A CROSS DISCIPLINARY INSIGHT FROM THE PHYSICS OF SUPER COOLED LIQUIDS INTO COMPRESSOR BLADING STALL MARGINS

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Summary

For compressor cascade blading operating in turbulent flow from upstream separations the differential equation (d.e.) of motion has significant similarities to a d.e. arising in the physics of the kinetic glass transition. Recent studies using mode-coupling analysis of collective dynamics of particles in super-cooled liquids show particular feedback mechanisms within a d.e. which is analogous to that of the oscillating blades in cascade. The kinetic glass transition exhibits two regimes of behaviour for a correlation function which separates two regimes of relaxation. One of these is a plateau in time and the other exhibits progressive decay along a range of individual pathways. Close similarity of behaviour in the blading case is revealed and suggests possible form of control parameters plus a method of implementing control of unstable blade motion. The study also offers an explanation as to why chaotic behaviour may not be exhibited to the extent often expected in such non-linear dynamical situations.

Introduction

When the mass flow in an axial compressor is reduced, the pressure rise increases beyond the desired operating point to a point at which the flow becomes unstable. Close to the critical point, a small change in the flow may initiate instability. If the compressor blading enters into rotating stall severe blade vibrations are set up which may lead to catastrophic results and recovery to the design condition is made difficult by a hysteresis effect.

The associated stall margins have been a matter of concern for many years. Theoretical predictions of stall have often been too optimistic because the disturbance field in turbomachinery is particularly complex and boundary layer transition and separation on blades may result from a variety of causes simultaneously or in competition

Our objective is to offer some cross-disciplinary information from the physics of kinetic transitions in super cooled liquids which may suggest future lines of investigation on the relationship of oscillatory blades in cascade and the initiation of undesirable processes which reduce stall margins. In outlining the utility and drawbacks of traditional approaches to turbulence Narasimha[1] has spoken of a "reconciliation" between the notions of ordered motion and statistical theory which will probably emerge from greater understanding of the apparently random occurrence of coherent events through a dynamical-systems approach which may use at least some traditional ideas perhaps from vortex dynamics. Some of this line of thinking is embodied in the work presented which also calls upon some of Kraichnan's earlier developments in his approaches[2] to turbulence and vortex dynamics.

Blade Motion in Separated Flow

A common type of motion exhibited by fluttering compressor blades is a bending vibration and the equation of motion, at a representative point on a blade, close to stall is of second order nonlinear forced oscillator form[3]. Although such equations are sensitive to initial conditions with all the peculiarities of strange attractors and bifurcation on the path to chaos, there has been some doubt concerning evidence of deterministic chaotic behaviour. Perhaps this is because there is both a large number of degrees of freedom in the practical system and a simple deterministic model is disturbed by stochastic perturbations or other time varying changes in the structure of the model, (for example, due to random forces from mode coupling interactions with other local blades) to investigate this matter. We take account of some of the foregoing effects and consider a stochastic differential equation for blade motion which allows for the autocorrelation function $M(k,t)$ for the local random force on a blade in the cascade. In such cases, the differential equation of blade motion at a point on the blade can be written in the form

$$\ddot{y}(k,t)+\gamma(\omega)\dot{y}(k,t)+\omega^2 y(k,t)$$
$$-\int_0^{t-\tau} M(k,t-t')f[\alpha(t'-\tau)]dt''=0 \qquad (1)$$

where f is the dynamical force coefficient given by the local slope of the lift curve and has the form $f(\alpha)=-K_1\alpha+K_2\alpha^2-K_3\alpha^3$, ω is the natural bending mode frequency, $\gamma(\omega)$ the damping coefficient and α is the effective angle of attack for a tapered, twisted blade in free stream velocity v.

$$\alpha=\alpha_i+\dot{y}/v-(\theta+n_0 b\dot{\theta}/v) \qquad (2)$$

where n_0 is a constant~1.5 at the 1/4 chord point and $\theta(t)$ is the small torsion motion of the blade due to its tapered-twisted form which gives a small orthogonal displacement to the main bending displacement. The phase lag τ is the lag of aerodynamic force behind the blade motion. Previous observations from industry[4a,b] have shown that τ is a function of the reduced frequency $\omega b/v$ and not a simple function of the slope of the static lift curve.

Analogy to the Kinetic Glass Transition

Eq. (1) is an extended type of generalised stochastic differential equation of Langevin for the density autocorrelation function $\phi(k,t)$ for particles in suspension given[5] as

$$\ddot{\phi}(k,t)+\gamma(k)\dot{\phi}(k,t)+\Omega^2(k)\phi(k,t)$$
$$+\int_0^t M(k,t-t')\phi(k,t')dt'=0 \qquad (3)$$

This corresponds to Eq. (1) with $\tau=0$ and f would be $-\phi(k,t')$. The forcing function term describes the retarded friction characterised by the "memory function" $M(k,t)$ which is actually the correlation function of a dynamical variable, the so called "random force" $R_k(t)$ used in studying the suspension of particles exhibiting irreversible Brownian dynamics due to the suspending fluid. We then note that at sufficiently high concentrations, such suspensions in a liquid may exhibit glassy states as well as the usual fluid and crystalline phases. The correlation function actually measured[6] over time-scales of a few seconds shows the features of Figure 1. In the case of the glass transition physics, the differential equation is heavily damped. The stochastic background for the relevance of Eq. (1) in a cascade of blades and the basis on which our analogy is drawn is covered in Appendix A. The mode coupling approximations in that application show that the function M reduces to a form quadratic in $\phi(k,t)$ which thus provides a nonlinear feedback mechanism.

In the case of the compressor blade model, a similar feedback situation occurs due to the presence of y in the expression for α as seen from Eq. (2) and Eq. (1). It is therefore suggested that the $y(k,t)$ blade motion may also exhibit several different regimes with a discontinuous change from ergodic to non-ergodic behaviour at a defined control parameter in an analogous way to temperature in particle suspension physics where one relaxation regime is separated from the step to final relaxation by a defined plateau.

Termination of Pathways to Chaos

So we are now prompted to examine more closely, the effects from the last term in Eq. (1) to possibly classify and hopefully identify control parameters of blade response. The study herein shows that even though the number of degrees of freedom in an actual physical system is very large, a progression to chaotic behaviour may possibly develop. As we shall also see, the pathway to chaos may be prematurely terminated because it requires particular conditions to progress to do so. If the conditions are met, the energy input supplied from upstream wakes can then lead to destabilisation of the system via a range of possible pathways and rates of decay of autocorrelation. The two modes of behaviour are related to two modes of the autocorrelation function ϕ in the glass kinetics of Eq (3) but the damping is "dead beat" in the latter case.

To identify the controlling parameters, we need to know the character of the dynamics, especially when the system is subjected to periodic disturbances from the upstream wakes of the cascade, apart from the local mode coupling interactions, in the blade's progression towards entrainment. The foregoing is directly relevant to a proposal for extending pre-stall margins by feedback injection of re-scaled wakes with spectral changes of wave number (k) which can be a controlling factor in preserving stability. We therefore need to know under what conditions entrainment is possible or not possible prior to such injection of such "coloured noise" to be effective.

So we need to look more closely into the nature of the governing differential equation Eq. (1). At present we do not know the exact form of the function M and therefore cannot write the exact d.e. However, the close analogy with the physics of the kinetic glass may be of help, as follows.

Limits on the Model

The mode-coupling approximation for the kinetic glass transition yields a quadratic in $\phi(k,t)$ in the function M thereby providing a non-linear feedback mechanism leading to heavily damped (dead-beat) response in the glass transition model but not necessarily so damped in the aerodynamic case. However, in Appendix A, we see that to achieve a reasonably realistic analysis in the presence of developing turbulence, the k and blade frequency parameter ω must conform to contain conditions. Fortunately, these seem realistic in cascades and correspond to a low loss of correlation of a Green's function for a response function in a Kraichnan-modified approach[2] to unsteady Navier Stokes using renormalisation methods from quantum mechanics. As also discussed in Appendix A, he also showed the need to incorporate the quantum mechanics of a particle in a random potential in order to preserve reality for the diffusion coefficient. (It is important to point out the latter connection to the glass transition model studied by Götze et al[8] where the same type of quantum mechanics model was used, as mentioned in Appendix A.)

Whereas in the glass transition mode-coupling model the feedback response is provided for by the feedback in the d.e. for blade motion with translations (y) and torsions (θ) is provided via the combined functions $M(k,t)f(y,\theta,\dot{\theta})$ in the last term in the d.e. where the function f is cubic with combination terms in $y,\theta,\dot{\theta}$. Also, if the constraints in our Kraichnan-modified Euler method regarding the spectrum of k (Appendix A) are satisfied, phase decorrelation can be avoided and the autocorrelation function M is expressible in terms of $y(k,t)$ of the form $\exp[-\Gamma(y-y_0)^2$ and approximated by a quadratic in $y(k,t)$.

Conversion to General Form of Excitable Systems

If we take the Laplace transform of Eq. (3) a simplification results from the removal of the lagged argument by virtue of

$$\mathcal{L}\int_0^{t-\tau} M(k,t-t')f(t'-\tau)d(t'-\tau) = \overline{M}(s)\overline{f}(s)$$

the bars denoting the transform of the individual functions.

Eq. (1) can then be expressed in the more general form

$$\dot{y} = \mathcal{F}(\overline{\theta},\overline{y}) \qquad \dot{\theta} = \mathcal{G}(\overline{\theta},\overline{y}) \tag{4}$$

where $\mathcal{F}$ is

$$\mathcal{F} = A\overline{\theta} + B\overline{\theta}\overline{y} + [C\mathcal{H}(\overline{y}) - D]\,\overline{y}$$

$$\mathcal{G} = E\overline{y} + F\overline{\theta}\,\overline{y} \tag{5}$$

and $\mathcal{H}(\overline{y})$ is a distribution function of the form

$$\exp -\Gamma[\overline{y} - \overline{y}_0]^2 .$$

Eqs. (4) and (5) represents the local mode-coupling between translational and torsional motion of the blade in the presence of turbulence from upstream separations. The terms $A\overline{\theta}, E\overline{y}, B\overline{\theta}\overline{y}$ and $F\overline{\theta}\overline{y}$ describe a Lotka-Volterra dynamics and the term $C[\mathcal{H}(\overline{y}) - D]y$ exhibits a type of Van der Pol process, i.e. competition between non-linear gain $C\mathcal{H}(\overline{y})$ and the linear loss process $D\overline{y}$. For $C > D$ a stable limit cycle exists as shown typically in Reference (9) for a generalised equation of this form. Furthermore, there can exist several unstable solutions. Unfortunately, this theoretical information is not very useful in providing relevant control parameters in practice.

Reverting to the kinetic glass transition, we can see an analogy to the two modes of behaviour observed in experiments shown in Figure 1 where, in that case, the stable state corresponds to a plateau of ϕ near unity in the upper ("α relaxation") region and the region below it ("β relaxation") corresponds to individual progressive decays of correlation paths with time corresponding to our unstable states for blade motion (M decreasing) with the possibility of individual pathways in time, progressing to increased instability. In the glass kinetic steady state, the control parameter separating the two states is temperature T but in our Eq. (1), it is not easy to isolate a practical control parameter from an analytical approach and the previously mentioned condition $C > D$ even in the general form is not very useful for this purpose. We therefore look back to the glass transition analogy to try to obtain an analogous parameter(s) to T which, at a critical temperature $T = T_0$, separates the β relaxation process from the α relaxation process by a plateau which becomes longer as the temperature T drops. Comparison of parameters for this condition suggest the parameter $\frac{mS(k)\omega^2}{kk_B}$ where $S(k)$ is a "structure factor" and m is blade mass per unit span. The parameter suggests the 3D geometry of the blade and/or the grouping geometry in the cascade as being key structural parameters apart from the k wavenumbers in the flow. In dimensional terms, the parameter has a strong dependence on blade mass distribution and on k (dimensionally 1/time)

Opportunity for Control

The foregoing suggestion for possible analogous parameters is not inconsistent with our earlier study[3] of stability under simpler conditions for Eq. (1) (with the absence of the autocorrelation factor). The control was seen to be dependent upon the aerodynamic lag time τ which, in turn, involves the parameters K, L, M (lift curve coefficients) m (blade mass per unit span) and b (semi-chord).

The opportunity for control under these conditions suggests that k, as well as the blade lift parameters is a key parameter and may be the only parameter which can be altered during operating conditions. We have previously[3] proposed a means of doing this by rescaling[3,7] the wake turbulence from unstable (fluttering) blade motion and feeding it back into the incident flow. In that study, we used the dual input describing function method to determine the stable range as governed by the phase angle $\omega\tau$ and a variable radial length r of the Nyquist incremental open loop gain locus where the combination of $r(\omega\tau)$ and $\omega\tau$ govern windows of stability as the Nyquist locus rotates with $\omega\tau$.

Evidence of improvement in pressure rise coefficient from the inclusion of a flow separator ahead of a compressor rotor shows the advantages of using a physical recirculation channel. One of the processes involved[10] there was the removal of low energy fluid near the fan casing. However, when recirculation vanes are deployed, these operate on, and transform, the length scale and swirl of the recirculated turbulence. Enstrophy manipulation results in a quite different frequency domain representation of incoming flow (coloured noise) with potentially quite different frequency cascading and dynamic behaviour. An example of results from Ziabasharhagh et al[11] is shown in Figure 2 and demonstrates the remarkable improvement in stall margin which can be achieved using only moderate treatment.

Whereas the previous analysis, we believe, is still valid if blades are in diverging motion in separating flow and progressing to chaotic motion, the analogy with the glass transition model suggests that blade instability will not necessarily progress to chaos except under the conditions similar those examined below.

If a periodic disturbance $F_0 \cos \lambda t$ is applied to a blade operating in the stable limit cycle corresponding to Eqs. (4) and (5) and whose frequency is ω_{LC}, it has been shown[9] that if $\lambda \ll \omega_{LC}$ no entrainment can be reached by increasing the amplitude of the disturbance F_0. The oscillatory motion will collapse before entrainment is reached and this precludes the development of chaos. An example of progressive collapse is shown in Reference (9). Therefore, it could be expected that similar restrictions apply in actual cascade blading where the Fourier components of the upstream wakes incident on the blade have significant frequency domain separation. The periodic components are strong and may fulfil the above condition for no entrainment and no progress to chaotic motion.

Nevertheless, it is also quite possible that the restrictive condition is not fulfilled in some cases and chaos could ensue after the steps of entrainment and quasi-periodic motion. Figure 3 shows part of the progressive entrainment after a feedback injection into a divergent blade oscillation of Eq. (1) when M is 0(1). The occurrence or nonoccurrence of chaos will be random in practice and this relates to Cox and Isham's description[12] of turbulence as a random collection of point occurrences in time or space with events of different magnitude and duration occurring at various instants of time rather than as a superposition of harmonic waves. Narasimha[1] considers this episodic view which embodies the random occurrence of coherent events, to be consistent with a dynamical system approach to turbulence. According to such a scenario, there is a number of bifurcation pathways and unexplained effects prior to eventual "collapse" leading to the onset of a response signal[9] or travelling wave. It has been recently reported that for period-doubling bifurcating pathways to chaos in nonlinear systems, the process may easily break down and suddenly reverse[13] giving rise to period-halving bifurcations. Such reversals may act to control and even prevent the onset of chaos. It is suggested that the foregoing features may explain why evidence of chaotic dynamics is often hard to come by or confirm.

Kaiser's results[9] show that the addition of stochastic noise to the chaotic state arising from an equation of the form of Eqs. (4) and (5) will cause the chaotic state to continue. However, we must point out that our suggestion of feedback of the rescaled **coloured** noise to the incident flow to blades in chaotic motion should cause progression along a ***preferred*** path to an ordered state (e.g. Figure 4) in which no order at all would be normally expected. At first sight, this might appear to be "order through fluctuations" or "order by noise" as introduced by Prigogine[14]. Although this may be the case, the underlying mechanisms for coloured noise are completely different and not yet fully elucidated. In the response to "coloured noise" the trajectories formed such as in Figure 4 can be interpreted[15] as non-normalised probability densities in the state space of $\bar{y}$ and $\bar{\theta}$ and it is important to note that these preferred trajectories appear simultaneously and not in the form of travelling waves.

The feedback channelling technology for the feedback process proposed is available and has been demonstrated theoretically[16] (active control) and experimentally (passive) in industrial fans. Extension to more complex geometries of compressors may be attainable and is available from the recent computational method[17] for the inverse problem of determining complex branching flows in multiple connected domains and which can include rotational flow effects and viscous effects and be extended to turbulence conditions.

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Appendix A

We consider a local region of volume V of the cascade containing a small group of blades and so have cyclic spatial boundary conditions. For the non-equilibrium Navier Stokes system there are severe difficulties for theoretical treatment as discussed at length by Kraichnan in considering the evolution of such a system. As noted by Kraichnan[2] none of the really important qualitative nor quantitative results in turbulence theory are derived from first principles of fluid dynamics such as the Navier Stokes (N.S.) equations. The Kraichnan stochastic models embody most of the structural properties of N.S. and lead to (closed "master equations") for mean quantities thereby avoiding the formidable closure problem of the original N.S. equations. By using a renormalisation procedure, generalised from quantum theory, he shows that after introducing a small periodic forced term $f_i(k)$ in the presence of isotropic turbulence, the equations of motion of the infinitesimal response matrix contains a term representing a dynamical damping with "memory" resembling viscous damping. The approach has some severe deficiencies but for certain conditions on k (see below) it does have the advantage of describing in a natural way, the physical phenomenon observed in turbulence that small scales of turbulence react on larger scales like a dynamical or eddy viscosity that augments the molecular viscosity.

To overcome some of the deficiencies which arise in other regards, he shows that if we take physically appropriate initial k spectra in which the excitation per mode falls off rapidly with k then the viscosity term η measures both amplitude decay and energy decay just like an ordinary viscosity provided that k is small compared to all appreciably excited wave numbers of the turbulence. But if substantial energy lies in wave numbers below k then η may have little relation to energy-transfer dynamics, largely because of phase decorrelation of the Green's function governing the response matrix.

The conditions in our cascade model would seem to fit the former case and avoid any severe decorrelation. This condition is being utilised in our attempt to find the asymptotic value of the autocorrelation function M under these conditions . The form of M is expressible as $\exp-\Gamma(y-y_0)^2$ when forming Eqs. (4) and (5). We recall that y is $y(k,t)$ and so our original expression $M(k,t)$ is consistent. An exponential form for M is possibly not unexpected in view of Narasimha's comments[2] (and examples) on the prevalence of exponential forms in turbulence studies.

The relationship which we suggest and utilise, of the kinetics of glass transitions to the above outlined Kraichnan models, is given below.

Kraichnan points out that if k_{max} (in his truncated Euler model of N.S. evolution where the initial statistical ensemble is Gaussian) is large compared to the characteristic viscous cut-off wave number then the problem is well posed and the quadratic energy constant can be decreased only by the viscous damping term. It is then worth noting that if k_{max} is taken as a kind of intermolecular spacing scale or mean free path, then the truncated system constitutes a non-trivial model of a molecular liquid. (However, there are some deep-lying anomalies which can arise in such a liquid model as Kraichnan points out.) We are nevertheless reminded of a connection to the glass transition model. Furthermore, in his progress to try to resolve the N.S. evolution difficulties, Kraichnan invoked an analogy of a quantum mechanical particle in a random potential where the particle undergoes a diffusion process in momentum space. The renormalised perturbation expansion for this problem is closely analogous to those for the convection of a passive scalar field by a random velocity field and resemble those for the turbulence problem. Unfortunately the diffusion coefficient which results in the WKBJ limit is given incorrectly unless the model is reworked using a generalised Schroedinger evaluation involving unequal time arguments. This suggests the extension to several particles moving in random potentials as a possible method for modelling in turbulence. There is then a tempting connection with the kinetics of glass transition model where Götze et al[8] developed the early mode-coupling model based on the theory of quantum particles moving in random potentials.

Appendix B

In the kinetics of the glass transition model, the nature of M with respect to ϕ arises from projecting the so-called random force $R_k(t)$ on to pair products of conserved variables (microscopic particle density and current) and factorising the resulting 4-point correlation function into products of 2-point correlation functions similar to $\phi(kt)$. This retains only pair products of the density and the memory function M to a form quadratic in $\phi(kt)$. Then, in the aerodynamic oscillations of blades in a cascade, the analogous conserved variables corresponding to "density" and "flow" are taken in a volume (cell) element V of the cascade containing several blades at time $t=0$. "Density" corresponds to $1/V$ times the mass of the blade motions moving in and out of the boundary of the volume cell over a period T_P of oscillation, i.e. $\frac{1}{V}\oint dm$ (dimensions of density). "Flow" corresponds to $\frac{1}{T_P}$ times the rate of change of the amount of blade mass which passes into and out of the volume element during a cycle, i.e. $\frac{1}{T_P}\oint\left|\frac{dm}{dt}\right|dt$ (dimensions of flow)

This concept of mass flow through a volume V can be more readily appreciated for rotating stall (which is the first limiting instability incurred). (See figure 5.)

There is an analogous relationship arising from the foregoing which might be drawn between $\phi(kt)$ and the blade motion $y(kt)$ of representative points on blades in a group in terms of the Van Hove functions $G(r,t)$. The Fourier transform of $\phi(kt) = G(r,t) = G_S + G_D$ where $G_S\ (r,t)$ is the probability of finding a (fluid) particle at time t at distance r from its original (origin) position resulting from diffusion and hydrodynamic-like processes and $G_D\ (r,t)$ is the probability of finding a second particle at r' at time t. The function $G_D(rt)$ characterises the correlated time-displaced motion of the two fluid particles in the bath of all other particles. The analogy for the blades in individual motions in a local region of a cascade can be visualised in regard to points represented by $y(k,t)$ on the blades in the cascade.

We also note that in the kinetic glass transition the hydrodynamic-like behaviour governed by the diffusion equation changes rapidly to a succession of jumps throughput a "frozen" disordered structure according to computer simulation[18].

In our blade model the analysis of the dynamics refers to pathways of bifurcations (analogous to jumps) in the progress of (some) solutions for $y(k,t)$ to chaos under certain conditions given in the text, i.e. progressive decay of correlations with time along different pathways similar to the decay paths of $\phi(k,t)$ in Figure 1.

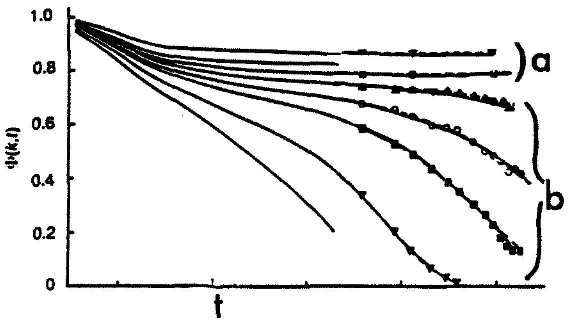

Figure 1

Two regimes indicated for $\phi(k,t)$ in particle suspensions. The upper regime ϕ tends to an ordered plateau state. The lower b decays to zero at time t increases (Ref. 6).

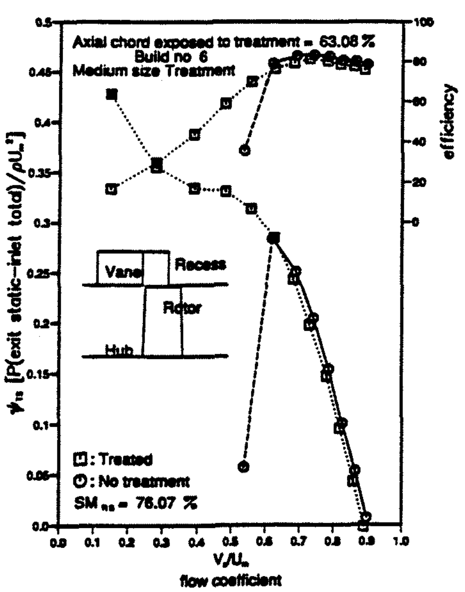

Figure 2

Results in Reference 11 for improvement in pressure coefficient using recessed vane passive stall control.

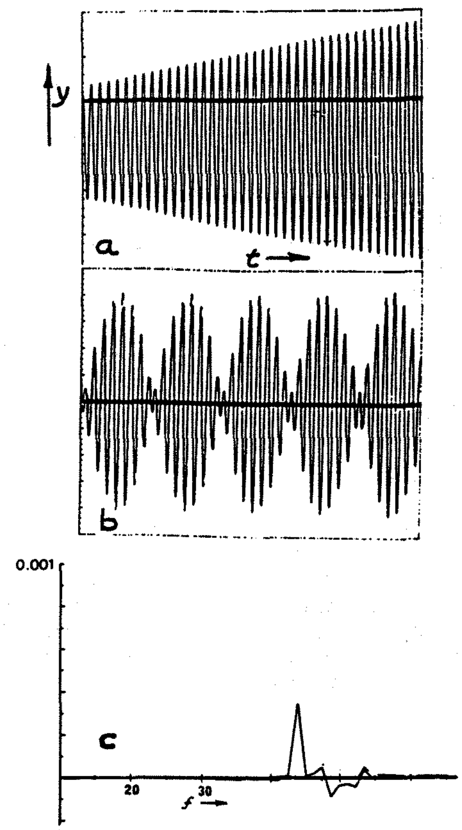

Figure 3

Progressive entrainment (b) of unstable blade oscillation (a) after injection of perturbation. Final stabilisation Fourier transform (c) for version of Eq. (1) without *M*. For Eq. (4) final entrainment may not necessarily occur because beat oscillations may collapse.

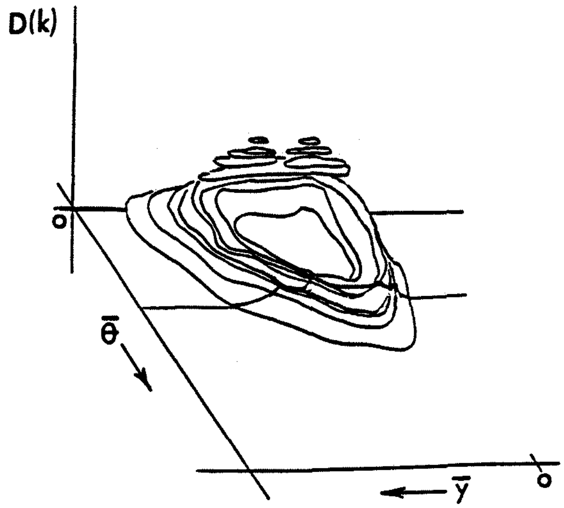

Figure 4

Example of an ordered structure with spectral density contours $D(k)$ at time t resulting from adding "coloured noise" to a Lotka-Volterra based model for $\bar{y}, \bar{\theta}$ in chaotic motion, e.g. $C = D$ in Eq. (4) or if $M(k,t)$ is rendered "coloured" by recirculation of selected k-spectra.

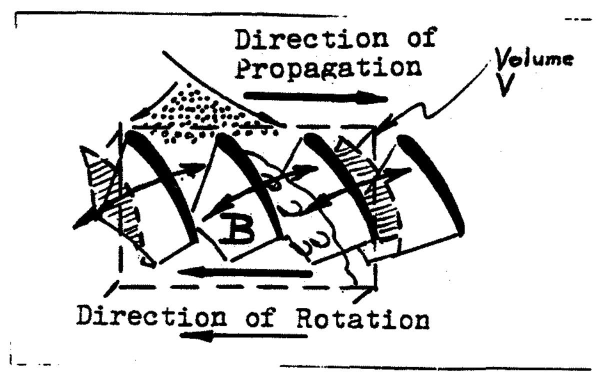

Figure 5

Blade B and adjacent blades oscillating (shaded) beyond volume V during rotating stall. Time-varying blade mass within volume V is $m(t)$.