STUDY OF FLY-BY-WIRE SYSTEM STABILITY

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ABSTRACT

A study of three problems about the stability for a fly-by-wire system: gain changing low, harmonic oscillation and phase compensation has been completed.

Based on the research of the variation in stabilator effectiveness with different flight conditions, the gain scheduled respectively by indicated airspeed and altitude were selected and then the larger one of the two was taken.

Through the spectral analysis and experimental investigation, a special sawtooth oscillation of control surface excited by the odd harmonic of the triangular ware formed by the actuator back-and-forth movement was discovered. The frequency features of the oscillation and the measures to suppress stabilator buffet were developed.

Confroted with the serious fact that the stability margins of the system could not meet the requirements for 78% flight conditions, the researcher established the relationship between the crossover frequencies (0 dB and -180^0) of the open-loop frequency responce curves and the corner frequencies of the corrective network. Four fixed paramenters in lag-lead network over the entire flight envelope are given at a time, so that all the stability margins and the handling qualities can meet the requirement of specifications.

The research results have all been demonstrated by ground and flight tests.

GAIN CHANGING LAW

Feedback Configuration

The longitudinal control system demonstrated in a flighter is a analogous fly-by-wire system in which aircraft motion is the controlled parameter. In order to control the aircraft effectively and safely throughout the flight envelope and to provide the wide closed-loop handwidth desired for satisfactory response characteristics, a high-gain control system is expected.

In the design of the longitudinal control law, the blended normal acceleration and the pitch-rate feedback is selected. Normal acceleration feedback is needed in order to reduce the stick force in steady state per g that varies with aircraft velocity, pitch rate feedback is used to increase damping ratio of short-period movement and to improve handling qualities.

Normal acceleration feedback, while greatly enhancing the stick force per g properties has a tendency to reduce system stability margins at some flight conditions. For this reason, normal acceleration to pitch rate feedback ratio of 4.5 : 1 was selected for the fly-by-wire system, meeting the stick force per g requirement.

C* Blended Feedback Voltage

High gain closed-loop system is composed of forward loop, airframe and feedback. A second order notch and first order lag filter for structural mode attenuation were included in the forward path, where a lag-lead network to provide phase conpensation for the FBW system is also used.

In order to design a gain changer which changes the forward loop gain to compensate the aircraft stabilator effectiveness variation due to changes in flight conditions, C* feedback voltage for unit surface deflection, composed of the blended normal acceleration and pitch rate feedback, as a function of altitude and Mach number throughout the flight envelope was studied and showed in Fig. 1. It is noted that the feedback value has a wide variation range. The maximum gain is 20 times as large as the minimum (for $H=0$ and $M=0.9$, $K_\delta^V=-1.7739$; while for $H=20$ km and $M=1.8$, $K_\delta^V=-0.0893$). It is very difficult to synthesize such system without a proper gain changing law.

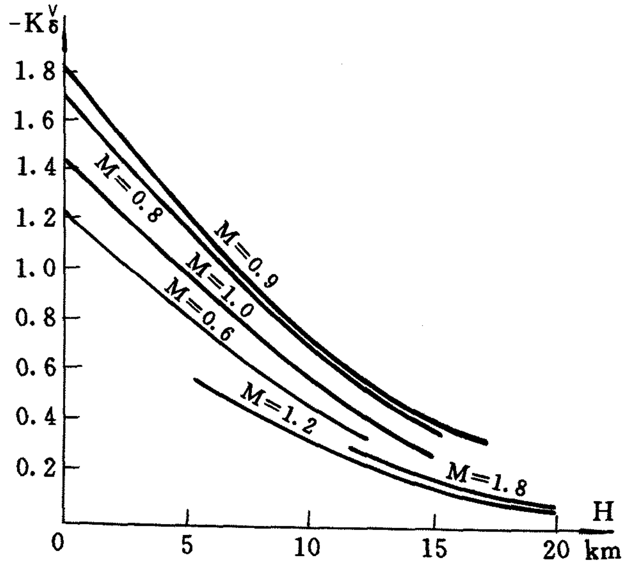

Fig. 1 C* Feedback Voltage for Unit Surface Deflection

Gain Changing Law

A gain changing law as function of dynamic pressure was initially adopted:

$$K_q=\begin{cases}1 & q\leqslant 10^3\ \mathrm{kg/m^2}\\ 1-0.12(10^{-3}q-1) & 10^3<q<6\times10^3\ \mathrm{kg/m^2}\\ 0.4 & q\geqslant 6\times 10^3\ \mathrm{kg/m^2}\end{cases} \quad (1)$$

The result of this gain changing was not ideal, C* blended feedback voltage variation range was still relatively large. The maximum gain is 11 times as large as the minimum in the flight envelope.

As can be seen from Fig. 1 , the C* blended feedback voltage for unit surface deflection attenuates with altitude increase throughout the flight envelope. For subsonic region the gain values increase with Mach number, whereas the gain values go down for Mach number of more than 1.2 .

Based on these variations resulted from the different flight conditions, the forward path gain scheduling was reconfigured with indicated airspeed and altitude respectively and then the larger one of the two gains is taken for each flight condition:

$$K_C=\mathrm{Max}\ [K_V,\ K_H] \quad (2)$$

$$\text{where } K_V=\begin{cases}1 & V_i\leqslant 100\ \mathrm{m/s}\\ \dfrac{100}{V_i} & 100<V_i<500\ \mathrm{m/s}\\ 0.2 & V_i\geqslant 500\ \mathrm{m/s}\end{cases} \quad (3)$$

$$K_H=0.2+0.04\,H \qquad [H]=\mathrm{km} \quad (4)$$

This gain changing law is presented in Fig. 2 .

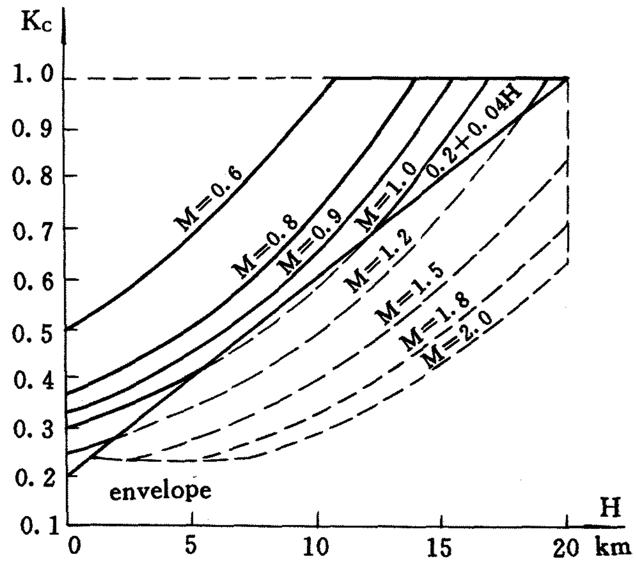

Fig. 2 Gain Changing Law With Indicated Airspeed and Altitude

As shown in this figure, all Kc values increase with H in the entire flight envelope. For subsonic region, Kv is selected as Kc which increases with H on parabola, while after $M>1.2$ the Kv is replaced by K_H which is a linear line being larger than Kv. This gain changing law perfectly compensates the changes in stabilator effectiveness due to changes in airspeed and/or altitude as shown in Fig. 1 . The change of the open-loop gain with the changes in flight conditions becomes smooth and closed by using this gain changing law. The maximum is only 5 times as large as the minimum. It provided a base for the design of the fly-by-wire system stability.

HARMONIC OSCILLATION

Buffet Characteristics

After completing the synthesis, design and simulation of the longitudinal fly-by-wire system, a kind of harmonic oscillation appeared in the ground physical tests. It is a special variable-frequency oscillation that we were not familiar with in the initial design. The frequency and energy of this oscillation are so high that the closed-loop system could not work. The designers had to modify the control law. This causes the system quality to go down.

The reference 1 mentions a kind of harmonic oscillation in nonlinear systems. The frequency of the excited signal is integer times that of the harmonic oscillation. The harmonic oscillation discovered in the paper is an frequency-increasing socillation. Its frequency is odd times the frequency of the excited signal.

The buffet phenomena appeared in the longitudinal fly-by-wire control system are shown in Fig. 3 . Where (a) is the tailplane buffet excited by shaking stick when the system is in open-loop operation; (b) is the sawtooth wave on a nonlinear-limit-cycle oscillation of this closed-loop system; (c) is tailplane buffet with control stick under closed-loop operation.

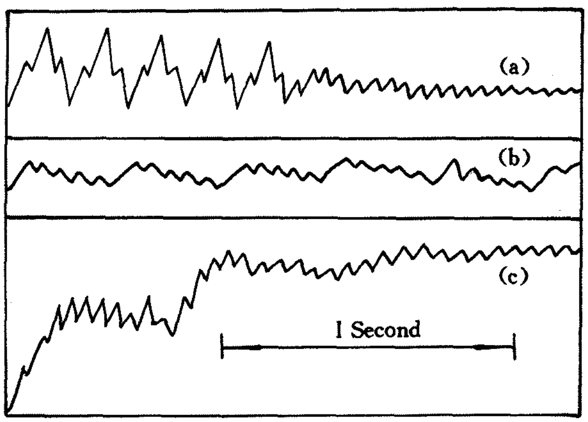

Fig. 3 Buffet of a FBW System

These sawtooth oscillations depend on the initial conditions of the system, that is, they can not happen themselves. It is not an inhernet charactristic of the colsed-loop system, because it could happen under open-loop conditions. This sawtooth oscillation has three charaeteristics on frequency:

1. The buffet frequeney is the resonance frequency of the tailplane.
2. The buffet frequency is odd times that of the fundamental wave.
3. The buffet is a variable-freequency subharmonic oscillation.

Excited Test

1. Sweep Frequency Test

In open-loop operation situation, a variable-frequency (from 1 to 50 Hz) sine signal, the amplitude of which corresponds to 1 g, is applied to the point where the control signal and blended feedback are synthesized and the tailplane response to the sweep signal is shown in Fig. 4 .

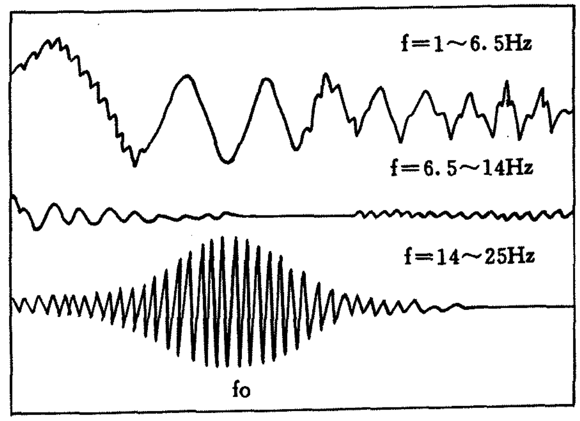

Fig. 4 Sweep Response of Tailplane

Keeping the amplitude of the input signal invariable, the amplitude of the tailplane response is attenuated when the frequency of the input signal increases. At the structural coupling frequency, the amplitude of the tailplane response approches to zero because there is a structural filter in the forward path. At the resonance frequency of the tailplane, the respone amplitude reaches the maximum. At the low-frequency section, beyond all imagination, there are the fifth-order and the third-order sawtooth harmonic oscillations.

2. Shock Test

In closed-loop situation, we shock the rate turn table, the shock wave of the gyro, the triangle wave of the actuator as well as the taiplane buffet are shown in Fig. 5 .

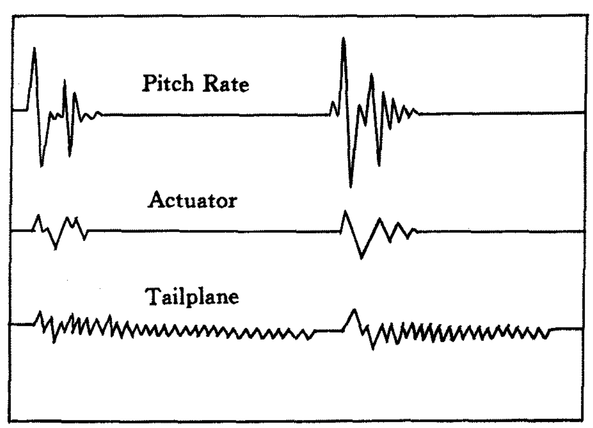

Fig. 5 Tailplane Buffet Execited by Interference of Pitch Rate

The frequency of the sawtooth wave is equal to that of the tailplane resonance.

It is seen from the test that, since the gain of the forward path in fly-by-wire system is very high, the back-and-forth motion of actuator produces triangular waves. The subharmonic component of actuator triangular wave, the frequency of which drops into the resonance frequency range of the tailplane, would excite the tailplane to produce resonance. The frequency, at which the actuator would output triangular wave, can given by reference 2:

$$\omega_n = \frac{\text{Actuator Rate Limit}}{4 \times \text{Demand Amplitude}} \times 2\pi \qquad (5)$$

Fourier Series of Triangular Wave

Suppose f(t), the period of which is T, is a continous triangular wave fuction(Fig. 6).

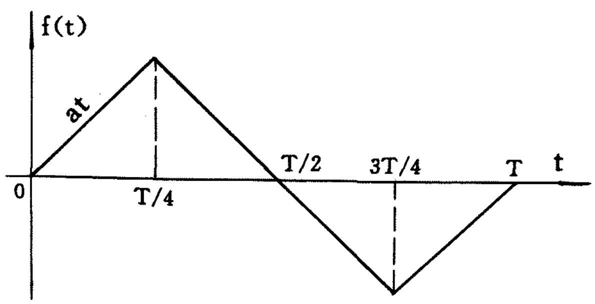

Fig. 6 Typical Triangular Wave Function

$$f(t) = \begin{cases} at & 0 \leqslant t \leqslant T/4 \\ -a(t-T/2) & T/4 \leqslant t \leqslant T/2 \end{cases} \qquad (6)$$

Expanding this function as Fourier series, we obtain:

$$f(t) = \frac{2at}{\pi^2}(\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t - \frac{1}{7^2}\sin 7\omega t + \frac{1}{9^2}\sin 9\omega t - \cdots) \qquad (7)$$

where $\omega = 2\pi/T$. From the expression (7), we can see that harmonic oscillation frequency is only the odd times that of the fundamental wave.

Frequency Characteristics of Harmonic

From a great deal of the tailplane buffet curves we can not find any even-order harmonic of fundamental frequency(see Fig. 7, where A — Actuator, T — Tailplane). The sawtooth number on one fundamental wave will enlarge when the period of triangular wave increases, but the number of the sawteeth in one second are always the same as that of the tailplane resonance frequency value.

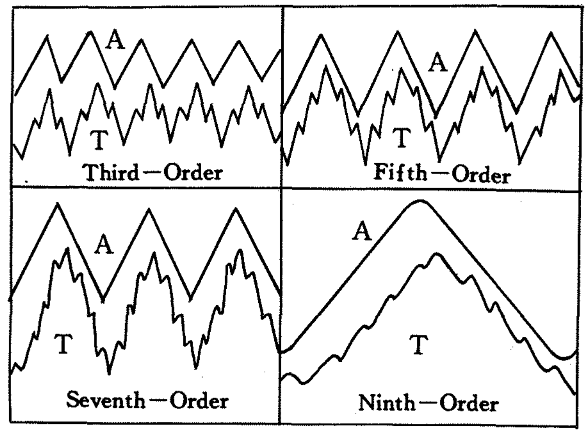

Fig. 7 Sawtooth on Fundamental Wave Excited by Triangular Wave of Actuator

Suppose the tailplane resonance frequency is f_o, the frequency of triangular wave of actuator is f_T, and the number of sawtooth on one fundamental wave is k_o, we can establish the following frequency characteristics:

$$\begin{cases} k_o \cdot f_T = f_o \\ k_o = 2n + 1 \\ n = 0, 1, 2, \cdots \end{cases} \qquad (8)$$

It should be pointed out that the triangular wave is the necessary condition to effect the buffet. The triangular wave, whose frequency does not satisfy (8), could not excite the tailplane to produce resonance. In addition, the buffet is related to the initial condition of system. The sawtooth could not keep itself without the fundamental signal.

Measures Against Harmonic Oscillation

As mentioned above, the essence of the buffet in fly-by-wire system is that the odd-order subharmonic of the quasi-triangular wave of the actuator loop output excites the tailplane resonance. The overcoming measures should provent triangular wave output from the actuator.

1. Enlargement of Linear Range of Actuator loop

The gain in the actuator loop concerns the linear range and the bandwidth of the actuator loop directly. Reduction of this gain will enlarge the linear range of the actuator loop and reduce the bandwidth only at the small signal. Selection of the amplifier gain in the actuator loop should consider the quality of the actuator loop, and at the same time should prevent triangular wave output from the actuator as fully as possible.

2. Addition of the Gyro Filter

The gyro is an important feedback element in fly-by-wire system. We find in test that the gyro path gets disturbance easily and effects the harmonic oscillation, therefore it is necessary to add a filter to this path. We selected a first-order filter in the gyro path. It has good filtering effect against the gyro disturbance. The filter in the feedback path also speeds up the reaction of the closed-loop system.

3. Arrangement of Filter in Forward Path

Arranging a forward filter is an effective measure to hinder abrupt signal through the high-gain forward path in the fly-by-wire system. A low-pass filter was placed in front of the actuator loop to weaken the signal shock and to smooth the signal leading edge for preventing the actuator from triangular wave output. The filter[3] in the forward path also has an active effect for restraint of structural couling oscillation.

4. Lowering Feedback Gain of Pitch Rate

It is an active and effective measure to lower the feedback gain of the pitch rate for restrainting tailplane buffet. However, the blended ratio of the normal acceleration to the pitch rate will be increased, so that the phase margin will be reduced[4]. In addition, the reduction of the gain of pitch rate feedback will affect the stability augmentation fuction of the system. We should consider these synthetic factors while using this method.

5. Lowering main Gain in the Forward Path

Reduction of the maim gain in the forward path is one of the effective measures for prevention of the harmonic oscilation. The high-gain is necessary for assuring handling qualities of fly-by-wire system. Lowering the main gain will degrade the handling qualities and enlarge the amplitude of limit-cycle oscillation in some flight conditions. So, it is not a good method against the tailplane buffet.

It should be point out that using a single measure above can not thoroughly eliminate the buffet of tailplane. We eclectically adopted the first three methods to overcome the buffet. It has been verified by the rig-test and on board test as well as the flight test that the tailplane buffets were completely eliminated.

PHASE COMPENSATION

Stability Margin Without Compensation

The method to overcome the buffet decreased 8—12 degrees phase margin and 1 dB gain margin. We selected 18 flight conditions in the envelope, of which 14 states did not satisfy the phase margin requirement, they occupied 78% of the whole states and 5 conditions did not reach 10 dB gain margin, they were 28% of the whole states (see Fig. 8) .

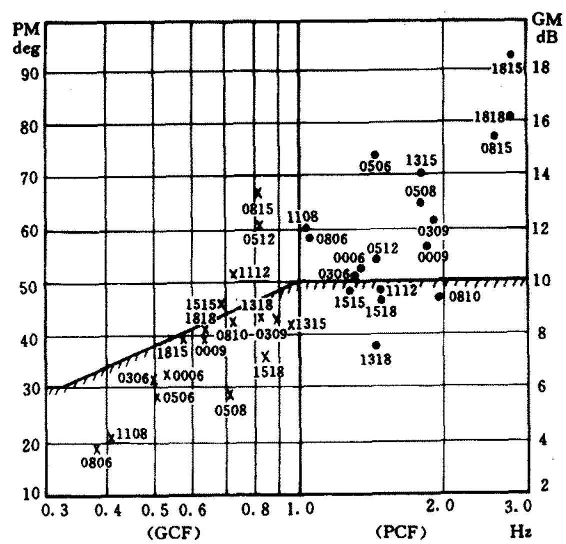

Fig. 8 Gain and Phase Margins of the System Without Compensation

A few situations, such as 0806 — 8 kilometer, 0. 6 Mach number, had only 19 degress of phase margin. To completely compensate the phase margin of the fly-by-wire system, the open-loop frequency characteristic curves in the whole envelope were measured through a ground testing and plotted together in Fig. 9 . We can see from this figure that the phase margins for most subsonic situations such 0006, 0306, 0506, 0508, 0806, 1108 etc. their phase frequency characteristics are smooth before the 0 dB crossover frequency, so that we could not raise the phase margin by means of lowering the main gain in the forword path. From the test results, the gain crossover frequency:

$$GCF = 0.41 - 0.96 \text{ Hz} \quad (9)$$

and the phase crossover frequency:

$$PCF = 1.06 - 2.75 \text{ Hz} \quad (10)$$

The phase margin and the gain margin could not have satisfactory results because the (PCF) min and the (GCF) max were very approached. Therefore, such problem that how to compensate the phase in a large part of the envelope for the fly-by-wire system has been offered.

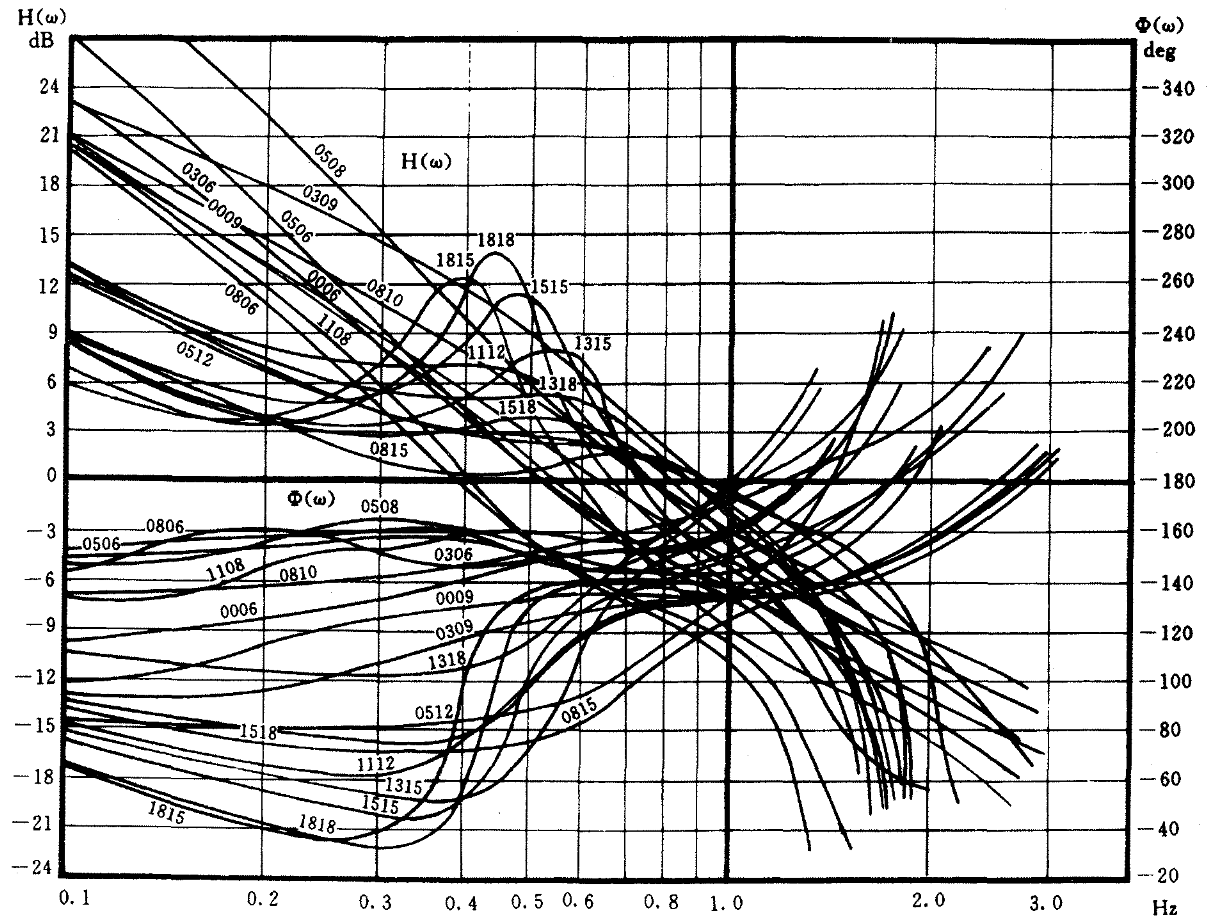

Fig. 9 Open-Loop Frequency Characteristic Curves of a FBW System

Eight Formulas in Frequency-Domain About Lag-Lead Compensative Network

Suppose the four corner frequency of a lag-lead corrective network with any two pairs of poles and zeros are separately a, b, c, d, in which a<b<c<d, in rad/sec. The transfer function of the network is simply noted as:

$$W_c = [a, b, c, d] \quad (11)$$

It represents following transfer function

$$W_c = (s/b+1)(s/c+1)/(s/a+1)(s/d+1) \quad (12)$$

It is a general expression of two order lag-lead corrective network with any two poles and two zeros. The parameter characteristics in frequency domain of this network are as following:

(1) phase-ferquency characteristic

$$\Phi(\omega) = -\mathrm{arctg}(\omega/a) + \mathrm{arctg}(\omega/b) + \mathrm{arctg}(\omega/c) - \mathrm{arctg}(\omega/d) \quad (13)$$

(2) Magnitude-frequency characteristic

$$H(\omega) = 10\{-\lg[1+(\omega/a)^2] + \lg[1+(\omega/b)^2] + \lg[1+(\omega/c)^2] - \lg[1+(\omega/d)^2]\} \quad (14)$$

g〔(3)〔 Maximum lag-phase frequency

$$\omega_1 \leqslant \sqrt{a\,b}, \quad \omega_1 \approx \sqrt{a\,b} \quad (15)$$

(4) Maximum lead—phase frequency

$$\omega_2 \geqslant \sqrt{c\,d}, \quad \omega_2 \approx \sqrt{c\,d} \tag{16}$$

(5) Zero—phase frequency

Let formula (13) be equal to zero and take tangent of phases with same sign, then get

$$\omega_0 = \sqrt{[ad(b+c)-bc(a+d)]/(b+c-a-d)} \tag{17}$$

When the geometry of the network is symmetrical, we get ad=bc, so

$$\omega_0 = \sqrt{a\,b} = \sqrt{b\,c} \tag{18}$$

(6) Maximum lag-phase

$$\varphi_1 = \varphi(\omega_1) \tag{19}$$

(7) Maximum lag-phase

$$\varphi_2 = \varphi(\omega_2) \tag{20}$$

(8) Maximum magnitude attenuation

$$H\,min = H(\omega_0) \tag{21}$$

Compensation for a Set of Frequency Curves

To seek the relationships between the corner frequences of the compensator and the crossover frequency bands of the frequency characteristic curves is a key to compensate the open-loop frequency curves in Fig. 8 .

1. Zero-phase frequency of compensator

For the fly-by-wire system, not only the phase margins do not satisfy the specification requirement under most flight conditions, but the gain margins also do not reach the 10 dB at all (see Fig. 8). The phase compensations are all needed for the range of the GCF = 0.41-0.96 Hz. Therfore, the zero-phase frequency of the lag-lead network should be located near-by the minimum of the GCF:

$$\omega_0 \approx (GCF)min = 0.41\ Hz = 2.57\ rad/s \tag{22}$$

2. Maximum Phase-lead Frequency

For separating the GCF and the PCF and raising the minimum of the PCF, the maximum phase-lead frequency of the compensator should be a little larger than the minimum of the PCF:

$$\omega_2 > (PCF)min = 1.06\ Hz = 6.66\ rad/s \tag{23}$$

3. Maximum Corner Frequency

The higher the last corner frequency of the compensator is, the larger the lead-phase contributed by the compensator is, but it would effect the gain margin in the high-frequency reqion. The fourth corner frequency of the compensator should be located at the maximum phase crossover frequency, that is:

$$d \approx (PCF)max = 17.28\ rad/s \tag{24}$$

4. Separations Among Corner Frequencies

For the lag-lead corrective network with symmetry of geometrical relationship, there are following equalities:

$$m = b : a = d : c, \quad a\,d = b\,c \tag{25}$$

The bigger the m is, the larger the maximum lead-phase of the compensator is. However the same lag-phase would be induced in low frequency region and the maximum magnitude attenuation would become large at the zero-phase frequency, the system bandwith would be low, so that the response of the system would become slow. The test results indicate that, taking m=4, 26 degrees of maximum phase-lead can be attained and the requirements of stability margins can be achieved. The separation between the second and the third corner frequencies should not be too large, otherwise the region affected by lag-phase would become large and the climb would occur in transient. Referring the parameters of [4], where b=4 and c =7, we take c : b=2. The relative separations among the corner frequencies of the compensator are following:

$$d : c = b : a = 4, \quad c : b = 2 \tag{26}$$

5. Design of the Compensator Parameters

To sum up, the parameters of the compensator should satisfy the following relationships:

$$\omega_0 = \sqrt{a\,b} = \sqrt{b\,c} \approx (GCF)min = 2.57\ rad/s \tag{27}$$

$$\omega_2 \approx \sqrt{c\,d} > (PCF)min = 6.66\ rad/s \tag{28}$$

$$\omega_1 \approx \sqrt{a\,b} = \omega_0^2/\omega_2 = 0.992\ rad/s \tag{29}$$

$$d \approx (PCF)max = 17.28\ rad/s \tag{30}$$

$$a\,d = b\,c, \quad a : b = c : d \tag{31}$$

$$d : c = b : a = 4, \quad a : b = 2 \tag{32}$$

Solving the equations (27)to(32), the parameters of the compensator are designed as:

$$a = 0.5, \quad b = 2, \quad c = 4, \quad d = 16 \tag{33}$$

simply noted as:

$$W_c = [\ 0.5,\ 2,\ 4,\ 16\ ] \tag{34}$$

It expresses following trasfer function of the compensator:

$$W_c = (s/2+1)(s/4+1)\ /\ (2s+1)(s/16+1) \tag{35}$$

Its frequency characteristics are:

$$\omega_1 = 1\ rad/s, \quad \omega_0 = 2.83\ rad/s, \quad \omega_2 = 8\ rad/s \tag{36}$$

Test Results

1. Increasing of Stability Margins

Through the rig-test in Aircraft the phase margin of the FBW system has been enlarged by 13.84 degrees on an average. For 0806 situation, the phase margin changed from 19 degrees to 51.6 degrees, increased by 32.6 degrees. The gain margin increased by 2.12 dB on an ave rage, they all reached 10 dB gain margin requirement. The open-loop phase crossover frequency increased by 0.59 Hz on an acerage. The phase crossover frequency bands changed from 1.06—2.75 Hz to 1.81—2.84 Hz, 0806 got excellent result, its PCF changed from 1.06 Hz to 2.55 Hz, the net increase of 32.6 degrees on phase margin has been attained. Comparing the Fig. 10 with Fig. 8, the separtion between the GCF and the PCF enlarged from 0.1 Hz to 0.82 Hz. The reason for this is the compensator offered larger phase-lead in this region, so the gain and the phase margins of the system completely satisfy the specification. They are presented in Fig. 10 .

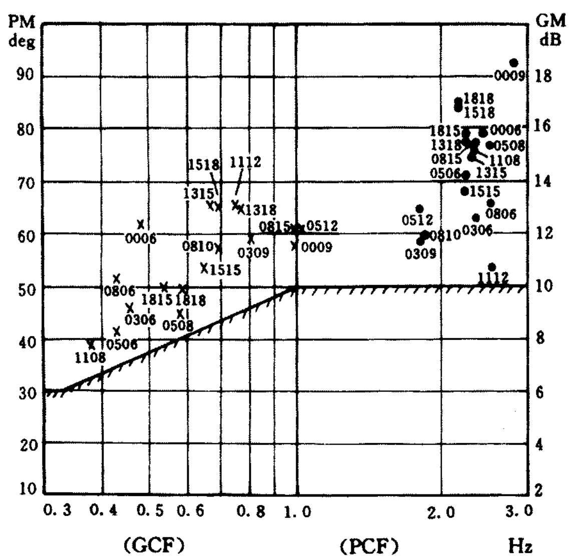

Fig. 10 Gain and Phase Margins of the System After Compensation.

2. Improving of Handling Quality

From a great deal of the test data, with equvalent match the self-oscillation frequencies without damping have been universally increased, by 0.42 rad/s on an average. The relative damping ratios have been entirely increased by 0.095 on an average, and the equivalent match delay has been reduced by 0.05 second.

SUMMARY

The forward loop gain changing law formed from changing the gain respectively with indicated airspeed and altitude, and then taking the larger one can excellently compensate the variations in aircraft stabilator effectiveness due to changes in airspeed and/or altitude.

The harmonic oscillation is the key technique in the development of the fly-by-wire system. The discovery of physical cause of tailplane buffet and the proposal of methods to ovecome the buffet may provide important references for the design of modern flight control system.

The emoloyment of the compensator designed by using the method presented in the paper to compensate a set of frequency curves over the entire flight envelope can make both the gain and phase margins of the fly-by-wire system completely meet the requirements, and the handling quality is also improved.

The solution of the above-mentioned key problems made the fligths of the fighter employing the FBW system very successful. The demonstration program had been will accomplished.

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