MULTIPLE HYPOTHESIS FAULT DETECTION FOR AN AIRCRAFT SENSOR SYSTEM

H. Göllinger
Technical University of Braunschweig
Institute for Flight Guidance and Control
Rebenring 18, 38106 Braunschweig
Federal Republic of Germany

Abstract

This paper adresses the problem of online fault detection. A statistical test, the Sequential Probability Ratio Test (SPRT) is extended to determine between several hypotheses. This test is used to investigate the innovations of a bank of Kalman filters. Each filter is designed to cope with a dedicated sensor fault. By evaluating the innovations the extended SPRT algorithm decides, which Kalman filter fits best to the measured data.

This method is applied to fault detection of sensors measuring the vertical motion of an aircraft. A changing number of sensor signals has to be considered. This is done by use of a time variant structure of the output equations. Simulated and measured data of test flights with simulated faults show the performance of this test.

Introduction

In the last years large efforts were made particularly in the field of aeronautics to improve sensor fault detection algorithms. A fault can either be detected by use of hardware or software redundancy. Hardware redundancy uses identical instruments which are monitored by a comparison scheme. In opposite, software redundancy uses the inherent redundancy of different sensors in an aircraft. These sensors measure different states that are related to each other by a mathematical model. One powerful approach is the so called multiple-hypothesis detection scheme(6).

This approach uses a bank of Kalman filters. Each of them is designed to describe the plant with one specific fault. A test algorithm is used to decide, which of the filters fits best to the measured data. This is seen from the Kalman filter innovations, e.g. the difference between predicted and measured values.

The algorithm investigates a time window of the innovations to decide, which of the Kalman filters fits best. The window size has a strong influence on the test performance. On one hand, a small window size results in fast fault detection and a high probability of false decision. On the other hand, a large window size increases detection time, but decreases the probability of false decision.

The disadvantages of a test with fixed window size are avoided by a test that increases the time window until it could decide with a given probability of false decision. This test is known as Sequential Probability Ratio Test (SPRT)(5). It was extended by P. Armitage to the case of multiple hypothesis testing(1).

In this paper, the SPRT is presented first. Then, the SPRT is extended to multiple hypothesis testing and a simple formulation is presented. This test is applied to the detection of sensor faults in the vertical motion of an aircraft. One of the sensors signals is not available at each sample step. It is shown how these informations could be integrated into the algorithm by just changing the output equations of the state space formulation. At last, the performance of this test is shown with simulated and with real flight test data that are falsified by simulated faults.

Sequential Probability Ratio Test

The Sequential Probability Ratio Test is a statistical test that investigates a given time series of data and chooses between two alternative hypotheses(3). In contrary to a test with fixed sample size the SPRT examines one observation at a time. The test stops if a hypothesis is selected, otherwise the test is continued by examining the next observation. This test is optimal in the sense that the decision is made in minimum time with respect to given error probabilities.

Suppose there are two hypotheses H_i and H_j. At sample step k, the probabilities $P_i(k)$ and $P_j(k)$ are calculated. They are the probabilities to obtain the measured data of sample steps 1 to k under hypothesis i and j.

Now, the log-likelihood ratio

$$\lambda_{ji}(k) = \log \frac{P_j(k)}{P_i(k)} \qquad (1)$$

is calculated. The decision algorithm is:

$$\lambda_{ji}(k) \le \log B_{ji} \quad : \text{accept } H_i$$

$$\lambda_{ji}(k) \ge \log A_{ji} \quad : \text{accept } H_j$$

$$\log B_{ji} < \lambda_{ji}(k) \le \log A_{ji} \quad : \text{take next sample } k+1. \qquad (2)$$

The logarithm is introduced here for ease of computation because Gauss distributions will be treated. The error probability β_{ij} is introduced to calculate the acceptance thresholds A_{ji} and B_{ji}. β_{ij} is the probability that hypothesis H_i is accepted though hypothesis H_j is correct. The thresholds are

$$A_{ji} = \frac{1 - \beta_{ij}}{\beta_{ji}} \qquad B_{ji} = \frac{\beta_{ij}}{1 - \beta_{ji}} \ . \qquad (3,4)$$

Assume H_i to be the fault-free hypothesis and H_j to be the hypothesis with a fault. Then the error probability β_{ji} corresponds to the probability α of a false alarm and β_{ij} corresponds to the probability β of a missed alarm.

<u>Extension to multiple hypothesis test</u>

Assume there are N hypotheses $H_1, H_2 \ldots H_N$. To achieve a global decision for one hypothesis H_m, every hypothesis has to be compared to all the others(7). So there are a total of N(N-1)/2 likelihood ratios that are to be tested by SPRTs. The global test should terminate and accept hypothesis H_m if every SPRT concerning H_m decides for this hypothesis.

The algorithm mentioned above could be formulated in a manner that only one of the two thresholds is used:

> Hypothesis H_m (j,m = 1..N) is accepted, if $\lambda_{jm}(k) \le \log B_{jm}$ holds for all $j \neq m$. The next observation at sample step k+1 is taken into consideration if none of the hypotheses H_m is accepted. (5)

Only thresholds B are used here. It could be shown that this algorithm is equivalent to using the thresholds A:

> Hypothesis H_m (i,m = 1..N) is accepted, if $\lambda_{mi}(k) \ge \log A_{mi}$ holds for all $i \neq m$. The next observation at sample step k+1 is taken into consideration if none of the hypotheses H_m is accepted. (6)

<u>Properties of the Kalman filter</u>

The state space description of a linear stochastic dynamic system is:

$$\underline{x}_i(k+1) = A_i\, \underline{x}_i(k) + B_i\, \underline{u}(k) + \underline{w}_i(k) + \underline{g}_i(k)$$

$$\underline{y}_i(k) = C_i(k)\, \underline{x}_i(k) + \underline{v}_i(k) + \underline{f}_i(k) \ . \qquad (7,8)$$

$\underline{u}(k)$ is the system input vector, $\underline{x}_i(k)$ is the state vector and $\underline{y}_i(k)$ is the output vector of model i. The state vector could be of different dimensions for different models i. Also the dimension of the output vector $\underline{y}_i(k)$ could change in relation to the number of output signals measured at sampling step k. This is also taken into consideration by the time varying dimensions of matrix $C_i(k)$. $\underline{g}_i(k)$ and $\underline{f}_i(k)$ are deterministic functions that are introduced to represent faults like bias. Faults affecting the dynamic behaviour of the plant are considered by individually choosing the differential equations for each model i.

Process noise $\underline{w}_i(k)$ and sensor noise $\underline{v}_i(k)$ are assumed to be Gaussian white noise with

$$E\{\underline{w}_i(k)\} = \underline{0} \qquad E\{\underline{w}_i(k)\ \underline{w}_i(l)^T\} = Q_i\ \delta_{kl}$$

$$E\{\underline{v}_i(k)\} = \underline{0} \qquad E\{\underline{v}_i(k)\ \underline{v}_i(l)^T\} = R_i\ \delta_{kl} \ . \qquad (9\text{-}12)$$

The Kalman filter algorithm consists of the following equations:

$$
\begin{aligned}
\underline{\gamma}_i(k) &= \underline{y}(k) - C_i(k)\, \underline{x}_i(k,k-1) - \underline{f}_i(k)\\
V_i(k) &= C_i(k)\, P_i(k,k-1)\, C_i^T(k) + R_i(k)\\
K_i(k) &= P_i(k,k-1)\, C_i^T(k)\, V_i^{-1}(k)\\
\underline{x}_i(k,k) &= \underline{x}_i(k,k-1) + K_i(k)\, \underline{\gamma}_i(k)\\
P_i(k,k) &= [\, I - K_i(k)\, C_i(k)\,]\, P_i(k,k-1)\\
\underline{x}_i(k+1,k) &= A_i\, \underline{x}_i(k,k) + B_i\, \underline{u}(k) + \underline{g}_i(k)\\
P_i(k+1,k) &= A_i\, P_i(k,k)\, A_i^T + Q_i \; . \qquad (13\text{-}19)
\end{aligned}
$$

The innovation $\underline{\gamma}_i(k)$ is the difference between the measured output and the predicted output of the Kalman filter corresponding to model i. The innovation is normally distributed with zero mean und covariance $V_i(k)$, if the differential equations implemented in Kalman filter i correspond to the plant. The probability density of an innovation vector $\underline{\gamma}_i(k)$ of time step k is:

$$
P_i(\underline{\gamma}_i(k)) = \frac{1}{(2\pi)^{n/2}\, |V_i(k)|^{1/2}} \cdot \exp\!\left(-\frac{1}{2}\, \underline{\gamma}_i^T(k)\, V_i^{-1}(k)\, \underline{\gamma}_i(k)\right)
$$

with n : dimension of $\underline{\gamma}_i(k)$. (20)

The probability density of the time series of the innovation $\Gamma_i(k) = [\underline{\gamma}_i(1) \ldots \underline{\gamma}_i(k)]$ is obtained by:

$$
P(\Gamma_i(k)) = \prod_{j=1}^{k} P(\underline{\gamma}_i(j)) \; . \qquad (21)
$$

So the log- likelihood ratio (1) of the actual step k could be calculated recursively:

$$
\lambda_{ji}(k) = \lambda_{ji}(k-1) + \log \frac{P_j(\underline{\gamma}_j(k))}{P_i(\underline{\gamma}_i(k))} \; . \qquad (22)
$$

Application: Measurement of the vertical motion

The plant

The extended SPRT is used to detect faults in the measurement of the vertical motion of an airplane. Generally, the three-dimensional motion is described by kinematic differential equations of motion, which include the three-dimensional vectors of acceleration, velocity and position[2]. Here, these equations are reduced to the one-dimensional vertical motion.

It is assumed that there are several sensors that could be used to determine the vertical motion: an Inertial Navigation System (INS) that is used here for measuring the accelerations, a barometric altimeter and a satellite navigation system (GPS).

The INS measures the turn-rates and the acceleration in body-fixed coordinates. With these data, the Euler angles and the vertical acceleration a_{zg} can be determined.

According to [4] the dynamic behaviour of the barometric altimeter used here is approximated by a low pass of first order:

$$
H_{alt}(s) = \frac{1}{1 + T_b s}\, H(s) \; . \qquad (23)
$$

GPS is a navigation system that uses runtime differences of signals from several satellites to calculate the position of the receiver every 0.6 sec. It is assumed that the GPS is used in differential mode for high accuracy. The calculated position refers to WGS84 (World Geodetic System 84), whereas the barometric altitude refers to sea level. It is assumed that the offset of the measured altitude between sea level and WGS84 is known. Then, one could compensate for the offset, so that these two altitudes coincide.
The structure of the plant is shown in Fig. 1.

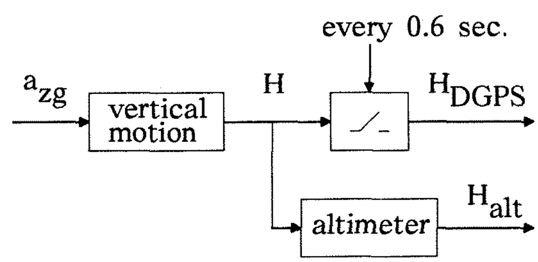

Fig. 1 : Block diagram of the plant

Modelling of sensor faults

Assuming a flat non-rotating earth, the vertical movement is described by the relation between the altitude H and the acceleration in vertical direction a_{zg}:

$$
\ddot{H} = -a_{zg} \; . \qquad (24)
$$

Now the state space equations are introduced that are used in the four Kalman filters. The state vectors $\underline{x}_i$ (i=1..4) all consist of the altitude H, the vertical velocity $\dot{H}$ and the barometric altitude H_{alt}.

The state space equations of the vertical motion without sensor fault (hypothesis H_1) are given by (7,8) with i=1:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1/T_b & 0 & -1/T_b \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad g_1(k) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \underline{f}_1(k) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{25}$$

The state space equations of the vertical motion with a bias fault in the measurement of the barometric altitude (hypotheses H_2 and H_3) are the same as those of hypothesis H_1:

$$A_2 = A_3 = A_1, \quad B_2 = B_3 = B_1$$
$$C_2 = C_3 = C_1, \quad g_2(k) = g_3(k) = g_1(k) . \tag{26}$$

The output equations differ due to the sensor faults. A bias of plus and minus 1 m is assumed:

$$\underline{f}_2(k) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{f}_3(k) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} . \tag{27}$$

The state space equations of the vertical motion with additional noise in the measurement of the barometric altitude (hypothesis H_4) are the same as those of hypothesis H_1:

$$A_4 = A_1 , \quad B_4 = B_1 , \quad g_4(k) = g_1(k) ,$$
$$C_4 = C_1 , \qquad \underline{f}_4(k) = \underline{f}_1(k) . \tag{28}$$

The covariance matrix R_4 is changed due to the increased measurement noise.

Time variant structure of the output equations

The barometric altitude is measured every 0.02 sec., but the GPS position informations are only valid every 0.6 sec. This is considered in the Kalman filter algorithm as follows. Only the first line of the output equations is used, if only the barometric altitude is measured. Both lines are used if barometric and GPS altitude are available. The size of the innovation vector $\underline{\gamma}_i(k)$ and the dimension of the covariance matrix $V_i(k)$ change according to the number of measured sensor signals. But this is already taken into account in (20), so that the probability density could be calculated and there is no change in the extended SPRT algorithm.

Adaption of the Kalman filters after a decision

Consider hypothesis H_a to be true. Kalmanfilter A copes very well with the measured data, but the other filters might diverge because the differential equations of plant and filter do not conincide. Now, a fault occurs that corresponds to Kalman filter B. Because of the diverged states, filter B needs a lot of time to adapt to the new situation. A long detection time would be the result. So an adaption algorithm is used to speed up the detection. Every time the extended SPRT has decided for a hypothesis, all the other Kalman filters are initialized with the states of the chosen Kalman filter.

Fig. 2 shows the structure of the fault detection algorithm, that consists of a bank of Kalman filters (KF) and the extended SPRT.

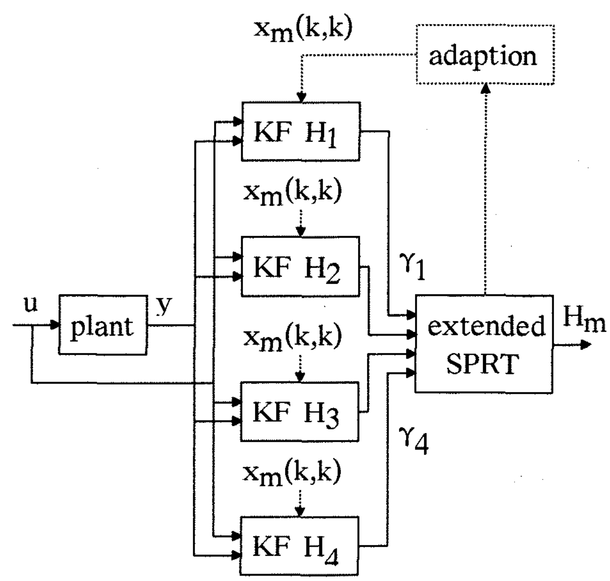

Fig. 2 : Structure of the detection algorithm

Simulation results

The performance of the proposed test is shown with simulated data. The matrices of hypothesis H_1 (without fault,(25)) are used to describe the plant.

The sensor noise standard deviations are assumed to be: $\sigma_{azg} = 0.05\ ^{m}/_{sec^2}$, $\sigma_{Halt} = 0.5$ m, $\sigma_{DGPS} = 0.2$ m, $T_b = 0.3$ sec. and the error probability is assumed to be $\beta_{ij} = 10^{-4}$ with i,j=1..4, i≠j.

At t = 10 sec., a bias fault of +1 m in the measurement of the baromatric altitude is introduced. The extended SPRT detects this fault very quickly (Fig.3a). At t = 20 sec., the bias fault is removed and the algorithm decides for hypothesis H_1 again. The peaks at the bottom of the diagrams show the time instants when the decision algorithm decides for one of the hypotheses.

The extended SPRT is used now to detect a bias of +0.65 m (Fig. 3b). The fault is detected correctly, but the algorithm needs more time for detection and to switch back to hypothesis H_1 compared to the bias fault of 1m.

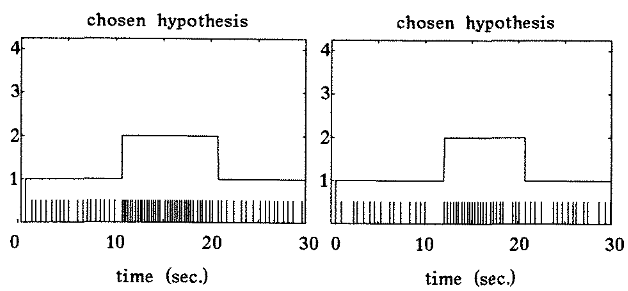

Fig. 3 : Detection of a bias of 1 m (a) and detection of a bias of 0.65 m (b)

A bias of -1 m in the measurement of the barometric altitude is introduced at t = 10 sec. and removed at t = 20 sec. As expected, the bias fault is easily detected (Fig. 4a).

Now, increased noise in the measurement of the barometric altitude is to be detected (hypothesis H_4). The faulty sensor noise has got a standard deviation of $\sigma_{Halt} = 1$m, and the Kalman filter covariance matrix R_4 is set accordingly. As before, the fault is introduced at t = 10 sec. and removed at t = 20 sec. The detection results could be seen in Fig. 4b.

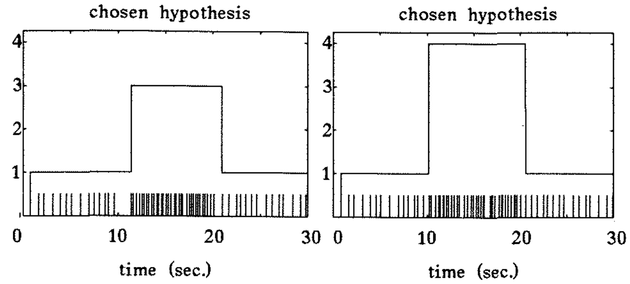

Fig.4 : Detection of a bias of -1 m (a) and detection of increased noise (b)

Flight test data

The extended SPRT is now applied to real flight test data, obtained during flights of the research aircraft of the Institute of Flight Guidance and Control, a twin-engine Dornier DO 128. The offset between sea level and WGS84 is known for Braunschweig airport, where the trials took place. The DGPS positions are corrected so that they refer to sea level.

The rotating earth is taken into consideration in the matrices A_i with:

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ k & 0 & 0 \\ 1/T_b & 0 & -1/T_b \end{bmatrix}$$

$$\text{with } k = 3.092 \cdot 10^{-6} \qquad (29)$$

Fig. 5 shows the measured altitudes with a fault of 2 m in H_{alt} introduced at 5 sec and removed at 10 sec. The measured altitude H_{DGPS} is shown as single points, when a new position is calculated.

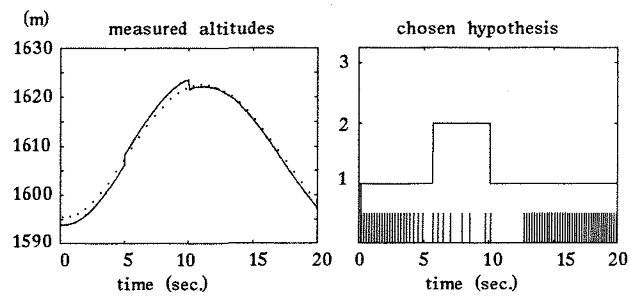

Fig. 5 : measured altitudes H_{DGPS} and H_{alt} (a) and chosen hypotheses (b)

Due to the simple model of the barometric altimeter and the resulting model uncertainties the noise covariances are increased. The extended SPRT chooses between three Kalman filters that are designed as follows: no fault (H_1), a bias of 2 m (H_2) or -2 m (H_3).

The fault is correctly detected, but due to the reasons mentioned above, the decision algorithm needs about 2.5 sec. to decide for hypothesis H_1 at t = 10 sec.

Conclusions

In this paper a fault detection algorithm is proposed that consists of a bank of Kalman filters and an extended SPRT, that evaluates the innovations. The changing number of sensor signals is taken into consideration in the Kalman filters by the use of time variant output equations.

The extended SPRT has been used to detect faults in the measurement of the vertical motion of an aircraft. Simulated and measured data show the performance of the test.

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