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SECOND LAW OF THERMODYNAMICS COMPLEMENTARY WITH FIRST LAW FOR THE OPTIMISATION OF A HYPERSONIC AIRBREATHING PROPULSION SYSTEM.

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Abstract

Irreversible thermodynamics and the second law allow determination of an optimal system and comparison of different losses.

First, the impact of a variable supersonic diffusion and of prewarming of hydrogen fuel on ISP and on the propulsion efficiency which are, respectively, the optimisation parameters used in hypersonic propulsion for the first and the second law analysis is examined. The enthalpy ratio ψ across the compression process is held constant for all speeds above about Mach 6.5 with a variable contraction ratio of the supersonic diffuser. This contraction ratio has to be chosen in such a way that enthalpy at the end of combustion process gives an ISP as high as possible. It has been checked using second law that this contraction ration gives also the minimum entropy rise.

Secondly, the influence of the angle of attack i_o for a given spaceplane forebody angle δ_o, which characterises the precompression, is also examined through the calculation of the irreversible entropy production and the results are compared with results from a first law analysis.

Along all these calculations, as predicted by P. Czysz, irreversible entropy calculation is shown to be a very efficient guide in the choice of design parameters for a Single Stage To Orbit vehicle propulsion system: it represents the indispensable complement for the enthalpy calculation.

Symbols

C_f	friction loss coefficient
δ	angle or deflection
d	fuel-air mixture
ER	equivalence ratio
F	thrust per unit span
h	enthalpy
i_o	angle of attack
ISP	Engine Specific Impulse
L	length
m_a	air mass flow
m_f	fuel mass flow
M	Mach number
P	static pressure
Q	dynamic pressure
Q_a	Brayton cycle heat
Q_c	fuel heat of combustion
S	entropy
T	static temperature
V	speed

Subscripts

o	free flow
1	diffuser inlet
2	diffuser exit
3	combustor inlet
4	combustor exit
6	engine exhaust

1. Introduction

The key to success of a combined cycle propulsion system for a S.S.T.O. spaceplane is a thermally integrated engine where you have to maximize the energy conserved and minimize the energy expended.

The optimisation of this propulsion system can be done on basis of the first law of thermodynamics or of the second law. The problem of first law analysis is that two different types of energies are taken into account. The first one is the energy which can be transformed in effective work (exergy) and the other one is the unavailable energy which will never be transformed any more due to an irreversible entropy rise. If this energy is not captured when available, it is lost for the rest of the cycle.

Two problems will be studied here: first the choice of the optimum supersonic diffusion, i.e. the static pressure at the inlet of the combustor and secondly the influence of the precompresssion, i.e. the total angle of attack (angle of attack plus forebody deflection).

2. Spaceplane and engine

The side view of the vehicule is shown in **figure 1**. The forebody provides a bow shock deflection of 6 deg and the three segment ramp an extra 7.50 deg (δ_a = 1.25, δ_b = 2.50, δ_c = 3.75). For the design conditions, Mach 15 & i_0 = 4 deg, near focalisation of the bow shock & of the three ramp shocks is achieved just above the diffuser cowl lip, as illustrated by **figure 2**. Total spaceplane length is 50 m incl. the 11.50 m ramp.

The whole calculation will be done for a spaceplane Mach number of 15. The scramjet is hydrogen-fueled with a temperature at the entrance of the injectors of 1000 K following its use as a coolant for active cooling of the vehicle and engine walls.

A real gas (9 elements) 1.5-D aerothermodynamics code taking into account shocks, skin friction, energy radiated, convective heat transfer is extensively used along the paper.

3. Optimum supersonic diffusion

A classical use of second law analysis is the study of the optimum diffusion with the calculation of the propulsion efficiency as done by Professor P. Czysz (1) & C. Builder (9).

a. Optimum compression

As we called ψ the ratio of the enthalpies at entrance of the combustor and at the entrance of the inlet, it can be shown that:

(1) the degree of diffusion or the speed at the entrance of the combustor is given by:

$$\left(\frac{V_3}{V_0}\right)^2 = 1 - \frac{\psi - 1}{K} \quad (1)$$

where $K = V_0^2/2h_0$. The maximum diffusion is reached as $V_3 = 0$, then the static enthalpy ratio ψ is equal to $\psi_{stagn} = K+1$.

(2) the optimum value of ψ, i.e. the value which gives the highest value of the speed at the exit of the engine V_6 is given by:

$$\psi_{opt} = \left[\frac{\eta_g}{1-\eta_g} * C\right]^{1/2} \quad (2)$$

ψ_{opt} is only a function of the global efficiency η_g (product of compression efficiency and expansion efficiency) and of C ($C = ER*Q_a/h_0$).

A critical speed V_{crit} can be defined: it is the aircraft speed V_0 at which it is desirable to begin diffusion to supersonic speed. This speed is reached when the stagnation enthalpy ratio is equal to the optimum enthalpy ratio:

$$\psi_{stagn} = K+1 = \psi_{opt}.$$

This means that for every speed V_0 above V_{crit}, ψ_{opt} is smaller than K+1 and thus that the kinetic compression is larger than the optimal one. Diffusion of the flow till a stagnation condition is unfavourable for the system and the diffusion must be limited. We speak about optimum compression and V_3 is supersonic.

As the optimisation has to be made at Mach 15, ψ has to be equal to ψ_{opt}. The determination of ψ_{opt} amounts to the determination of h_{3opt} or P_{3opt}

b. First law analysis

For a dynamic pressure of 60 kPa, the pressure P_3 which gives the maximum ISP following application of the first law is between 95 and 100 kPa (ISP = 1143 s at P_3 = 100 kPa, see **figure 3**). This is a relative low pressure what gives little constraints from a structural point of view. The contraction ratio RA_{02} is equal to 22.63 what is not so far from the result of 25.23 given by a Billig's relation (5).

c. Second law analysis

We have to check if this pressure of 100 kPa gives also the smallest entropy production or, in other words, the highest propulsion efficiency.

Propulsion efficiency is defined as the ratio of propulsion power over fuel injected power:

$$\eta_p = \frac{F \times V_0}{m_f \times Q_c} = \frac{F \times V_0}{m_a \times Q_a} = \frac{ISP \times V_0}{Q_c} \quad (3)$$

As the effect of the pressure and of the surfaces at the ends of the engine can be neglected, the equation (3) can also be written as follows:

$$\eta_p = \frac{m_a \times (V_6 - V_0) \times V_0}{m_a \times Q_a} = \frac{\left(\frac{V_6}{V_0} - 1\right) \times V_0^2}{Q_a} = \frac{2K}{C} \times \left(\frac{V_6}{V_0} - 1\right) \quad (4)$$

In the case of an optimum compression, equation (4) gives:

$$\eta_{opt} = \frac{2K}{C}\left\{\left[1 + \frac{\psi_{opt} - 1}{K}\left[\eta_g\left(1 + \frac{C}{\psi_{opt}}\right) - 1\right]\right]^{1/2} - 1\right\} \quad (5)$$

or $$\eta_{opt} = \frac{2K}{C}\left[\left(1+\eta_g \frac{C}{K}\left(\frac{\psi_{opt}-1}{\psi_{opt}}\right)^2\right)^{1/2} -1\right] \quad (6)$$

With these two last expressions, it is possible to calculate the propulsion efficiency.

A correction factor has been introduced by the Professor P. Czysz (1): in all these equations the hydrogen mass, which can be very important in the case of a non stoichiometric fuel-air mixture (like in this case at Mach 15), was never taken into account. This positive effect can be included in the equation (6) as follows:

$$\eta_{opt,f} = \frac{2K}{C}\left[(1+d_{stoe}\times ER)\left(1+\eta_g \frac{C}{K}\left(\frac{\psi_{opt}-1}{\psi_{opt}}\right)^2\right)^{1/2} -1\right] \quad (7)$$

(with hydrogen, $d_{stoe} = 0.0292$).

A last effect which can be very positive is the kinetic energy of the hydrogen at the injection. As V_f is the injection speed of the hydrogen, the expression of η_p is as follows:

$$\eta_{opt,fke} = \eta_{opt,f} + \frac{K}{C}\times ER \times d_{stoe} \times \frac{V_f^{\,2}}{V_0^2} \quad (8)$$

These two effects have a major influence on the performances of a scramjet at very high Mach (1).

d. Application to the engine Hyperjet Mk.3

We can apply the equations (6), (7) and (8) to our engine with P_3 = 100 kPa with the intention to:

(1) control that the value of the propulsion efficiency is acceptable;

(2) see the influence of hydrogen mass and injection speed.

In this case, the equivalence ratio ER is equal to 2.328 and K to 45.356. With V_3 = 3987 m/s and V_f = 4000 m/s, we calculate the following efficiencies:

$$\psi = 13.648$$

$$\eta_g = 84.77\ \%$$

$$\eta_p = 65.10\ \%$$

$$\eta_{p,f} = 87.98\ \%$$

$$\eta_{p,fke} = 87.98 + 6.70 = 94.68\ \%$$

We have through the fuel mass an increase of 35 % and an extra increase of 10 % through the injection speed what gives us a propulsion efficiency of nearly 95 % *without taking into account the internal losses.*

We may conclude that the pressure P_3 = 100 kPa is a good choice even if it is not yet the optimum. If we calculate η_p for different pressures, we see on **figure 3** that a maximum exists around 100 kPa. For $\eta_{p,f}$ in function of P_3, there is an asymptotic value of about 0.89 due to the positive effect of ER which becomes more and more important and compensates the decrease of η_p.

Following the second law, 100 kPa is almost the optimum solution.

4. Influence of precompression

A very original way to use the second law analysis is the study of the precompression as executed hereafter.

a. Foreword

Forebody precompression is of particular importance for an airbreathing Single Stage To Orbit vehicle which transitions from scramjet to rocket at rather high Mach number, say Mach 15. Provided by the combination of forebody deflection δ_0 & angle of attack i_0, precompression affects both thrust, through airflow, & ISP through friction drag and heat transfer (6).

b. First law

It can be shown that, due to the intimate coupling between aerodynamic and propulsive forces that exists on hypersonic vehicles, a high precompresssion improves thrust, net specific impulse and effective specific impulse through a higher airflow through the engines and a better 'amortization' of surface frictions & heat transfers. It is seen on **figure 4** that an optimum ISP is reached with a total angle of attack of about 11 deg.

c. Second law - Generalities

Through application of the second law, we shall try to find once again this optimum which, in this case, would be a mininum of the irreversible entropy production.

Along with the energy conservation which leads to the first law, there is the principle of conservation of entropy:

$$S_2 - S_1 + \Delta S^{**} = \theta \quad (J/K) \quad (9)$$

with S_i the entropy of the system in the state i, ΔS^{**} the entropy change of the passive environment and θ the entropy generation (or production) term. Equation (9) is a statement of the second law applied to an isolated system and conceived as a union of the system proper and the passive environment. The difference between the initial and final entropy of such an adiabatic system, defined as the entropy production, must be positive or zero depending on whether the process 1-2 is irreversible or reversible, thus $\theta \geq 0$. In the case where the only interactions between the system and the passive environment are through rigid diathermal walls, we have that:

$$\Delta S^{**} = -q/T^{*} \quad (J/K)$$

with T^* the wall temperature and q the heat transfer flux. ΔS^{**} is very often called the irreversible entropy production, noted ΔSirr.

Another term will be used to characterise the irreversible process, it is the lost work or dissipation:

$$\phi = T^{*} \times \Delta Sirr \quad (W) \quad (10)$$

d. Second law and precompression

In the case of the precompression, the system we consider is the flowtube between the nose of the aircraft (just behind the bow shock) and the inlet of the combustor where the flow conditions are almost the same for every angle i_0 (we work always at the pressure P_3 = 100 kPa). The passive environment is the hydrogen tank which maintains a wall temperature T^* of 1000 K through a variation of the coolant flow (thus of the ER). The complete system is considered as a huge heat exchanger.

In a heat exchanger, there are two sources of entropy production: the heat transfer and the viscous dissipation. For a Newtonian fluid, we can write that ΔSirr is given by:

$$\Delta S_{irr} = -q \times \frac{\Delta T}{T^2} + \frac{1}{T} \times \frac{1}{2} \times C_f \times m_a \times V^2 \times \frac{L}{R_h} \quad (11)$$

with q the heat transfer flux (q>0), ΔT the difference between the wall temperature T (or T^*) and the mean temperature of the flow, V the mean speed of the flow, L the exchange length and R_h the hydraulic radius.

It is now possible with the equations (10) and (11) to calculate the lost work ϕ in function of i_0 with a constant forebody deflection δ_0. This curve has normaly to show a minimum for the value of i_0 which gives the maximum ISP.

e. Application to the engine Hyperjet Mk.3

The calculations were done for δ_0 = 6 deg and a constant maximum total deflection ($\delta_0 + \Sigma\delta$) = 19 deg. The flow tube has to be splitted in two parts: one from the nose to the inlet cowl lip and the other one from this lip to the entrance of the combustor. This is necessary due to the facts that the difference in the shock strengths is especially important on the forebody and that the expressions of L/R_h are totally different in these two portions.

The **figure 5** gives the variation of the lost work ϕ in function of the angle of attack i_0 (δ_0 = 6 deg). It is seen that a minimum exists at a value which is in good agreement with the results of the first law analysis. This is also an indication that the engine energy management is good.

5. Conclusion

The application of the second law of thermodynamics is shown to be a fantastic optimisation process. Second law analysis has in the study of the optimum compression and in the study of the influence of the precompression shown a good agreement with first law analysis. It is a good complementary tool for first law analysis.

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Figures

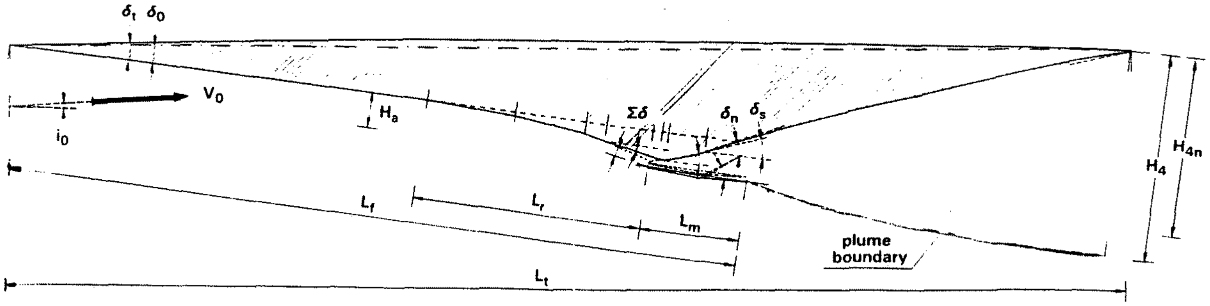


Figure 1

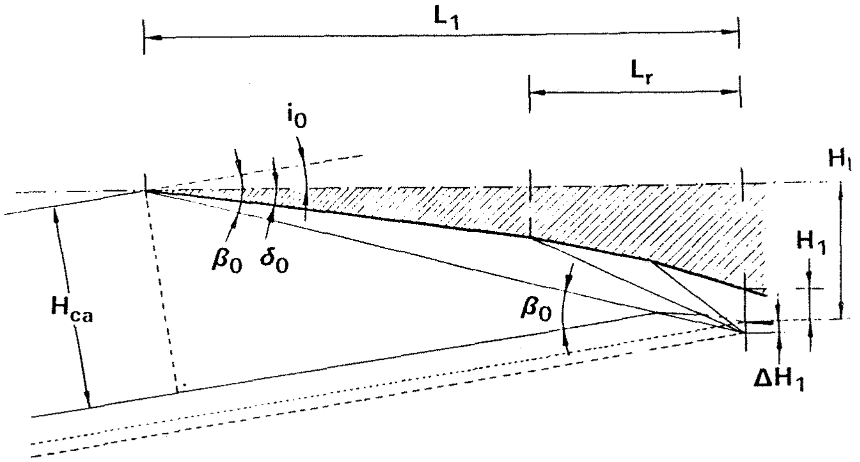
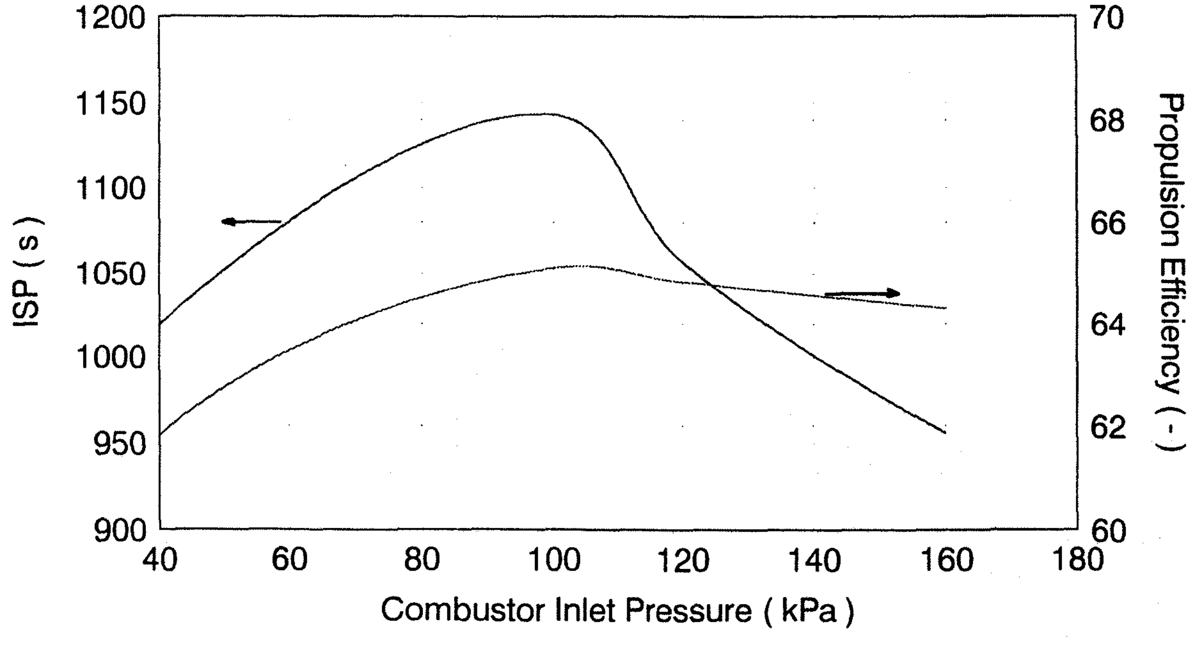


Figure 2



Mach 15 & Qo = 60 kPa & io = 4°

Figure 3

First law analysis

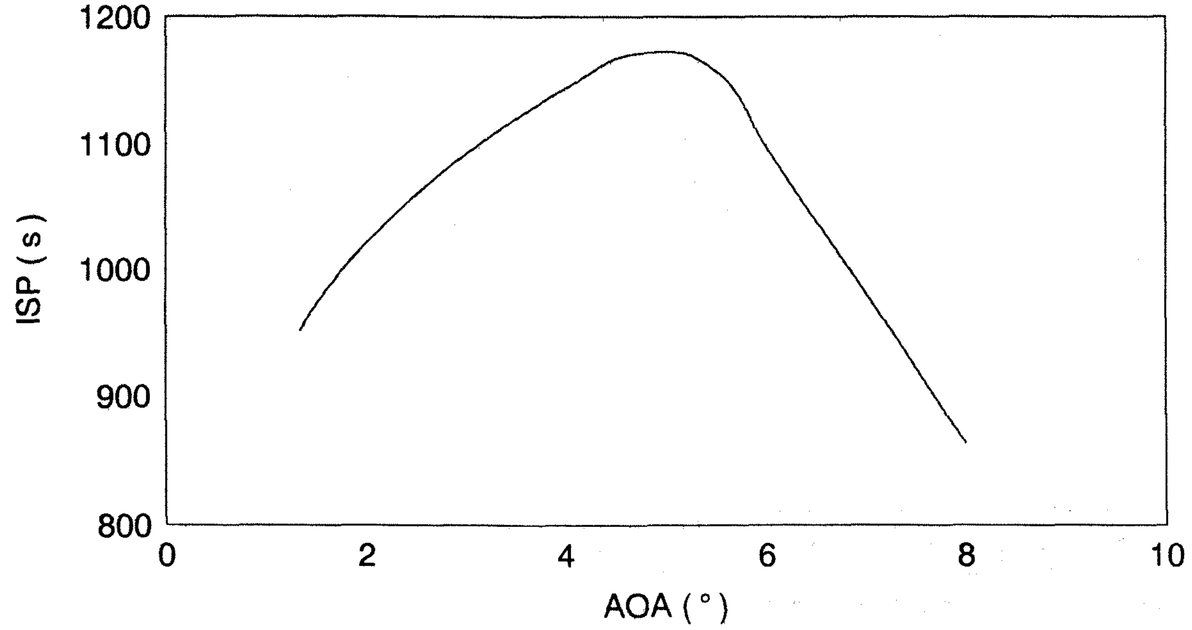


Figure 4

Second law analysis

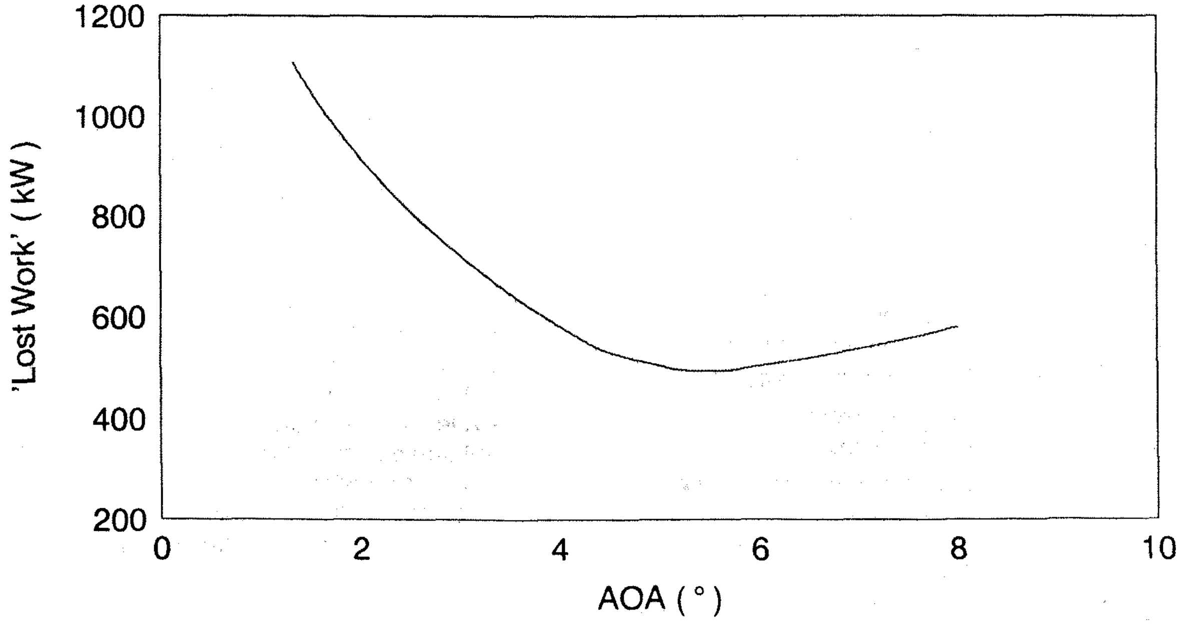


Figure 5