THE MANOEUVRE EFFECT IN SKI-JUMP TAKE-OFF TRAJECTORY

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Abstract. Ski-jump take-off for aircraft has existed as a technique for a considerable time now. Using a ski-jump for the take-off of the aircraft in itself leads to a gain in take-off length since the trajectory, from the moment of take-off, becomes semi-ballistic. If, in addition to the ski-jump, there is the possibility of rotating the thrust, the benefits in terms of rolling become considerable.

In previous works, we have determined, given the main aerodynamic, propulsive, weight and geometrical characteristics of the aircraft, the vectored thrust angle and the jump exit angle which define the overall minimum length of the runway necessary for take-off. In all previuos works the manoeuvre procedure both for rotating thrust and longitudinal control didn't be taken into account: we always considered a standard manoeuvre procedure only. Now the aim of this work is to study the most suitable ways for longitudinal maneouvre and for the thrust to return to axis after the aircraft has left the ski-jump. These two types of intervention, which are obviously left up to the pilot, have consequences on the type of trajectory which the aircraft assumes after take-off. It is well known that this trajectory, especially for security reasons, must have certain geometric characteristics.

This study have therefore saught to satisfy two requirements, both important for the ski-jump take-off: obtain the minimum global runway length (landing lane and ski-jump) necessary for take-off and to fulfil the safety standards required by the post-take-off trajectory.

For the manoeuvre concerning the thrust rotation we have considered in particular the ways in which the thrust must return to its axis: the beginning of the returning rotation and the thrust rotation velocity. For the longitudinal manoeuvre we have concentrated our attention on the type of manoeuvre and in particular, for the gradual linear one, on the variation speed of the elevator angular position, taking into account the influence that flight speed has on the effectiveness of the manoeuvre itself.

The results are presented in two forms: like comparison of trajectories, each one caracterized by a particular manoeuvre procedure and in the shape of three-dimensional diagrams reporting the minimum global runway length necessary for take-off as a function of the vectored thrust angle and the jump exit angle.

Introduction

The ski-jump has been used over the last decennaries as an instrument which allows to take-off with a certain overload.

Even using a ski-jump only during take-off of an aircraft already shows its benefits in launch distance, because the tipical trajectory during the take-off becomes semi-ballistic. In this way, the aircraft can detach from the ski-jump with a certain speed, that still does not allow it the aerodynamic sustenance, but the vertical component that it gains during runway on the ski-jump, allows it to accomplish a trajectory, which is at first with a vertical negative acceleration, therefore with a concavity towards to low, and during which the aircraft itself gains enough speed for sustaining itself aerodynamically, thanks to the thrust action.

If, besides the ski-jump you add the possibility of rotating the thrust in the aircraft, you may have more considerable benefits in terms of runway length. The benefit due to the presence of this second factor is obvious, if you think about the usefullnes of having a force sustenance component during the detachment from the ski-jump, when the only aerodynamic sustenance is not sufficient. The presence of both these benefits (ski-jump and thrust rotation) at the same time, allow a take-off that in best conditions can be schematized by Fig.1.

The equations that define the aircrafts' motion during take-off differ, according to which take-off phase we are considering. We have to take into consideration the two main phases: the first one that

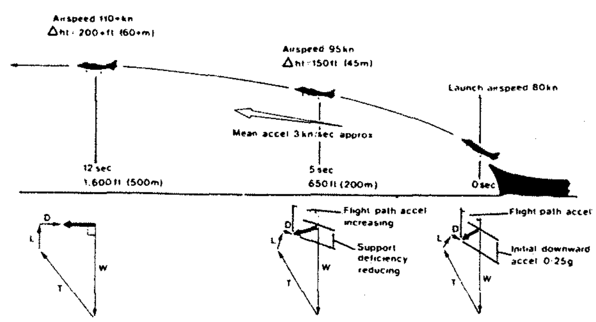

Figure 1: Tipical take-off trajectory with ski-jump and thrust rotation.

lasts until the aircraft is still on the runway and the following one of flight, from the take-off onwards.

As far as the running time is concerned, it would be better to obtain the moto equations starting from the balance force equations, with reference to the ramp of the ski-jump. The simplifications that these equations undergo are immediately noticeable in the horizontal part of the runway preceeding the ski-jump.

During the flight phase, that is from take-off from the ski-jump onwards, the forces involved are substantially the same, but the drag is by now only aerodynamic (that is, the rolling drag is not present anymore) and you do not pay attention to the centrifugal force because trajectories with very large bending radius have been enforced. Adding the centrifugal force (in the same or adverse way of the lift) to the polygonal of the setting forces wouldn't represent a problem anyway, because we know the aircrafts' trajectory every instant.

Subsequently, we will simply have to integrate the relations containing th horizontal resultant force F_o and the vertical resultant force F_v:

$$\frac{d^2X}{dt^2} = F_o\,\frac{g}{W}\,; \qquad \frac{d^2Y}{dt^2} = F_v\,\frac{g}{W} \tag{1}$$

to obtain the components X and Y, horizontal and vertical respectively, of the space s covered in the direction of movement.

For these studies two calculation programs have been worked out, where particular attention has been given to the adaptability of them to the variuos situations that may occur. This requirement has been satisfied leaving a large number of inputs in the programs. The file of input data is the same for both the calculation programs. First of all we find the inferior and superior limits of the variable quantities that have to be studied and their variations, whose effects want to be seen. These quantities are: the exit angle γ from the ski-jump, the final thrust rotation angle β and the total launch distance $Lski$. Obviously, also the number of the values between the minimum and maximum limit are given.

Than there is the quantity that characterises the ski-jump that is, the ramp length L_{ramp}.

Amoung the inputs of the programs, there are those quantities that characterises the aircraft studied and that directly intervene on the take-off length. These are: the zero altitude static thrust T_{S0}, the take-off total weight W, the static by-pass ratio of the turbo-jet τ_{by}, the take-off minimum drag coefficient C_{D0}, the Ostwald factor e that intervenes on the quadratic approximation of the aerodynamic aircraft polar, the runway angle of attack α_{rw}, function of the opening percent of the flaps, the take-off angle of attack α_{to}, determined by the maximum rotation angle of the aircraft on ground, the aspect ratio A, the wing load W/S and the maximum load factor n_{lim} that we admit to reach (usually on the ramp).

Another category of inputs is the one that characterises the type of manoeuvre made, that may be either the longitudinal control of the aircraft or the thrust rotation; this subject will be deeply dealt with in the following paragraphs.

Then, there are those quantities that depend upon the operative conditions: take-off standard altitute z_{to} considering that the operations occur in a International Standard Atmosphere, the height at which the thrust control starts, that is the height of return rotation thrust H_{con}, the minimum accepted height of the trajectory H_{min} (if we foresee the possibility of accepting a partial relapse of the aircraft on the trajectory after take-off), the height of a fictitious abstacle corrisponding to the total take-off length H_{obs} (even if according to the regulation it should be fixed at 35 ft), the operating altitude z_{oper} which the various trajectories tend to follow because we foresee the fact that the aircraft ought to reach as quickly as possible a certain flight altitude, and the rolling drag coefficient that has been considered as a function having a constant part and a part having a function of speed and of load on the wheels.

In both the programs, first of all the normal take-off length L_{ref} of the aircraft is calculated, that means the length necessary for the considered aircraft to take-off if it does not use the ski-jump or the thrust rotation. This part of the program has been developed using the classical treatise of take-off [2] and it has been inserted into the program because it supplies a useful comparison term, besides the value for adimensionalizing the take-off lengths with the ski-jump: $Alski = Lski/L_{ref}$.

The first program also calculate and draws the trajectories of the aircraft relatively to the type of ski-jump (that is, to the exit angles γ and to the launch distances $Lski$) and to the thrust rotation angles β chosen. With these results you immediately understand the influence that the parameters considered have on the type of trajectory and, above all you can underline the criterion that allows to choose the best trajectories that are those that permit a take-off without dangers and that satisfy the rules imposed.

The second program instead, provides a way of considering (for a certain aircraft with a certain configuration) all the ski-jump geometries ($Lski$ e γ) and all the maximum thrust rotation angles β, that we intend considering within a certain interval of values. For every couple (β, γ), the trajectories are calculated and you may choose, between those in accordance with regulations, the trajectory which corresponds to the minimum launch distance $Lski$ with which take-off is possible. In this way it is possible to find the best situation, that is the one that allows to take-off with the minimum total launch distance: horizontal length plus the ramp. Only one precise couple of values, one linked to the ski-jump geometry (β) and the other to the thrust rotation angle (γ), characterises the absolute minimum value of launch distance ($Lski$) that allows take-off for that particular aircraft with that particular configuration.

Studies Carried Out on the Matter

During the first studies on the ski-jump take-off [8] the best values of the physical parameters characterising it were determined, that means those that allow the minimum launch distance $Lski$. At the same time, also the program that calculates and draws the trajectories with certain values of β, γ and $Lski$ was set. These trajectories are always compared to those followed by a normal take-off, without ski-jump or thrust rotation: thus determines the L_{ref}, as explained in the previous introduction.

You will find an example of a three-dimensional diagram, represented by Fig.2. The $Alski$ can become bigger than 1 for small values of β and γ, because during the ski-jump take-off the longitudinal manoeuvre is carried out only after that the aircraft has detached from the ski-jump, whilst during a normal take-off you pass from α_{rw} to α_{to} before the take-off from the runway.

This three-dimensional diagram is therefore simply characterized by the type of aircraft (that is, by its aerodynamic, propulsive, weight and geometrical characteristics) and by those parameters that have

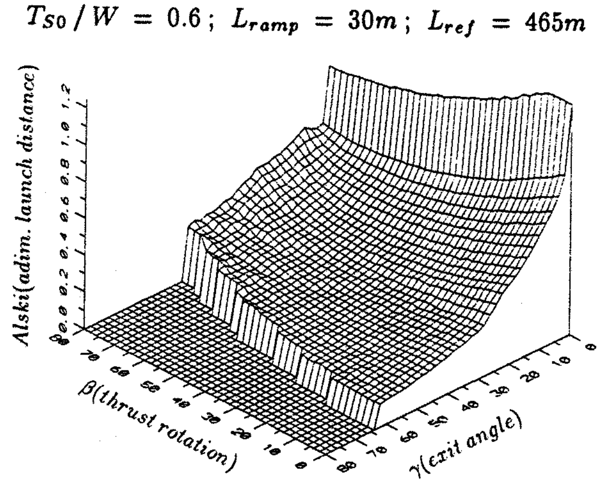

Figure 2: Three-dimensional diagram of the adimensional launch distance.

been considered constant or that simply depend upon the hypothetical situation which the aircraft is in. From this diagram is immediate determining the minimum launch distance necessary for the take-off of the aircraft considered and the corresponding values of exit angle γ and the maximum thrust rotation angle β, that determine the best value of $Lski$.

Another physical feature that has to be considered and that has been drawn in these first studies, is the maximum load factor n_{MAX} usually reached on the ramp. As known, the load factor n is given by the following relation:

$$n = \frac{F_c}{W} + \cos\gamma_i$$

Which is calculated every instant, and determining it's maximum value n_{MAX}, a three-dimensional diagram can be drawn, similar to the one already done for $Alski$. An example of such a diagram is represented by Fig.3.

If the n_{MAX} value overcomes the n_{lim}, given in the input file, the program itself signals that anomaly and therefore, the trajectory examined as well as the relative $Lski$, are not considered.

Please note that these three-dimensional diagrams have a limit moving to a decreasing γ for an increasing β: this is due to the fact that the program automatically excludes any inopportune trajectories; that are those that climb too much. Besides, in these situations, very high load factors are reached on the ramp because, besides the high speed, for high γ, the bending of the ramp also increases.

During a second study [9], always using these kinds of diagrams, the influence of some parameters characterising both the aircraft and the operative sit-

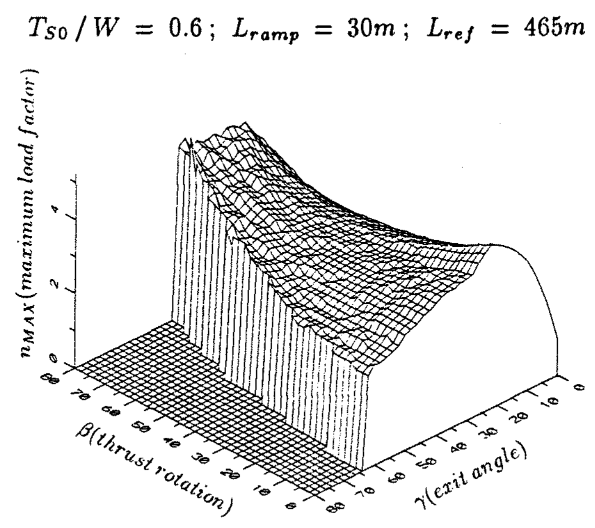

Figure 3: Three-dimensional diagram of the maximum load factor.

uation were taken into consideration; which determine γ and β, and thanks to these we can find the minimum launch distance $Lski$. These parameters are: the thrust-weight ratio T_{SO}/W, the ramp length L_{ramp}, the take-off standard altitute z_{to}, the minimum accepted height of the trajectory H_{min} and the flaps percentage.

During a third study on this subject, which is going to be issued on an italian megazine, I have introduced a new factor in optimal take-off length determination: the fuel consumption during the take-off fase and the following climb to the operating height. In this study, I have introduced in addition, two other parameters that must be taken in mind in the real situation: the ship speed in the case of on board aircrafts and the wind speed.

Longitudinal Manoeuvre

As regards the longitudinal manoeuvre of the aircraft to the pilot, two different treatments have been developed for the ground rotation during a normal take-off or for the longitudinal manoeuvre in flight, carried out after the aircraft has left the ski-jump.

Ground Rotation

The ground rotation is the manoeuvre during a normal take-off to pass from α_{rw} to α_{to}, angles defined by aircraft geometry, by its own aerodynamic features and by the percent of flaps and data in inputs file. The difference between α_{rw} and α_{to} is given by the possible rotation angle α_r, just defined by air-

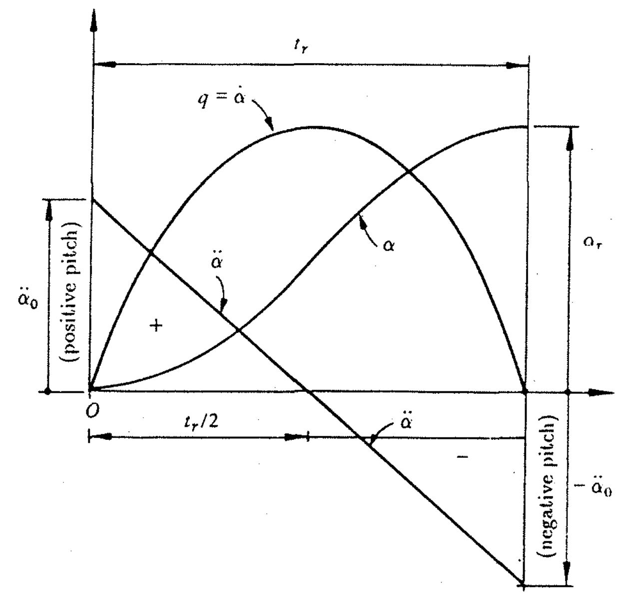

Figure 4: Pitching acceleration and speed and angle of attack during the ground rotation.

craft geometrical features:

$$\alpha_{to} = \alpha_{rw} + \alpha_r \tag{2}$$

Rotation time t_r depends on the modality of the manoeuvre carried out by the pilot because it originates the pitching moment and so the pitching acceleration that makes aircraft rotate around landing gear wheels contact point. The acknowledgment of this time is determinated to calculate the relative covered space and to map out the trajectory of normal take-off; trajectory used, said, as term of comparison for take-off trajectories with ski-jump.

According to the treatment in [2], if we suppose that at the end of t_r time the pitching speed q of the aircraft is again null (as it was at the beginning), the pitching acceleration diagram as a function of t must have null area on the whole. In order to simplify the treatment we can ascribe to it (acceleration) the form shown in Fig.4 with a almost real proceeding. According to the graphic of this figure, the maximum positive pitching acceleration $\ddot{\alpha}_0$ is applied to step when $t = 0$. During t time increase, the acceleration steps down linear until it fades with $t = t_r/2$ time, then reaches the value $-\ddot{\alpha}_0$ on $t = t_r$ manoeuvre final time. This progressive acceleration reduction in time is partly due to the aircraft pitch rotation that changes its pitch angle, and changes the angle of attack of the horizontal tail, but must partly be created by the pilot who, after carrying the stick to positive pitch limit in the beginning, manoeuvres to negative pitching gradually.

Supposed this law by $\ddot{\alpha}$, we have a parabolic trend by $\dot{\alpha}$ (that may be equal to q because, on this horizontal route part, we may confuse pitch angle θ for angle of attack α) and a cubic trend by α: from its expression, with $\alpha = \alpha_r$, we have:

$$t_r = \sqrt{\frac{6\alpha_r}{\ddot{\alpha_0}}} \qquad (3)$$

Now we have to calculate the initial acceleration $\ddot{\alpha_0}$ function of the pitching moments that may be carried out on the aircraft. Altogether these moments are indicated by M. M is given by an aerodynamic pitching moment M_A, by a moment due to the mass forces M_W and by a manoeuvre moment M_M:

$$M = M_A + M_W + M_M \qquad (4)$$

All these moments are function of main aerodynamic, propulsive, weight and geometrical characteristics of the aircraft, but to determine M_M we have to suppose pilot kind of action on stick, too: for our calculation we have supposed, said, that he carry out a step initial manoeuvre to positive pitch limit.

So, we may calculate the initial acceleration $\ddot{\alpha}_0$ using Newton law, knowing the global initial pitching moment M_0:

$$M = B\,\ddot{\alpha} \qquad (5)$$

Where B is the moment of inertia of the aircraft round the rotation axis on the ground. According to Huygens-Steiner theorem this pitching moment of inertia is given by:

$$M = \frac{W}{g}\,(\rho_y^2 + r_{C.G.}^2) \qquad (6)$$

Flight Manoeuvre

The flight manoeuvre is the longitudinal manoeuvre of the aircraft left to pilot after leaving the ski-jump. The aim is reaching established operative altitude z_{oper} (in program inputs) as fast as possible. The program considers the aircraft is controlled to pass the stalling speed (since during the leaving of the ski-jump this speed is not reached usually), then to stay and keep on a climb trajectory of rapid ascent.

To reach and pass as fast as possible the stalling speed it is accepted that the aircraft carry out a descending trajectory after the first climbing tract just left the ski-jump. The trajectory is not accepted (then is rejected the relative runway lenght $Lski$) if it may not keep above the minimum accepted height H_{min} because of aircraft insufficient speed.

When the aircraft reaches the right speed it runs, said, on a climb trajectory to reach z_{oper}.

For all these manoeuvres the pilot use the elevator and, to get aircraft answer when elevator angle δ changes we have started from the equation general system of 3-variable longitudinal motion: ΔV (speed variation), $\Delta\alpha$ (angle of attack variation) e θ (pitch angle) and with assigned variation, δ time function: $\delta = f(t)$. As equilibrium equation along x-axis we have:

$$\dot{V} = -h'_{11}\Delta V - h'_{12}\Delta\alpha - h'_{13}\theta - h'_{14}\delta(t) \qquad (7)$$

Along this way the term $-h'_{14}\delta(t)$ can be left because useless along x the tail term. Along z-axis:

$$\dot{\alpha} = -h'_{21}\Delta V - h'_{22}\Delta\alpha - h'_{23}q - h'_{24}\delta(t) \qquad (8)$$

and around y-axis:

$$\theta = -h'_{31}\Delta V - h'_{32}\Delta\alpha - h'_{33}q - h'_{34}\delta(t) \qquad (9)$$

So, it is: $d\theta/dt = q$ with q pitching speed.

h'_{ij} coefficients are functions of some aerodynamic forces and moments, of some aerodynamic derivatives, of aircraft mass and pitching moment of inertia of the aircraft.

The events of a step manoeuvre: $\delta(t) = \Delta\delta = cost$ and of a linear manoeuvre: $d\delta/dt = cost$ are easy to solve also without $V = cost$ (not accepted in our event) and $h'_{24}\delta(t) = 0$ term simplifications in the second equation (negligible variation of horizontal tail lift by the manoeuvre).

In our case, where we have anyway considered linear manoeuvres with different gradient values, the solution is obtained through numerical integration of these equations, by considering time intervals Δt (about 1/10 sec.) which can however be changed according to the gradient value because are a part of the program inputs. After calculating variation $\Delta\alpha$, variation ΔV and the new pitch angle θ after the interval Δt considered, the new polygon of forces can be drawn and, through the (1), the new position of the aircraft can be calculated. Of course, you need to take into account the changes occurring in the coefficients h'_{ij}, because some of their entities may vary in time.

The gradient values of the involved linear manoeuvres may be given by the program. In this way δ angle changes in a predetermined range, in program input file, too and lower than mechanically possible maximum variation range. The program automatically, at the end of a manoeuvre effect and after valuing consequences, make carry out to the aircraft the following manoeuvre to the direction of the aim to be reached.

Thrust Rotation

Being fixed the length of the ramp, for each total runway length it is fixed the length of the horizontal runway (horizontal tract length). The rotation of the thrust, from null β angle direction according to the longitudinal axis of aircraft to the maximum one that has to be considered by the program, is carried out with an established speed determined by construction choice. This is one of the several inputs which may be assigned to the program.

This rotation starts so as to it is completed at the moment of ski-jump aircraft take-off: in this way for the maximum possible tract the aircraft is subjected to the maximum acceleration thrust. Only in the event of the runway is so short to be insufficient for a complete rotation of the thrust with the established constant gradient, we have the finishing of the rotation at required value, after ski-jump take-off.

The return thrust rotation along the x axis is carried out through a lower rotation speed, but established and in program inputs, too, once passed H_{con} height thrust control, also given by the program. These values comes from the experience, but may change with wide intervals and one aim of this work has been the one to look into the effect of their variation on take-off trajectory.

A particular treatment of the problem is, as said, on the ramp for the delicacy of this tract of take-off runway. The program thickens the calculation points of this tract automatically and assumes very reduced time intervals.

Results

As for previous works about this subject, we have considered an aircraft with Harrier' features and, in particular, a ratio between zero altitude static thrust and the take-off total weight: $T_{S0}/W = 0.6$, characteristic, if referred to Harrier, of a condition of an almost maximum operative weight. Of course, the thrust used in the equilibrium equations is the effective thrust T, the one correct because of altitude and speed. The wing area is: $S = 48m^2$, the aspect ratio is: $A = 4.5$, the ramp length is: $L_{ramp} = 30m$, the standard take-off altitude is: $z_{to} = 0m$, the operative altitude to which tend the various trajectories is: $z_{oper} = 600m$, while the minimum accepted height of the trajectory is: $H_{min} = 15m$. the aerodynamic characteristics of the aircraft are concentrated in C_{D0}, α_{rw} and α_{to}, while in the diagrams of the adimensional runaway length $Alski$ will be referred also the runway length of the normal take-off L_{ref}

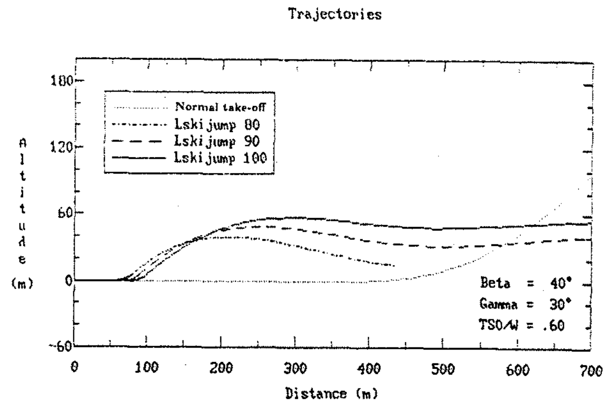

Figure 5: Trajectories with slow rotation of the elevator and slow rotation of the thrust.

with which are adimensionalised, as said, the runway length $Lski$. The figures relative to the trajectories will ever be corresponding to a maximum angle of thrust rotation $\beta = 40^o$, to an exit angle $\gamma = 30^o$ and the $Lski$ corresponding to each trajectory are on the same drawing.

In order to determine the manoeuvre result, aim of this work, we have examined the consequences of a variation of some parameters which characterize the manoeuvre. For the manoeuvre relative to the thrust rotation, keeping the speed rotation during the launch, we have considered two different thrust returns to the axis speeds, with two different speed gradients, one low: $d\beta/dt = 3^o/s$ called slow rotation of the thrust, and one high: $d\beta/dt = 15^o/s$ called fast rotation of the thrust. Always referring to the manoeuvre relative to the thrust rotation, we have considered two different heights where starts the return rotation thrust: $H_{con} = 30m$ and $H_{con} = 70m$.

For the longitudinal manoeuvre we have considered two rotation speeds of the elevator with constant gradient. One low: $d\delta/dt = 4^o/s$ called slow rotation of the elevator, and one high: $d\delta/dt = 15^o/s$ called fast rotation of the elevator.

Longitudinal Manoeuvre Effect

The effect of $d\delta/dt$ is not so remarkable as by comparing, for the relevant aircraft, the corresponding trajectories to different $Lski$ in the event of slow rotation of the elevator (Fig.5) and of fast rotation of the elevator (Fig.6). For both events we are in the presence of slow rotation of the thrust but the trajectories, because of a different $d\delta/dt$, are almost the same.

Effect of the Speed Thrust Rotation

In this event the effect $d\beta/dt$ is very much remark-

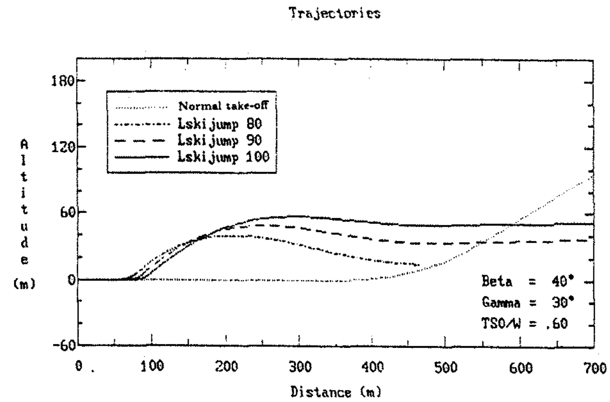

Figure 6: Trajectories with fast rotation of the elevator and slow rotation of the thrust.

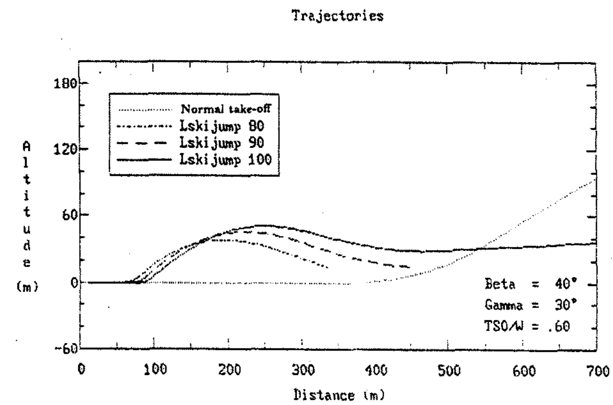

Figure 7: Trajectories with fast rotation of the thrust.

able, as by comparing the trajectories corresponding to a slow rotation of the thrust (Fig.6) and to a fast rotation of the thrust (Fig.7). For this speed rotation variation we are in the presence of a slow rotation of the elevator for both events, but we have pointed out that this factor is not so important, too.

We can see these two combined effects ($d\beta/dt$ e $d\delta/dt$) by comparing the two iso-level curves of *Alski* as a function of γ and β. These courses, surely different ones, refer to a slow rotation of the thrust and fast rotation of the elevator in Fig.8 and to a fast rotation of the thrust and slow rotation of the elevator in Fig.9.

In this event speed thrust rotation and speed elevator angle rotation effects are summed because, as shown in Fig.6 and summarized in Fig.8, the thrust return to the axis delay together with a faster manoeuvre of the elevator, keep the trajectories on a higher minimum height with the same *Lski*.

Height Control Effect The different height thrust control H_{con} is important when there's a fast rota-

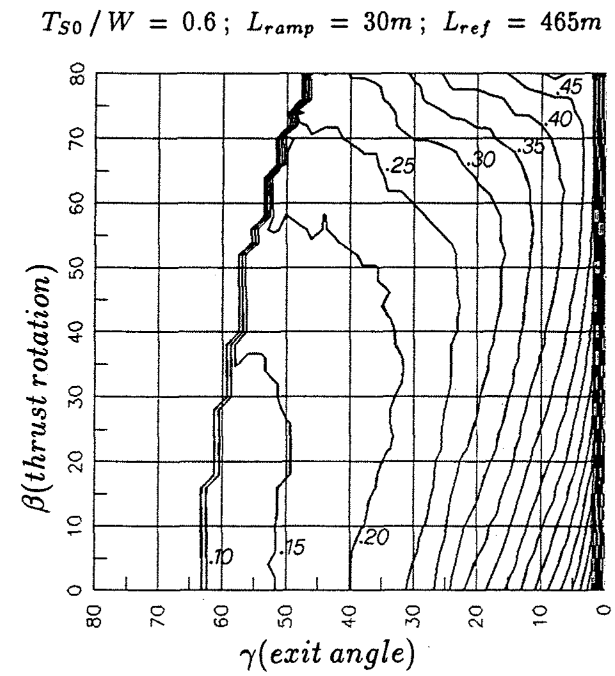

Figure 8: Iso-level curves of *Alski* for slow $d\beta/dt$ and fast $d\delta/dt$.

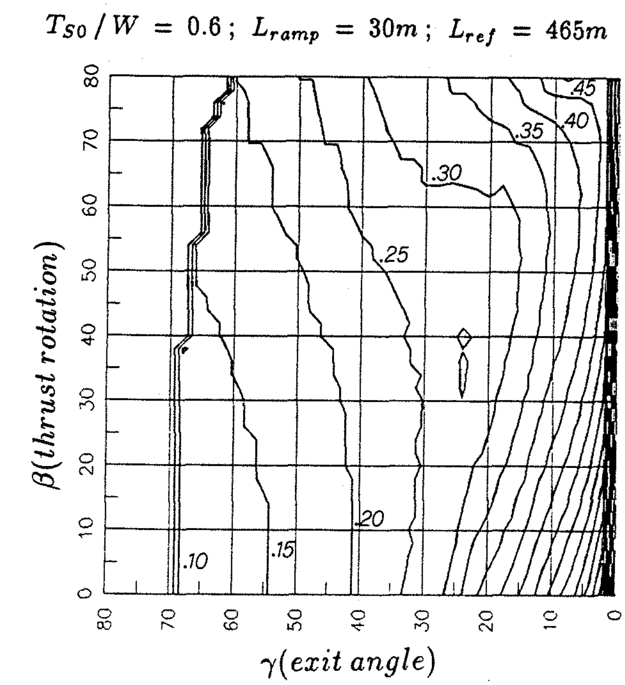

Figure 9: Iso-level curves of *Alski* for fast $d\beta/dt$ and slow $d\delta/dt$.

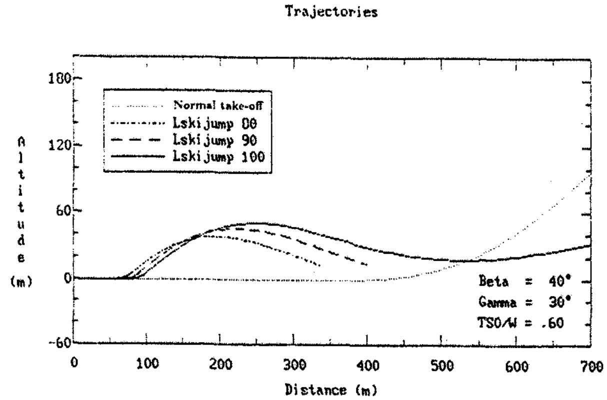

Figure 10: Trajectories with low height control.

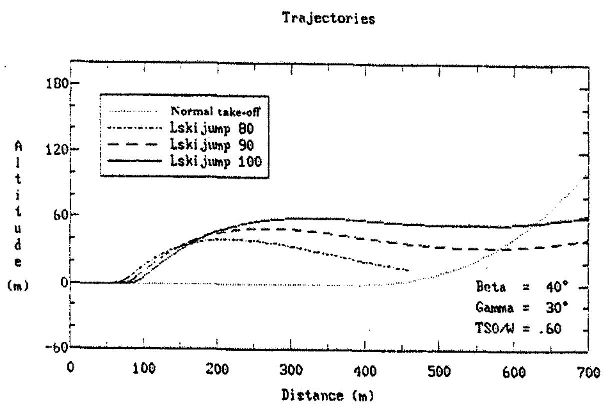

Figure 11: Trajectories with high height control.

tion of the thrust. A low height control of the thrust rotation ($H_{con} = 30m$) is quoted in Fig.10, while in Fig.11 there's an high height control ($H_{con} = 70m$), always with a fast rotation of the thrust. The bad effect of an early return to the axis of the thrust is clear in Fig.10, furthermore the speed rotation of the thrust is high.

Conclusions

As regards the results of this work, the most important parameter during post take-off manoeuvre is the return speed of the rotation thrust to the axis and the relative initial altitude of such a manoeuvre. The longitudinal manoeuvre through the elevator is not so important in post take-off manoeuvre, and in determining γ and β that bring *Alski* to the minimum. On this subject we have to consider that to obtain the minimum absolute value of the launch distance we should choose *Alski* in the lower zone of the three-dimensional diagram γ, β, *Alski* (or in the corresponding zone of the relative two-dimensional diagram with iso-level curves). This zone is characterized by low values of β final thrust rotation angle and by high values of γ exit angle, so the derivable trajectories would have excessive angles of climb and would be inappropriate.

This is the motive to choose other γ and β values, usually we position in a quite regular and flat surface area, characterized by β and γ values of 40^o that, even if do not correspond to absolute minimum zone, allow a peaceful take-off. Besides, in this zone the utilization safety state in different operating conditions (i.e. with other thrust/weight ratios) is kept.

The best procedure to follow, as regards the longitudinal manoeuvre, is a slow return to the axis of the thrust, beginning from a relatively high altitude and, as regards an action on the elevator, its speed rotation faster as possible.

This praxis satisfies two requirements, both important for the ski-jump take-off: to obtain the minimum global runway length necessary for take-off, and to fulfil the safety standard required by the post take-off trajectory and in particular the minimum height of the trajectory post take-off.

The other characteristic parameters both of the aircraft and of the operative situation, the consideration of the fuel consumption during the take-off phase and the following climb to the operating altitude and of required relative time, do not change the conditions that, considering the manoeuvre in this work, optimize the take-off.

With this last work on the ski-jump we have set up these two calculation programs. The first program provides the calculation and the drawing of the trajectories, and the second can determine the optimum conditions of ski-jump take-off according to environment situation or the contingent parameters which have to be considered. Besides these two programs are very complete because, through inputs file, we can consider every real situation.

List of Symbols

A	Aspect ratio
$Alski$	Adimensioned launch length
C_{D0}	Minimum aerodynamic drag coefficient
D	Total aircraft drag
e	Ostwald factor
F_c	Centrifugal force
F_o	Horizontal resultant force
F_v	Vertical resultant force
g	Acceleration of gravity
H_{con}	Height thrust control
H_{min}	Minimum accepted height of the trajectory

H_{obs}	Height of the feigned obstacle
k_{S0}	Zero altitude static specific consumption
L	Aircraft lift
$Lski$	Launch distance
L_{ramp}	Length of the ramp
L_{ref}	Normal take-off length (reference length)
M	Pitching moment
n	Load factor
n_{lim}	Limit load factor
n_{MAX}	Maximum load factor
q	Pitching speed
$r_{C.G.}$	Distance between C.G. and rotation axis
S	Wing area
s	Co-ordinate in motion direction
t	Generic time
t_{oper}	Time to reach the z_{oper}
t_r	Rotation time
T	Thrust of the aircraft
T_{S0}	Zero altitude static thrust
V	True speed of the aircraft
W	Aircraft weight
W/S	Wing loading
x, y, z	Body axes
X	Horizontal co-ordinate
Y	Vertical co-ordinate
z	Generic altitude of the aircraft
z_{oper}	Operative altitude
z_{to}	Take-off altitude
α	Angle of attack
α_r	Rotation angle
α_{rw}	Runway angle of attack
α_{to}	Take-off angle of attack
β	Maximum angle of thrust rotation
γ	Exit angle
γ_i	Angle of climb or descent
δ	Elevator angle
Δt	Time interval
θ	Pitch angle
ρ_y	Radius of inertia of the aircraft round y axis
τ_{by}	By-pass ratio of the turbo-jet

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