A COMPARISON BETWEEN AN OPTIMAL CONTROL LAW DESIGN AND A POLE-PLACEMENT CONTROL LAW DESIGN WITH RESPECT TO STABILITY CHARACTERISTICS AND GIBSON DROPBACK CRITERION

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ABSTRACT

A design have been carried out for a rate command-attitude hold control law including a proportional plus integral controller acting on pitch rate and angle of attack, both feedbacked to elevator. The proportional feedback enables the rate command characteristcs to be tailored as required and the integral feedback drives the error signal to zero obtaining good longer term holding characteristics, also a feedforward acting on the reference input is included in the controller allowing to shape the response as required. The objective of the design is to obtain a flight control system that meets the Gibson dropback criterion and also the stability requirements of MIL-F-8785C. The design have been performed by two methods, pole placement and optimal control (LQR). Both designs have been compared when an actuator dynamics are included and also when the phugoid model is included. The control effort and control rate effort are analyzed and compared, when the flight control system works as a regulator and when works with a pilot input, as required by the dropback criterion. The dynamic characteristics of the aircraft augmented with both designs have been obtained and analyzed. A simplification of the control law design was also studied. A robustness analysis of both designs was also studied when the gains are varied with respect to the nominal designed gains.

1 POLE PLACEMENT CONTROL LAW DESIGN

The aircraft model used was the Boeing B-747, and the appropriate data can be found on Heffley[1]. First the short period longitudinal reduced model is used as,

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \eta \quad (1)$$

with $x^T = [\ w \quad q\ ]$ (2)

it is possible to use

$$\dot{x} = A\,x + B\,\eta \quad (3)$$

The control law must ensure that the augmented aircraft satisfies the stability requirements contained on MIL-F-8785C[2] and the Gibson dropback criterion[3]. The control anticipation parameter, CAP, contained on MIL-F-8785C will take into account the stability requirements and the dropback parameter, DB, defined in Cook[4] will take into account the dropback criterion. So CAP is defined as,

$$CAP = \frac{g\, T_{\theta 2}\, \omega_{sp}^2}{V_e} \quad (4)$$

and dropback is,

$$DB = \frac{T_{\theta 2}\omega_{sp} - 2\zeta_{sp}}{\omega_{sp}} \quad (5)$$

The control law structure is showed on figure (1) and is simply a PI controller with a feedforward gain.

As can be seen it is necessary to add an extra state, that is, ε_q, defined as,

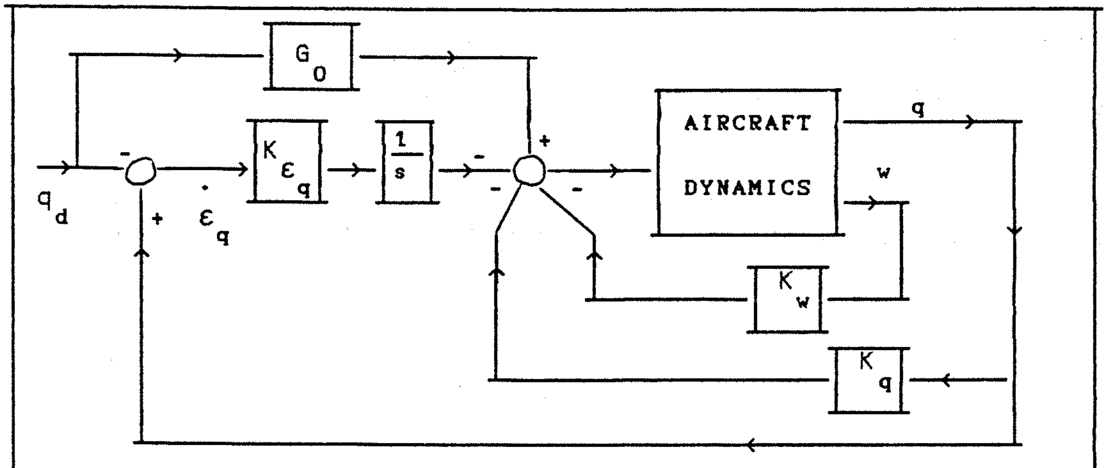

figure 1 - control law structure

$$\dot{\varepsilon}_q = q - q_d \quad (6)$$

in order to take into account the error. With the help of (4) and (5) it is possible to define ω_{sp} and ζ_{sp} with the constraint that both satisfy the CAP requirement and DB near zero as required by the Gibson criterion. The state space model including ε_q can be written as,

$$\dot{x} = A\,x + B\,\eta + E\,q_d \quad (7)$$

with now $x^T = [\, w \; q \; \varepsilon_q \,]$ (8)

and

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (9)$$

$$B^T = [\, b_{11} \;\; b_{21} \;\; 0 \,] \quad (10)$$

$$E^T = [\, 0 \;\; 0 \;\; -1 \,] \quad (11)$$

The control law is simply given by,

$$\eta = -G\,x + G_0\,q_d \quad (12)$$

with G a control vector given by,

$$G = [\, K_w \; K_q \; K_{\varepsilon_q} \,] \quad (13)$$

So the closed loop system is given by,

$$\dot{x} = (A - BG)x + (BG_0 + E)q_d \quad (14)$$

where G is simply obtained by defining the closed loop poles and G_0 is computed in order to obtain pole zero cancellation as in Friedland[5] The characteristic equation of the closed loop system is of order three and so it is necessary to define three poles, two closed loop poles are defined with the help of (4) and (5) and the third pole is defined in order that the integral term has a time constant of the same magnitude of the short period natural frequency and so not great changes are introduced in the system. As the short period natural frequency is around 1 rad/sec, the third pole have been choosed as -1, that is , s = -1, ω_{sp} = 1.20 rad/sec and ζ_{sp} = 0.83, for 20000 ft mach 0.70 which gives the complex poles -1.02 ± i 0.63. With the use of MATLAB[6] the gains are,

$$G = [\ 0.0012 \quad -0.889 \quad -1.183\ ] \qquad (15)$$

$$G_0 = \ 1.183 \qquad (16)$$

and G_0 is computed by,

$$G_0 = [C(A-BG)^{-1}B]^{-1}C(A-BG)^{-1}E \qquad (17)$$

and so the design is completed. Table (1) shows the closed loop poles choosed for this design, the CAP, ω_{sp} and ζ_{sp}.

In table (2) the gains obtained are listed.

2 OPTIMAL CONTROL LAW DESIGN

The same control law structure used in the pole placement design is now used but the design is now performed by optimal control law method, that is, by the LQR method. The state vector is given by (8) and the state space model is given by (7). The appropriate performance index is ,

$$V = \int_0^{\infty} (x^T Q\ x + \eta^T\ R\ \eta) d\tau \qquad (18)$$

Q is a (3x3) matrix and R is a scalar. The Q matrix and R can be choosed by means of a parametrically study and since only ε_q is of concern in the design than Q will be choosen as a diagonal matrix with the form:

TABLE 1

h	Mach	closed loop poles	CAP	ω_{sp}	ζ_{sp}
ft	rad		s^{-2}	rad/s	rad
1000	0.60	-1.08 ± i 1.11 , -1	0.117	1.55	0.70
20000	0.70	-1.02 ± i 0.63 , -1	0.101	1.20	0.85
40000	0.80	-1.61 , -0.449 , -1	0.086	0.85	1.21
10000	0.40	-0.58 ± i 0.59 , -1	0.092	0.83	0.70
30000	0.70	-0.86 ± i 0.25 , -1	0.087	0.90	0.96

TABLE 2

h	Mach	K_w	K_q	K_{ε_q}	G_0
ft	rad	$ft^{-1}s$	s	rad	s
1000	0.60	0.0012	-0.588	-1.219	1.219
20000	0.70	0.0012	-0.889	-1.183	1.183
40000	0.80	0.0011	-1.875	-1.697	1.697
10000	0.40	0.0026	-1.094	-1.270	1.270
30000	0.70	0.0013	-1.249	-1.252	1.252

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (19)$$

and $R = \rho$ which will be varied in order to find suitable gains in the way that the augmented aircraft meet CAP and dropback criterion. With A, B, Q and R it is possible to find the feedback gains solving the algebraic Riccati equation, that is, the classical LQR problem. To find G_0 the method given by Friedland[5] is used and the computation is given by,

$$G_0 = - R^{-1}B^T[\ A_c^T\ ]^{-1}M\ E \qquad (20)$$

where $A_c = A - B\ R^{-1}B^TM \qquad (21)$

and M is the solution of the algebraic Riccati equation, in the way that G is given by

$$G = R^{-1}B^T\ M \qquad (22)$$

So by performing a parametric study with the variation of ρ it is possible to find a set of gains that allow the augmented aircraft to meet CAP and dropback criterion. Table (3) shows the resulting closed loop poles, CAP, ω_{sp} ,ζ_{sp} and final ρ for this design.

In table (4) the gains obtained by this method are listed,

In figure (2) the performance of both designs with respect to the dropback criterion are ploted and as can be seen both satisfy the criterion quite well. Also can be seen that both satisfy CAP very well, as the CAP requirement is given by,

$$0.085 \leq CAP \leq 3.6$$

in the MIL-F8785C flying qualities requirements.

TABLE 3

h	Mach	ω_{sp}	ζ_{sp}	closed loop poles	ρ	CAP
ft	rad	rad/s	rad		rad	s^{-2}
1000	0.60	1.64	0.63	-0.23 , -1.03 ± i 1.27	10	0.130
20000	0.70	1.42	0.53	-0.27 , -0.75 ± i 1.20	5	0.141
40000	0.80	1.19	0.51	-0.24 , -0.61 ± i 1.03	1.5	0.168
10000	0.40	1.11	0.54	-0.19 , -0.60 ± i 0.93	5	0.165
30000	0.70	1.18	0.48	-0.21 , -0.56 ± i 1.03	5	0.150

TABLE 4

h	Mach	K_w	K_q	K_{ε_q}	G_0
ft	rad	$ft^{-1}s$	s	rad	s
1000	0.60	0.0002	-0.135	-0.316	1.290
20000	0.70	0.0003	-0.216	-0.447	1.286
40000	0.80	0.0005	-0.543	-0.816	1.724
10000	0.40	0.0006	-0.2803	-0.447	1.906
30000	0.70	0.0004	-0.257	-0.447	1.541

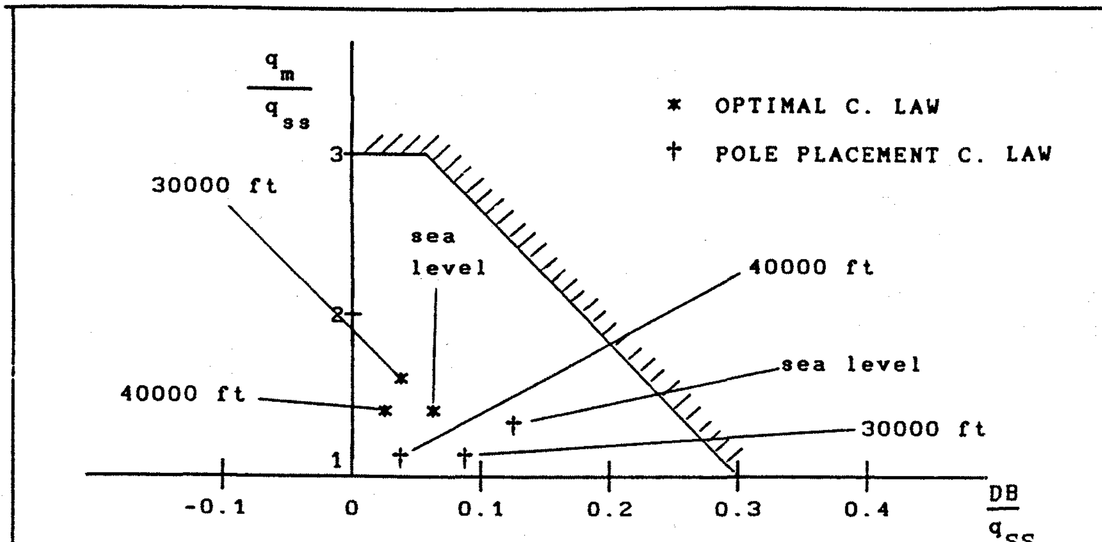

figure 2 - dropback criterion plot of both designs

3 COMPARISON OF BOTH DESIGNS

Having designed both control law an assessment of both have been carried out with respect to dynamic parameters with the help of the transfer function q/q_d. The findings of the study are summarized as :

i The feedforward gain of the optimal control law design is always greater than the feedforward gain of the pole placement control law design, that means higher control effort.

ii In the pole placement control law design K_{ε_q} and G_0 are the same, that fact simplifies the implementation.

iii The optimal control law design offers a greater settling time than the pole placement control law design.

iv The optimal control law design has a better performance with respect to CAP than the pole placement control law design.

v The pole placement control law design has a lower bandwidth compared to the optimal control law design.

The pole placement control law design offers also a lower resonant peak than the optimal control law design, and as specified in D'Souza[7] the optimum range is between 0.83 dB and 3.52 dB, so the pole placement control law design meets this requirement while the optimal control law design does not meet.

In figure (3) there is a time response comparison of both designs for flight condition 2.

4 INFLUENCE OF AN ACTUATOR

As the control law designs will be working with an actuator it is useful to analyze what happens if an actuator is included in the model. The actuator model is given by,

$$\begin{bmatrix} \dot{\eta} \\ \dot{v}_\eta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -14 \end{bmatrix} \begin{bmatrix} \eta \\ v_\eta \end{bmatrix} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} \eta_c \qquad (23)$$

with a damping ratio of 0.70 and a natural frequency of 10 rad/sec. So with the inclusion of this actuator into both designs an evaluation was carried out and the results can be summarized as: When the actuator is included the pole placement control law design still satisfying the

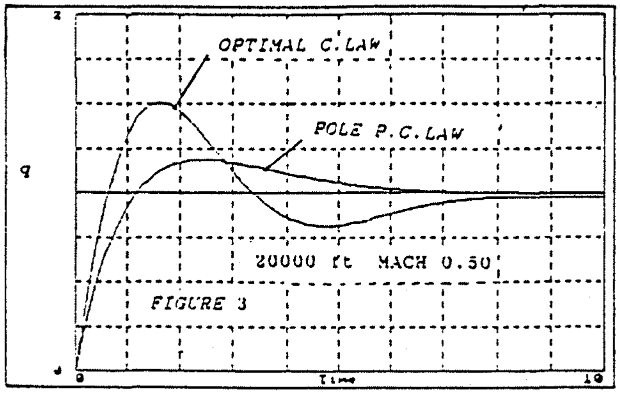

figure 3 - Comparison of pitch-rate time history of the augmented aircraft with both designs following a step input in q_d

dropback criterion for almost all flight cases while the optimal control law design no more satisfy the dropback criterion for almost all flight cases. In the aspect of CAP, the optimal control law design still satisfying CAP for all flight cases but the pole placement control law no more satisfy CAP at almost all flight cases. So it is obvious that the optimal control law design offers a better degree of robustness with respect to CAP and the pole placement control law offers a better degree of robustness with respect to dropback criterion.

5 INFLUENCE OF THE COMPLETE AIRCRAFT MODEL AND ACTUATOR

The design were carried out based on the reduced short period model, but it is important to investigate the performance of both designs when the phugoid model is also included, so the state vector to be considered is now the following,

$$x^T = [\ u\ w\ q\ \theta\ ] \qquad (24)$$

for the aircraft dynamics. With this state vector and the designed control laws an evaluation was also performed and the results can be summarized as

The results has shown that with phugoid model included as also actuator model the optimal control law design still satisfying the CAP for all flight cases, but not the dropback criterion, by the other side the pole placement control law design does not satisfy the CAP requirement more for pratically all flight cases, as also does not satisfy more the dropback criterion for some flight cases. Figure (4) shows the degradation that occurs in both control laws. It have been also noticed that the steady state error, that is the relationship of

$$[\ q/q_d]_{ss} \simeq 1$$

is pratically maintained in the pole placement control law design, but is lost in the optimal control law design. Certainly this can be atributed to the design method of the feedforward gain in each control law.

6 EFFECT OF A SIMPLIFIED CONTROL LAW

Looking for the obtained gains in each design it have been noticed that the magnitude of K_w is small compared to the others, so if K_w can be made as zero only two feedback gains will be used, and also it will be not necessary to use an angle of attack sensor, so the flight control system will be simpler ad cheaper. A study have been performed with both control law designs using $K_w = 0$, and the results have showed that in this case both designs do not meet the dropback criterion, but both still satisfying the CAP requirement very well. This can be atributed to the fact that the steady state error be dependent much more of K_{ε_q} and of K_w. Figure (5) shows what happens to both control law designs with $K_w = 0$

7 ROBUSTNESS WITH RESPECT TO DROPBACK CRITERION AND CAP WITH GAINS VARIATIONS

In order to get some idea of how robust are each control law design when the gains are varied from the nominal computed gains a study have been performed with both designs, varying the gains to + 20 % and to -20 %. In this study all the gains are varied at the same time. The study have shown that with respect to meet the dropback criterion the pole placement control law is more robust than the optimal control law when the gains are varied, that is, the pole

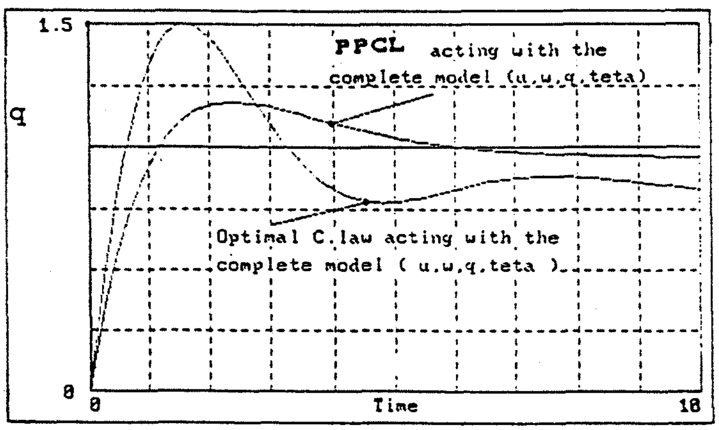

figure 4 - PHUGOID MODEL AND ACTUATOR INFLUENCE pitch rate time history comparison for a step input in q_d at 20000 ft, Mach 0.50

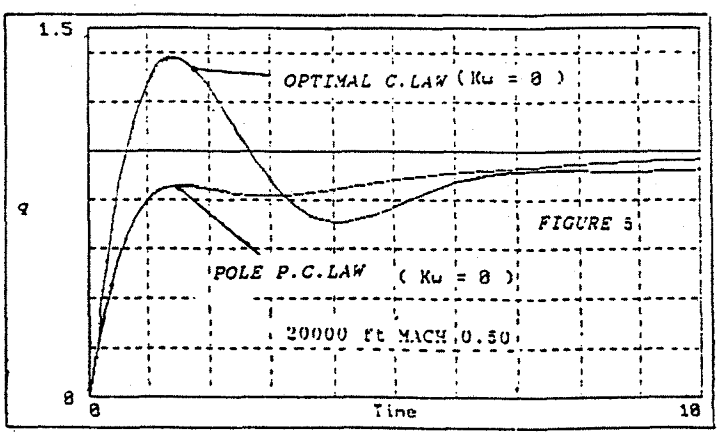

figure 5 - pitch rate time history of the augmented aircraft with both designs following a step input of q_d when $K_w = 0$ in both designs.

placement control law does not meet the criterion with this ammount of variation, but the deterioration is small compared with the deterioration that occurs in the optimal control law design. However with respect to CAP the optimal control law still satisfying CAP for all flight cases with both variations, that is + 20 % and – 20 %, instead the pole placement control law does not meet CAP for some flight cases with both variations. Figure (6) illustrates what happens for the pole placement control law design with gain variations and figure (7) shows the same for the optimal control law design.

8 CONTROL EFFORT AND CONTROL RATE EFFORT

It is very important to study the control effort and control rate effort required by each design since it is impossible to implement a design that. requires high control effort or high control rate effort with respect to the available actuator. This analysis is easily performed by obtaining the transfer function η/q_d from the state space model. The study have shown that both designs require about the same ammount of control effort, but the

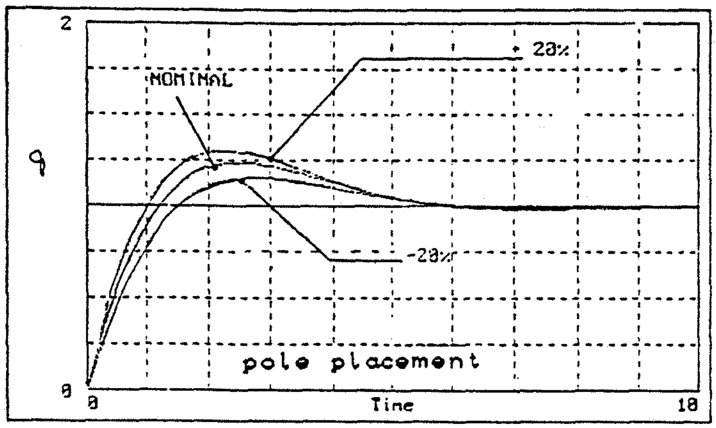

figure 6 - EFFECT OF GAIN VARIATION ON PPCL DESIGN
pitch-rate time history following a step input in q_d
at 1000 ft Mach 0.30

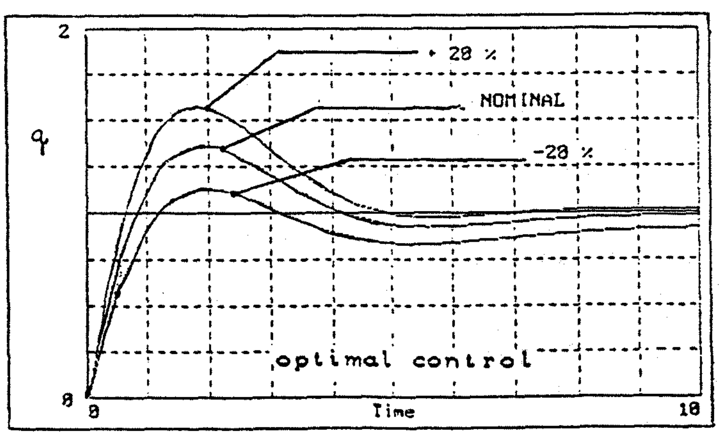

figure 7 - EFFECT OF GAIN VARIATION ON THE OCL DESIGN
pitch-rate time history following a step input in q_d
at 1000 ft Mach 0.30

optimal control law design is rerquiring more control rate effort than the pole placement control law design. Table(5) shows a comparative result obtained for a step input of q_d= 5 deg/sec for both control law designs.

TABLE 5					
h (ft)	Mach (rad)	$\dot{\eta}$ (deg/sec)		η (deg)	
		PPCL	OCL	PPCL	OCL
1000	0.30	42.8	54.6	-11.5	-11.2
20000	0.50	33.7	45.3	-10.9	-10.4
40000	0.70	43.6	45.7	-11.7	-11.3
10000	0.30	42.1	57.7	-12.4	-12.0
30000	0.50	38.6	47.7	-11.5	-10.9

PPCL = pole placement control law design
OCL = optimal control law design

9 THE REGULATOR PERFORMANCE OF BOTH DESIGNS

It is now useful to analyze the regulator performance of both designs, that is, the ability of each design to eliminate the effect of disturbances. The study was performed by simulating an angle of attack initial disturbance, that is a $w(0) \neq 0$, what is commonly called in the aeronautical industry as an alpha release simulation. The study was performed with the short period reduced order model, and the findings are : In this case as there is no pilot input, that is, the reference input q_d= 0 here, the pole placement control law design has required higher control effort and control rate effort than the optimal control law design, this was expected, since the feedback gains of the pole placement design are higher than the feedback gains of the optimal control law design. It have been also noticed that the pole placement control law design takes more time than the optimal control law design to restore w to zero.

10 CONCLUSIONS AND OBSERVATIONS

From the study carried out in this work it have been noticed that the optimal control law design is more robust with respect to meet the CAP requirement, as also is better with respect to regulator performance, that is, disturbance rejection. The optimal control law design also requires the same control effort when there is a pilot input, and lower control effort and control rate effort when working as a regulator. The disadvantage of the optimal control law is relative to control rate effort when working with pilot input, that requires higher control rate effort than the pole placement control law design. The pole placement control law design offers a better performance with respect to dropback criterion when the actuator is included or the phugoid model is included, however does not offer a reasonable level of robustness with respect to meet the CAP requirement (stability). It appears that the optimal control law design is more flexible to be redesigned in order to meet the dropback criterion again when the phugoid model is included or the actuator, in other words, is a design that can accept changes more easily than the pole placement control law design, and continuing to satisfy the requirements.

11 AIRCRAFT DATA

The aircraft data used in this work is summarized here and can be found on Heffley[1].

12 NOTATION

- u perturbed longitudinal aircraft velocity
- w perturbed normal aircraft velocity
- q perturbed aircraft pitch-rate
- θ perturbed aircraft pitch-attitude
- h aircraft altitude above the earth
- a_{ij} elements of the aircraft longitudinal state matrix A

flight condition 20000 ft - Mach 0.70

data for the complete model	data for the reduced short period model
$A = \begin{bmatrix} -0.0048 & 0.0596 & -21.528 & -32.18 \\ -0.1243 & -0.6660 & 732.76 & -0.9717 \\ 0.0001 & -0.0018 & -0.7070 & 0.0002 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$A = \begin{bmatrix} -0.6660 & 732.76 \\ -0.0018 & -0.7070 \end{bmatrix}$
$B^T = [\; 0.9783 \quad -33.543 \quad -1.9173 \quad 0.0 \;]$	$B^T = [\; -33.543 \quad -1.9173 \;]$

flight condition 30000 ft - Mach 0.70

data for the complete model	data for the reduced short period model
$A = \begin{bmatrix} -0.0035 & 0.0480 & -55.220 & -32.09 \\ -0.1140 & -0.4800 & 696.70 & -2.5840 \\ 0.0001 & -0.0014 & -0.5060 & 0.0003 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$A = \begin{bmatrix} -0.4800 & 696.70 \\ -0.0014 & -0.5060 \end{bmatrix}$
$B^T = [\; 1.9180 \quad -24.400 \quad -1.4190 \quad 0.0 \;]$	$B^T = [\; -24.400 \quad -1.4190 \;]$

flight condition 40000 ft - Mach 0.80

data for the complete model	data for the reduced short period model
$A = \begin{bmatrix} -0.0041 & 0.0516 & -60.510 & -32.10 \\ -0.0881 & -0.3703 & 768.50 & -2.5450 \\ 0.0000 & -0.0011 & -0.4434 & 0.0003 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$A = \begin{bmatrix} -0.3703 & 768.50 \\ -0.0011 & -0.4434 \end{bmatrix}$
$B^T = [\; 1.6389 \quad -20.980 \quad -1.2100 \quad 0.0 \;]$	$B^T = [\; -20.980 \quad -1.2100 \;]$

b_{ij} elements of the aircraft longitudinal control matrix B

η control effort

v_η control rate effort

x aircraft longitudinal state vector

η_c control input to the aircraft

A aircraft longitudinal state space matrix

B aircraft longitudinal control matrix

G vector of the feedback control gains

G_0 feedforward gain

ω_{sp} short period natural frequency

ζ_{sp} short period damping ratio

g gravity acceleration

$T_{\theta 2}$ numerator term of the transfer function q/η obtained with the reduced order short period model.

V_e steady state velocity of the aircraft

DB dropback

K_w feedback gain relative to w feedback path

K_q feedback gain relative to q feedback path

K_{ε_q} feedback gain relative to ε_q feedback path

q_d pitch rate demand to be tracked

CAP control anticipatiom parameter

ε_q integral of the pitch rate error

η control rate effort

A_c aircraft closed loop state matrix

M Riccati matrix

V performance index

Q state weight matrix

R control weight matrix

ρ control weight parameter

s laplace operator

s seconds

sec seconds

rad radians

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