L_d \approx L_q$  current  $i_d$  must be zero to minimise the stator cooper losses of the electric motor. This condition and the a vanishing position error are enough to define the voltage values to apply to the electric motor.

To assure a vanishing tracking position error it's enough to generate the  $\mathbf{u}_q$  value given by (4) where  $\mathbf{S}_q$  is defined by (5). Then the position tracking error dynamics is

given by (6) where the parameters  $K_j$  must be chosen in order to assure stability and a decaying position error. The pole assignment method may be used to determine these parameters.

$$u_{q} = \frac{1}{B_{\gamma_{c}}} \left[ A_{\gamma_{c}}(e_{y}, e_{v}, e_{\gamma_{c}}, e_{\dot{\gamma}_{c}}, e_{\dot{\gamma}_{c}}, t) + S_{q} \right]$$
 (4)

$$S_{q} = K_{\nu}e_{\nu} + K_{\nu}e_{\nu} + K_{\nu}e_{\nu} + K_{\nu}e_{\nu} + K_{\nu}e_{\nu} + K_{\nu}e_{\nu} + K_{\nu}e_{\nu}$$
 (5)

$$k_{y}e_{y} + k_{y}e_{y} + k_{y_{c}}e_{y_{c}} + k_{y_{c}}e_{y_{c}} + k_{y_{c}}e_{y_{c}} + k_{y_{c}}e_{y_{c}} + e_{y_{c}} = 0$$
 (6)

Equation (6) with adequate  $k_j$  parameters represents a desirable performance to the controlled system. Concerning current id a similar approach may be done.

# **B. Automatic Generation of Command Voltages**

The theoretical result (6) shows a robust controlled system. However, it is difficult to determine the voltage value (4), because the parameters and disturbance are not well known. Furthermore calculations are complex and time consuming. Sliding mode is a good solution to automatically generate the command voltage  $\mathbf{u}_q$ . This is achieved using the commutation law (7). This condition is equivalent to (6) when no initial errors exist and  $\mathbf{S}_q$  becomes almost equal to zero.

$$u_{q} = \begin{cases} V_{1q}(t) & \text{if } S_{q} > 0 \\ V_{2q}(t) & \text{if } S_{q} < 0 \end{cases}$$
 (7)

To assure that the command (7) is effective, that is  $S_q$  is near zero, the  $S_q$  time derivative is evaluated and the result is given by (8).

$$\dot{S}_{q} = \begin{cases} A_{\gamma_{c}}(i_{d}, y, v, \gamma_{c}, \dot{\gamma}_{c}, \ddot{\gamma}_{c}) - B_{\gamma_{c}} V_{1q} & S_{q} > 0 \\ A_{\gamma_{c}}(i_{d}, y, v, \gamma_{c}, \dot{\gamma}_{c}, \ddot{\gamma}_{c}) - B_{\gamma_{c}} V_{2q} & S_{q} < 0 \end{cases}$$
(8)

This result shows that it is possible to control the signal of the derivative using the commutation law (7), if relation (9) is verified.

$$\left|V_{iq}\right| > \frac{1}{B_{\gamma_c}} \left[ \left[ A_{\gamma_c}(i_d, y, v, \gamma_c, \dot{\gamma}_c, \ddot{\gamma}_c, t) \right] \right] \quad i = 1, 2$$
 (9)

A simple geometric interpretation enables to conclude that function  $S_q$  and its derivative may be opposite signs (10) if  $V_{qj}$  has adequate signal and high enough absolute value (9).

$$S_{q}(t)\dot{S}_{q}(t) < 0 \tag{10}$$

When condition (10) is verified the sliding mode operation is effective. The command voltage is generated

automatically by modulation to assure that equation (11) is verified.

$$S_q \approx 0$$
 (11)

For current id control a similar approach is done using as commutation function the following result:

$$S_d = -i_d = 0 \tag{12}$$

#### IV. IMPLEMENTATION

# A. Simplified Commutation Law

Implementation using commutation function (5) is difficult, because direct measures of derivatives of acceleration are not available. Moreover, the noise environment is an obstacle concerning derivate calculation. To overcome these difficulties a simplified commutation function (13) was proposed in <sup>(2)</sup> where the piston acceleration value was obtained using an observer.

$$S_{q} = k_{y}(y_{\text{Re}f} - y) - k_{y}v - k_{\gamma_{c}}\gamma_{c} - k_{\omega}\omega - k_{i,i}i_{q}$$
 (13)

The solution presented in <sup>(2)</sup> shows the feasibility of the sliding mode to command an electro-hydraulic actuator. However, with this commutation function a steady position error may exists. Considering equations (1) and (13) in a steady state one can evaluate the piston position error (14).

$$(y_{Ref} - y) = \frac{k_{i_q}}{k_y} \frac{d_b}{\phi_f C_p} F_x$$
 (14)

The result given by (14) shows that when the system is loaded a steady state position error appears. Moreover, it is difficult to implement very low gain (15) to constrains the position error.

$$\frac{k_{i_q}}{k_y} \cong 0 \tag{15}$$

To overcome this drawback another simplified commutation function is proposed.

As equation (16) assures a vanishing tracking error, this means that the piston speed must verify relation (17).

$$k_{\nu}(y_{ref} - y) + k_{\nu}(v_{ref} - v) + k_{\nu_c}(\gamma_{cref} - \gamma_c) = 0$$
 (16)

$$v = \frac{k_{y}}{k_{v}}(y_{ref} - y) + v_{ref} + \frac{k_{\gamma_{c}}}{k_{v}}(\gamma_{cref} - \gamma_{c})$$
 (17)

Considering no leakage and incompressibility of the hydraulic fluid, the rotor speed and piston speed verify relation (18). So, one uses (17) to define the electric motor speed that assures equation (16).

$$\omega = \frac{C_P}{d_b} v \tag{18}$$

Considering a motor speed controller that imposes equation (19), where  $\gamma$  is the rotor acceleration and uses as reference rotor speed the value given by (18) and (17) to obtain a commutation function.

$$k_{\nu}(\gamma_{ref} - \gamma) + k_{\omega}(\omega_{ref} - \omega) = 0$$
 (19)

Considering equal to zero the derivatives of the piston acceleration and it's reference value, equation (20) is obtained.

$$S_{q} = k'_{y}(y_{ref} - y) + k'_{y}(v_{ref} - v) + k'_{\gamma_{c}}(\gamma_{cref} - \gamma_{c})$$

$$-k'_{\gamma}\gamma - k'_{\omega}\omega + \frac{k'_{\gamma}C_{p}}{d_{b}}\gamma_{cref} + \frac{k'_{\omega}C_{p}}{d_{b}}v_{ref}$$
(20)

When dynamic position error is admissible, ones uses a simplied result as the one given by (21).

$$S_a = k_y (y_{ref} - y) - k_y v - k_y \gamma_c \gamma_c - k_y \gamma - k_\omega \omega$$
 (21)

#### **B.** A Motor Acceleration Observer

The simplified commutation function (21) includes piston acceleration value and motor acceleration value, which are difficult to measure when the signals contain a large frequency range spectrum. In a same way of reference <sup>(3)</sup> an observer for the motor acceleration can be used as follows.

Mechanical equation is put in form (22), that includes the parameters variation as an external torque load.

$$(J_N + \Delta J) \frac{d\omega}{dt} = (\phi_{f_N} + \Delta \phi_f) i_q - T_R$$

$$J_N \frac{d\omega}{dt} = \phi_{f_N} i_q - T_R' \tag{22}$$

The observer error of the equivalent motor torque (23) verifies the dynamic equation (24) where g is a free chosen parameter.

$$\tilde{T}_R' = T_R' - \hat{T}_R' \tag{23}$$

$$\frac{d\tilde{T}_R'}{dt} = -g\tilde{T}_R' \tag{24}$$

If motor torque signal has low level frequency components (25), a structure of the observer like, the one

presented by equations (26) may be used to determine the motor load torque and the motor acceleration (27).

$$\left| \frac{dT_R}{dt} \right| << \left| \frac{d\hat{T}_R}{dt} \right| \tag{25}$$

$$\dot{\delta} = -g\delta + g\phi_{f_N}i_q + g^2J_N\omega$$

$$\hat{T}_R = \delta - gJ_N\omega \tag{26}$$

$$\hat{\gamma} = \frac{\phi_{fn}}{J_N} i_q - \hat{T}_R \tag{27}$$

The two observers, piston acceleration and motor acceleration, enable us to use for the commutation law variable values that are measure in a easy way (position, piston speed, pressure, rotor speed and currents).

#### C. Simulation Results

In this point simulation results are presented to ilustrate the benefits of the proposed commutation law.

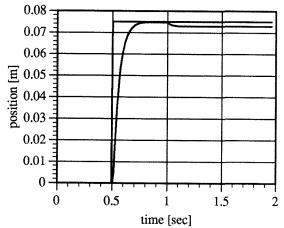


Figure 2 - Piston position and reference evolutions.

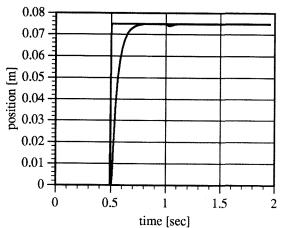


Figure 3 - Piston position and reference evolutions.

Figure 2 and figure 3 show the piston position evolution when a step reference input and a load step are applied. In figure 2 the controller uses the simplified commutation law (13) that considers the motor current. This result shows a steady state position error when the system is loaded. In figure 3 this error becomes null, because the controller uses the simplified commutation law (21), where the rotor acceleration is considered.

Figure 4 presents simulation results showing the evolution of piston position, when a sine curve reference position is applied to the system and commutation law is the one given by equation (21). In this situation there is a dynamic error, that becomes small when the commutation law (20) is used without the two last terms as shown in figure 5.

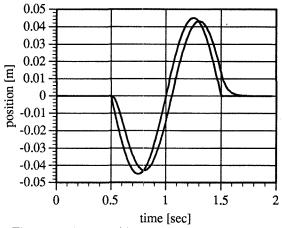


Figure 4 - Piston position and reference evolution.

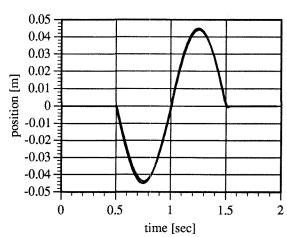


Figure 5 - Piston position and reference evolution

## V. CONCLUSIONS

This paper presents a sliding mode position controller for an electro-hydrostatic actuator. A simplified commutation law is proposed to overcome some implementatios problems, as the direct measures of variables and the steady state position error referred in a previous paper (2). Theoretical and simulation results show that good performances can be achieved in

tracking applications with the proposed commutation law. This solution enables an easy electronics integration and uses usual accessible variables (position, speed, pressure and electric currents). Moreover, the proposed sliding mode controller may include automatic limitation of variable values in a simple way, as explained in <sup>(3)</sup>.

#### **REFERENCES**

- [1] Utkin, V.I.; "Variable Structure Systems with Sliding Modes", *IEEE Trans. on AC*, Vol.AC-22, pp. 212-222, 1977
- [2] Barroso, Cláudio; Dente, J.A (1994); "Position Control Sliding Mode of an Electro-Hydrostatic Actuator System Using a Piston Acceleration Observer", ICEM 94- International Conference on Electrical Machines, 5-8 Sept. 94, Gif-sur-Yvette, France.
- [3] Maia, J.H.; P.J. Costa Branco; J. Esteves; Dente, J.A. (1993); "Robust Position Sliding- Mode Control of a Permanent- Magnet Motor Using a Load-Torque Observer", IEEE PESC'93, Power Electronics Specialists Conference, Washington USA, 1993.
- [4] Franhenfield, Tom; Stavrou, Paul (1993); "Developing Trends in Hydraulics Tied to Electronic Controls", Control Engineering, Vol. 40, No. 5, April 1993.
- [5] Yen, Y.; Lee, C.. (1992): "Robust Speed Control of a Pump-Controlled Motor System", *IEEE Proceedings-D. Vol. 139*, No. 6, November 1992.