ONE APPROACH OF ADAPTIVE CONTROL SYNTHESIS OF SYSTEMS WITH FLEXIBLE STRUCTURE BY USING ITS REDUCED DYNAMIC MODEL

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Abstract

Several methods for synthesis adaptive control of flexible systems are well known. Methods based on eigen-functions and corresponding natural shape modes are frequently used for mathematical description of flexible system motion. In this paper a different approach of synthesis adaptive control of flexible systems is presented. Method is based on the generalized method of decomposition and reducing mathematical model of elastic structure dynamics performed for systems of second order differential equations in direct procedure, which is used for synthesis complete mathematical model of flexible system motion. This method is useful for synthesis active control of elastic structure system vibrations. Main feature of this approach of mathematical modeling flexible systems is possibility of system control synthesis in several phases corresponding to the actual system state which is in this phase independent of other dominant shape modes. Main advantage of presented method of simplify computer oriented procedure for simulation of flexible system motion and corresponding control design are presented and described in the paper. This method includes numerical procedure for solving algebraic Riccati matrix equation with corresponding computer package which is main problem of complete presented method. Some performances of complete computer program package for simulation of flexible system motion, based on presented procedure of its mathematical modeling, are shown in the given example. Corresponding PID control synthesis are also presented. The concept of adaptive control synthesis of internal flexible structure dynamics is also presented in the paper. Internal dynamics of flexible system is usually described with large scale mathematical submodel corresponding to the model of system nominal motion. System control is assumed as same as for system with rigid structure, but with change control law corresponding to the internal dynamics state as a small disturbance of the whole system. Any additional generalized coordinates of system control are not included in the first phase of its design.

Introduction

Complete system can be separated onto four subsystems by using Kalman's decomposition as followed [2]: subsystem which is observable and controllable; subsystem which is not observable but controllable; subsystem which is observable, but not controllable; subsystem which is not observable and not controllable. The first of mentioned subsystems is of prime interest in problems of system control design and the subject of this paper only. Several methods for reducing subjected subsystem are known. They can be classified in six basic approaches and its combinations as followed [2]:

-Parameter Optimization approach;

-Aggregation approach;

-Singular Perturbation approach;

-Modal Dominance approach;

-Component Cost Analysis approach and

-Internal Balancing approach.

Discussions in the paper are related to a third of mentioned approaches in intention to define some possible ways for solving existing numerical and other problems which arises in its use.

First step in a system control synthesis is reducing of system dynamic model on usable dimensions, but without arbitrary lose in its accuracy. If system nominal control is defined completely, the problem is how to compensate internal flexible system dynamics as much as possible assuming it as a small disturbance around nominal motion. Another problem is synthesis of nominal dynamics in accordance to the minimization of system internal dynamics influence trough the whole phase of mentioned system motion. This problem presents the advanced system control. Internal dynamics can be used for different nominal motion synthesis, which optimizes the system performances in a measure greater as for the rigid one.

Several problems of control synthesis of flexible systems with distributed parameters can be described by coupled mathematical model between nominal and flexible internal dynamics. Dynamic systems with nominal and internal flexible motion are presented in a problems of anti-flutter control synthesis, synthesis of passive and active control of rotor head vibrations, control synthesis of flexible structure manipulators in a space robotics, control of vibrations of mechanical systems, etc. Usually, nominal dynamics is described by nonlinear dynamic model. Internal one is usually given in a linear mathematical form (except for the systems with large flexible displacements, which are not the subject of this paper). It

is useful to reduce mentioned dynamic submodel of internal flexible system dynamics on a minimal form which includes only modes of interest to be compensated, whose number is less of the number of dominant flexible modes. By using this procedure, problem of system control synthesis can be described in a more simplified mathematical form.

Mathematical model of complete nonreduced flexible system can be generally described in the following implicit form [3]:

$$\dot{X} = f[X, U, \varepsilon(X, \dot{X}, Y, \dot{Y})]$$
$$= g(Y, X, \dot{X}, U)$$
(1)

with corresponding initial conditions. X is the subvector of system nominal dynamics state, which is not the subject of system reducing procedure. Additional vector Y of system state corresponds to its internal flexible dynamics described by the second differential matrix equation of the relations (1). Parameter ε is a measure of internal system dynamic influence on its nominal motion.

Procedure of second order dynamic system reduction

Presented procedure is based on the method of first order dynamic system reduction [6]. Consider a dynamics system given in a matrix form by the following relation:

$$M\ddot{X} + D\dot{X} + KX = F$$
(2)

with corresponding initial conditions. X is the vector of flexible system state. Equation (2) can be transformed into a first order matrix form by following change of system coordinates:

$$\dot{X} = Y$$
(3)
$$= AY + BX + G$$

where system matrices are defined as:

$$A = -M^{-1}D \quad B = -M^{-1}K \quad = M^{-1}F$$
(4)

Matrix form (3) can be written in a following form:

$$\dot{Z} = PZ + Q$$
(5)

where subvectors of generalized coordinates are redefined in the form:

$$Z = \{Z_1, Z_2\}^T ; X = \{X_1, X_2\}^T ; Y = \{Y_1, Y_2\}^T$$
$$Q = \{Q_1, Q_2\}^T ; Z_1 = \{X_1, Y_1\}^T ; Z_2 = \{X_2, Y_2\}^T$$
(6)

Corresponding submatrices are given in the form:

$$P_{11} = \begin{vmatrix} 0 & I \\ B_{11} & A_{11} \end{vmatrix}; .. P_{12} = \begin{vmatrix} 0 & 0 \\ B_{12} & A_{12} \end{vmatrix}; .. P_{21} = \begin{vmatrix} 0 & 0 \\ B_{21} & A_{21} \end{vmatrix}$$
(7)
$$P_{22} = \begin{vmatrix} 0 & I \\ B_{22} & A_{22} \end{vmatrix}; .. Q_1 = \begin{vmatrix} 0 \\ G_1 \end{vmatrix}; .. Q_2 = \begin{vmatrix} 0 \\ G_2 \end{vmatrix}$$

where A, B and G are the cofactors of the corresponding matrices given by relations (4). By using procedure of first order matrix system reduction, system form (5) can be de coupled into two separated subsystems presented in the following form:

$$\dot{W}_1 = C_1 W_1 + E_1 \qquad \dot{W}_2 = C_2 W_2 + E_2$$
(8)

where are:

$$_1 = P_{11} - P_{12}L$$
$$_2 = P_{22} - LP_{12}$$
(9)
$$E_1 = Q_1 + N(Q_2 + LQ_1)$$
$$E_2 = Q_2 + LQ_1$$

Matrices L and N are the solutions of corresponding matrix algebraic Riccati equations:

$$P_{21} - P_{22}L + LP_{11} - LP_{12}L = 0$$
$$-P_{11}N + P_{12}(I + LN) + N(P_{22} + LP_{12}) = 0$$
(10)

Corresponding transformation of generalized system coordinates is given by matrix relation:

$$\begin{vmatrix} Z_1 \\ Z_2 \end{vmatrix} = \begin{vmatrix} I & -N \\ -L & I + LN \end{vmatrix} \begin{vmatrix} W_1 \\ W_2 \end{vmatrix}$$
(11)

Expanding presented relations by cofactor matrices we can determine matrices of the systems (8) in the following form:

$$C_1 = \begin{vmatrix} 0 & I \\ B_{11} - B_{12}L_{11} - A_{12}L_{21} & A_{11} - B_{12}L_{12} - A_{12}L_{22} \end{vmatrix}$$
$$C_2 = \begin{vmatrix} -L_{12}B_{12} & I - L_{12}A_{12} \\ B_{22} - L_{22}B_{12} & A_{22} - L_{22}A_{12} \end{vmatrix}$$

$$E_1 = \begin{vmatrix} (N_{11}L_{12} + N_{12}L_{22})G_1 + N_{12}G_2 \\ (I + N_{21}L_{12} + N_{22}L_{22})G_1 + N_{22}G_2 \end{vmatrix}$$
(12)
$$E_2 = \begin{vmatrix} L_{12}G_1 \\ L_{22}G_1 + G_2 \end{vmatrix}$$

Cofactor matrices are presented by the following relations:

$$A_{11} = M_{11}^{-1}[-D_{11} - M_{12}(M_{22} - M_2 M_{11}^{-1}M_{12})^{-1}(M_2 M_{11}^{-1}D_{11} - D_{21})]$$
$$A_{12} = (M_{11} - M_{12}M_{22}^{-1}M_{21})^{-1}(M_{12}M_{22}^{-1}D_{22} - D_{12})$$
$$A_{21} = (M_{22} - M_2 M_{11}^{-1}M_{12})^{-1}(M_2 M_{11}^{-1}D_{11} - D_{21})$$
$$A_{22} = M_{22}^{-1}[-D_{22} - M_{21}(M_{11} - M_{12}M_{22}^{-1}M_{21})^{-1}(M_{12}M_{22}^{-1}D_{22} - D_{12})]$$

$$B_{11} = M_{11}^{-1}[-K_{11} - M_{12}(M_{22} - M_{21}M_{11}^{-1}M_{12})^{-1}(M_{21}M_{11}^{-1}K_{11} - K_{21})]$$

$$B_{12} = (M_{11} - M_{12}M_{22}^{-1}M_{21})^{-1}(M_{12}M_{22}^{-1}K_{22} - K_{12}) \qquad (13)$$

$$B_{21} = (M_{22} - M_{21}M_{11}^{-1}M_{12})^{-1}(M_{21}M_{11}^{-1}K_{11} - K_{21})$$

$$B_{22} = M_{22}^{-1}[-K_{22} - M_{21}(M_{11} - M_{12}M_{22}^{-1}M_{21})^{-1}(M_{12}M_{22}^{-1}K_{22} - K_{12})]$$

$$G_1 = (M_{11} - M_{12}M_{22}^{-1}M_{21})^{-1}(F_1 - M_{12}M_{22}^{-1}F_2)$$

$$G_2 = M_{22}^{-1}[F_2 - M_{21}(M_{11} - M_{12}M_{22}^{-1}M_{21})^{-1}(F_1 - M_{12}M_{22}^{-1}F_2)]$$

Equations (8) can be expanded into second order matrix differential equation in a form:

$$\ddot{U} + A_R \dot{U} + B_R U = G_R \qquad (14)$$

where system matrices are presented in the form:

$$A_R = -(A_{11} - B_{12}L_{12} - A_{12}L_{22})$$
$$B_R = -(B_{11} - B_{12}L_{11} - A_{12}L_{21}) \qquad (15)$$
$$G_R = (B_{11} - B_{12}L_{11} - A_{12}L_{21})S_1 + S_2$$

S1 and S2 are the corresponding (in dimension) cofactors of vector E. Linear transformation of generalized coordinates of reduced system is given in a form:

$$\dot{U} = T_{21}X_1 + T_{22}\dot{X}_1 + N_{21}X_2 + N_{22}\dot{X}_2 \qquad (16)$$

where cofactors T21 and T22 are corresponding cofactors of the matrix T, given by the relation:

$$T = I + NL \qquad (17)$$

It is also of interest to define the complement submodel of reduced part of transformed system, which can be used in a two step method of system model synthesis, in a form:

$$\ddot{V} + A_E \dot{V} + B_E V = G_E \qquad (18)$$

where subsystem matrices are given in the form:

$$A_E = -(C_{11} + C_{12}C_{22}C_{12}^{-1})$$
$$B_E = -C_{12}C_{21} + C_{12}C_{22}C_{12}^{-1}C_{11} \qquad (19)$$
$$G_E = C_{12}R_2 + R_1$$

Cij are the corresponding (in dimension) cofactors of matrix C and Ri are the corresponding cofactors of matrix E. Linear transformation of generalized coordinates of complement subsystem form is presented as:

$$V = L_{11}X_1 + L_{12}\dot{X}_1 + X_2 \qquad (20)$$

Mentioned two step procedure can be used for decoupled system control synthesis. The first step is reduction of the whole system into representative reduced model. The second step is decoupling reduced form into two decoupled submodels, where one or both of them can be controlled with separate system control for slow and fast modes.

Method of numerical solving of matrix algebraic Riccati equation.

As it is well known, iterative methods for solving matrix algebraic Riccati equation are not too powerful because the initial point and convergence conditions for the method of stationary point usually can not be determined in general form. For that purpose generalized gradient method is effective for solving matrix algebraic Riccati equation for large scale problems, but local extremes (complex solutions) of the given Riccati matrix equation must be overlaid. Complete procedure and performances of computation corresponding solutions are presented in the following text.

Method of stationary point cannot be used without some transformation of mentioned matrix Riccati equation because it contains the parabolic algebraic equations, which satisfy the convergence condition if the initial point is inside the region bounded between possible solutions only. It produces this method unusable because generally is not possible to determine the initial point which satisfy mentioned conditions. One of the possible ways for performing given matrix equation into a form which satisfies the convergence condition for the wide region of initial points is to transform each of parabolic algebraic equation into two corresponding algebraic equations called as "upper" and "lower" form. The first one is define as a parabolic equation in the same form as it is defined in matrix equation, whose region is bounded from the upper side with the 'second' zero of the mentioned equation. The second one is defined with inverse parabolic equation bounded from the lower side with the 'first' zero of corresponding non transformed equation. Main problem which follows this procedure is in fact that 'upper' and 'lower' possible solutions depends of initial point, which generally cutting out some of the possible real solutions. Just because of that this method is not so effective for use in general form of the problem.

Method discussed here is combined general gradient-Newton's method in connection with method of minimal quadratic forms. In this case arises problem of local extremes which are not the real solutions of the given matrix Riccati equation. It will be shown that it is possible to separate quadratic form on two domains, with and without local extremes. It is also possible to transform given quadratic form to cut out some or all of local extremes included in procedure of gradient method. However, it depends of number of complex solutions of the given Riccati equation, because each of them corresponds to one local extreme. Method of scaling

matrices of given Riccati equation can be used for cutting out local extremes.

For solving general algebraic equations, given in a vector form as:

$$f(x) = 0 \qquad (21)$$

we can determine corresponding procedure for minimizing following functional, given in a form:

$$F = \sqrt[q]{\sum_{i=1}^{n} |f_i(x)|^q} \qquad (22)$$

whose minimum is equal zero. Numbers r and q are positive numbers. If r=1 and q=2 functional is completely differentiable for the each corresponding zeros of the given equation. In case r=q=2k functional has to faster convergence then the first one. For both cases the local extremes or saddle points are existed and makes some problems in computational procedure. Optimization problem of the second type functional is a problem with constraints, and just because of that it is more complicated than the first one. Procedure of localization of local extremes for the first functional are very sensitive, especially for the problems with large scale dimensions, and it is possible to cross over the local extreme. Problem of oscillation around local minimum or extreme is presented for the booth of cases.

For determination of norm minimum is used combined generalized gradient and Newton's method, corresponding to the value of q=2. Gradient is defined by next relation:

$$= \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{m} g_{ij}^2} \qquad (23)$$

were corresponding partial derivatives are given in a form:

$$g_{ij} = \frac{\partial \|E\|}{\partial(I_{ij})} \qquad (24)$$

Next value of unknown nonquadratic matrix L is defined by the assumed step coefficient θ, or in a form:

$$\Delta I_{ij} = -\theta \frac{\|E(I_{ij})\|}{G} g_{ij} \qquad (25)$$

Instead the first of mentioned matrix algebraic Riccati equations (10) we can determine the norm of the nonquadratic error matrix E, given by next matrix relation:

$$E = -LA_{12}L + LA_{11} - A_{22}L + A_{21} \qquad (26)$$

where matrices are with corresponding dimensions nxn, nxm, mxn and mxm, n dimension of reduced system form and m dimension of its complement subsystem. In a scalar form of coefficients equation (26) can be written as:

$$e_{ij} = -a_{ij}^{21} + \sum_{k=1}^{m} a_{ik}^{22} x_{kj} - \sum_{k=1}^{n} a_{kj}^{11} x_{ik} + \sum_{k=1}^{n} \sum_{l=1}^{m} a_{kl}^{12} x_{ik} x_{lj} \qquad (27)$$

Norm of error matrix E, defined in the form:

$$\|E\| = \sqrt[q]{\sum_{i=1}^{n} \sum_{j=1}^{m} |e_{ij}|^q} \qquad (28)$$

where n and m defines the matrix dimensions, and q=2k, k=1,2,... is the assumed exponent (in computer program q=2), must satisfy following condition:

$$E = 0 \qquad (29)$$

if matrix L is a solution of first of equations (10). Square of the norm of error matrix E is forth degree nonnegative polynom of nxm independent variables, whose coefficients can be presented in the following form:

$$e_{ij}(x_{ij}; x_{k\ne i, l\ne j} = k) = a_{ji}^{12} x_{ij}^2 + (a_{ii}^{22} - a_{jj}^{11}) x_{ij} - (a_{ij}^{21} - b_{ij})$$

$$e_{ij}(x_{i,k\ne j}; x_{p\ne i, r\ne k} = k) = (c_{ij} - a_{kj}^{11}) x_{ik} + (d_{ij} - a_{ij}^{21})$$

$$e_{ij}(x_{k\ne i, j}; x_{p\ne k, r\ne j} = k) = (a_{ik}^{22} + f_{ij}) x_{kj} + (g_{ij} - a_{ij}^{21})$$

$$e_{ij}(x_{k\ne i, l\ne j}; x_{p\ne k, r\ne l} = k) = h_{ij} - a_{ij}^{21}$$

where corresponding coefficients are:

$$b_{ij} = \sum_{\substack{k=1 \\ k\ne i}}^{m} a_{ik}^{22} x_{kj} - \sum_{\substack{k=1 \\ k\ne j}}^{n} a_{kj}^{11} x_{ik} + \sum_{\substack{k=1 \\ k\ne j}}^{n} \sum_{\substack{l=1 \\ l\ne i}}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$c_{ij,\delta} = \sum_{l=1}^{m} a_{\delta l}^{12} x_{lj}$$

$$d_{ij,\delta} = \sum_{k=1}^{m} a_{ik}^{22} x_{kj} - \sum_{\substack{k=1 \\ k\ne\delta}}^{n} a_{kj}^{11} x_{ik} + \sum_{\substack{k=1 \\ k\ne\delta}}^{n} \sum_{l=1}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$f_{ij,\delta} = \sum_{k=1}^{n} a_{k\delta}^{12} x_{ik}$$

$$g_{ij,\delta} = \sum_{\substack{k=1 \\ k\ne\delta}}^{m} a_{ik}^{22} x_{kj} - \sum_{k=1}^{n} a_{kj}^{11} x_{ik} + \sum_{k=1}^{n} \sum_{\substack{l=1 \\ l\ne\delta}}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$h_{ij,\delta\eta} = \sum_{k=1}^{m} a_{ik}^{22} x_{kj} - \sum_{\substack{k=1 \\ k\ne j}}^{n} a_{kj}^{11} x_{ik} + \sum_{\substack{k=1 \\ k\ne j}}^{n} \sum_{\substack{l=1 \\ l\ne i}}^{m} a_{kl}^{12} x_{ik} x_{lj} \qquad (31)$$

Expanding given relations (31) square of the norm of error matrix E can be presented in the following scalar form:

$$\|E(x_{ij}, x_{k\ne i, l\ne j} = k)\|^2 = \sum_{p=1}^{m} \sum_{q=1}^{n} e_{pq}^2 =$$

$$= e_{ij}^2(x_{ij}) + \sum_{\substack{p=1 \\ p\ne i}}^{m} e_{pj}^2(x_{ij}) + \sum_{\substack{q=1 \\ q\ne j}}^{n} e_{iq}^2(x_{ij}) + \sum_{\substack{p=1 \\ p\ne i}}^{m} \sum_{\substack{q=1 \\ q\ne j}}^{n} e_{pq}^2(x_{ij}) \qquad (32)$$

or in expanded form as:

$$\|E(x_{ij}, x_{k \neq i, l \neq j} = k)\|^2 = (\alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij})^2 +$$

$$+ \sum_{k=1}^{n+m-2} (\delta_{ij}^k x_{ij} + \varepsilon_{ij}^k)^2 + \sum_{k=1}^{nm-n-m+1} (\phi_{ij}^k)^2 = \qquad (33)$$

$$= (\alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij})^2 + (\delta_{ij} x_{ij}^2 + \varepsilon_{ij} x_{ij} + \phi_{ij})$$

It must be noted that the second brackets term are always nonnegative. Given polinom has no more than tree local extremes, and also must have one local extreme. If all of the possible polynoms corresponding to the each of coefficients of solution of matrix Riccati equation has only one local extreme which is equal zero, it means that the given problem has unique real solution of the problem (all of other possible solutions are complex and not of interest). It is of interest to define the conditions which satisfy that the mentioned form has only one local extreme. Question is can we satisfied that conditions by using procedure of scaling matrices of coefficients of given Matrix Riccati equation. Procedure of matrix scaling can be presented in the following form:

$$\mu E = -(\mu A_{21}) + A_{22}(\mu L) - (\mu L) A_{11} + (\mu L)(\mu^{-1} A_{12})(\mu L) = 0 \quad (34)$$

where μ is an arbitrary scalar. It means that the diagonal submatrices are unchanged, and nondiagonal submatrices are multiplied with factors μ and $1/\mu$. In the case of matrix scaling corresponding form of polinom coefficients given by relations (31) can be performed as:

$$b_{ij} = \sum_{\substack{k=1 \\ k \neq i}}^{m} a_{ik}^{22} x_{kj} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{kj}^{11} x_{ik} + \frac{1}{\mu} \sum_{\substack{k=1 \\ k \neq j}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$c_{ij,\delta} = \frac{1}{\mu} \sum_{l=1}^{m} a_{\delta l}^{12} x_{lj}$$

$$d_{ij,\delta} = \sum_{\substack{k=1 \\ k \neq \delta}}^{m} a_{ik}^{22} x_{kj} - \sum_{k=1}^{n} a_{kj}^{11} x_{ik} + \frac{1}{\mu} \sum_{\substack{k=1 \\ k \neq \delta}}^{n} \sum_{l=1}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$f_{ij,\delta} = \frac{1}{\mu} \sum_{k=1}^{n} a_{k\delta}^{12} x_{ik}$$

$$g_{ij,\delta} = \sum_{\substack{k=1 \\ k \neq \delta}}^{m} a_{ik}^{22} x_{kj} - \sum_{k=1}^{n} a_{kj}^{11} x_{ik} + \frac{1}{\mu} \sum_{k=1}^{n} \sum_{\substack{l=1 \\ l \neq \delta}}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$h_{ij,\delta\eta} = \sum_{k=1}^{m} a_{ik}^{22} x_{kj} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{kj}^{11} x_{ik} + \frac{1}{\mu} \sum_{\substack{k=1 \\ k \neq j}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{m} a_{kl}^{12} x_{ik} x_{lj}$$

$$\qquad (35)$$

If assume that μ is enough large number, corresponding relation (30) of norm can be written in a form:

$$\|E(x_{ij}, x_{k \neq i, l \neq j} = k)\|^2 = (\frac{1}{\mu} \alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij}')^2 +$$

$$+ \sum_{k=1}^{n+m-2} (\delta_{ij}^k ' x_{ij} + \varepsilon_{ij}^k ')^2 + \sum_{k=1}^{nm-n-m+1} (\phi_{ij}^k ')^2 = \qquad (36)$$

$$= (\frac{1}{\mu} \alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij}')^2 + (\delta_{ij}' x_{ij}^2 + \varepsilon_{ij}' x_{ij} + \phi_{ij}')$$

where tree local extremes can degenerate into only one local minimum defined with the term in second brackets added by the first order term into first bracket. Another question after discussions about existence of local extreme equal zero is what type is it. Enough condition for existence of local extreme equal zero is that each member of relation (30) must be equal zero in the same point, or each polinom of mentioned terms must have one zero same for all polynomial terms of relation (33). It means that the norm of error matrix E can be written as:

$$\|E(x_{ij}, x_{k \neq i, l \neq j} = k)\| = |x_{ij} - x_{ij}^*| \sqrt{(x_{ij} - x_{ij}^{**})^2 + \sum_{k=1}^{n=m-2} (\delta_{ij}^k)^2} \quad (37)$$

with corresponding relation in the form:

$$\sum_{k=1}^{nm-n-m+1} (\phi_{ij}^k)^2 = 0 \qquad (38)$$

where x*and x** are corresponding zeros of mentioned terms. Term in square root is always great than zero. It means that only one zero of the E norm exists and is equal x*. We must note also that the existing local zero is not a local extreme of E norm because it is not differentiable in that point. This is a main conclusion of this method on which the numerical procedure of computing local minimum equal zero is based. This fact makes procedure of computation too faster than usual generalized gradient method. Another quality of this computational method is that the local extremes corresponds to a different conditions for its existence than for the local minimum equal zero as a solution of subjected Riccati matrix algebraic equation. That is the reason why gradient method only for determination of local minimum can't be used. Performances of computational procedure are presented on a following figures. Presented method can make some problems of convergence in a domain of local extreme. There are two possibilities for solving this problem. One of them is to use method of matrix scaling, which can not be determined uniquely because the matrix equation of conditions for stationary points can not be solved. Another possibility is to define new functional which lies between given one which corresponds to the solution of Riccati equation and zero, or which has the same zeros as basic functional. This problem can be solved effectively and it arises from the equations (37). Main idea is to find some new functional without any stationary point which is not equal zero except local extreme as maximum. In that case Newton's generalized

method can be used as always convergent method, except in case of crossing over the local extremes as maximum. Mathematical formulation of this method can be written in a following form:

$$\left\| E'(x_{ij}, x_{k \neq i, l \neq j} = k) \right\| = \left| (\alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij}) \right| \leq$$

$$\leq \sqrt{(\alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij})^2 + (\delta_{ij} x_{ij}^2 + \varepsilon_{ij} x_{ij} + \phi_{ij})} =$$

$$= \left| (\alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij}) \right| \sqrt{1 + \frac{\delta_{ij} x_{ij}^2 + \varepsilon_{ij} x_{ij} + \phi_{ij}}{(\alpha_{ij} x_{ij}^2 + \beta_{ij} x_{ij} + \gamma_{ij})^2}} = \qquad (39)$$

$$= \left\| E(x_{ij}, x_{k \neq i, l \neq j} = k) \right\|$$

Term under the second of square roots is always greater then one. In that case arises one zero more for the norm E', which is not a zero of the norm of the initial corresponding functional of the Riccati matrix equation. Both of zeros are local minimums with nondifferentiable functional in that points. Convergence in that case will be too faster than for the basic procedure. Only one problem arises as the result of using this method. It is a problem of possible minimums which are not the solutions of the Ricati matrix equation. This problem can be solved by searching only, If some solution not satisfies given Riccati equation, whole procedure with new step or gradient or initial point must be repeated.

Main fact is that only one solution needs to be find by using this method because the functional is defined for any point with real coordinates. Nondiferentiability in each point which is a solution of the Riccati equation is effectively used for relatively fast convergence of given method, which is not a good performance of the gradient method. Problem of oscillating trough coming over the local minimum can slow the whole procedure.

Third approach for solving matrix Riccati equation can be expressed as optimization with constraints. Suppose that the norm of error matrix is presented in a following form (r=q=1) corresponding to the expression (22):

$$F(x) = \sum_{i=1}^{n} |f_i(x)| = \sum_{i=1}^{n} \lambda_i f_i(x) \qquad (40)$$

where are x corresponding n-dimensional vector of independent variables, λ_i corresponding scalars to the each cofactor of the error matrix defined as follow:

$$\lambda_i = \begin{cases} 1, f_i(x) \geq 0 \\ -1, f_i(x) < 0 \end{cases} \qquad (41)$$

It means that exists 2^n combinations of possible forms of given functional as a norm of the error matrix E equal F. Each of these combinations of vector coefficients corresponds to the one possible real or complex solution of

the given matrix Riccati equation. For any arbitrary point of independent variables we have exactly value of coefficients of vector λ. It means also that there are no explicit procedure for determination each initial point for calculation corresponding local minimum. Just because the fact that each local minimum corresponds to the cross section between two corresponding functional defined for the different values of vector λ, initial optimization problem can be expressed as a constrained optimization in the following form:

$$J(x) = F_i(x) \qquad (42)$$

with corresponding constraints in a vector form:

$$g_i(x) = F_i(x) - F_j(x) = 0 \qquad (43)$$

where are i and j corresponds to a values λi an λj. This formulation of the constrained optimization problem can be transformed into unconstrained one by using Lagrange multipliers in a form:

$$J' = F_i(x) + \mu[F_i(x) - F_j(x)] \qquad (44)$$

including corresponding constraint in a form (43). This formulation leads to the final form of transformed functional F as:

$$J' = J - (\frac{\partial g}{\partial x_k})^{-1} (\frac{\partial J}{\partial x_k}) g(x)$$

$$dx_k = -(\frac{\partial g}{\partial x_k})^{-1} \sum_{i=1}^{n-1} \frac{\partial g}{\partial x_i} dx_i \qquad (45)$$

with corresponding constraint (43). This optimization is n-1 dimensional optimization without any constraints. Main problem is how to define corresponding initial value and checking each of the steps values to satisfy the constraint. If we assume the corresponding dependent variable xk in quadratic form the problem can be solved easily if there are exists the real solutions of this expression. In opposite ,we must cheek another point does it satisfy the constraints.

Each of the presented methods can be used in some cases of the problem formulation. Combying them, it is possible to create a computer package with all mentioned options which can be used for this purpose. It must be noted that for the large scale problem this package can be used, but the possibilities for computing any of the solutions depends of problem formulation. For analyses and computation zeros of the Riccati matrix equation, which is a main computational problem of the presented procedure, are maked the computer package described in a following text.

Program package has several options, which can be of interest in a desired problem. This options and possibilities can be described as:
-adaptive step of iterations;
-adaptive step for gradient calculations;
-adaptive step of function searching in a domain of local extreme or minimum;
-adaptive step compensation of oscillations.
Another options are defined for the different type of functional which can be used. In a package all of described cases of functional types are presented.

Some performances of the used methods and corresponding results will be shown on the following graphs.

Description and performances of Computer package for numerical solving matrix Riccati equation

It is well known that complete optimization of nonlinear problem with high dimensionally can't be done without using direct search technique. As it is described this problem is nonlinear, but with some spatialties which leads to a spatial type of computational algorithm for local minimum determination of given functional which corresponds to the one of the possible real solutions of initial matrix Riccati equation. Presented procedure is based on combination of methods of minimal squares, generalized gradient method, generalized Newton's method and spatial search procedure performed for determination of local minimum of not differentiable functional. Procedure is generally not too fast but it can be used successfully in a several phases because the error of computing is not cumulative.

Presented procedure can be used for determination one of the possible real solutions of given Matrix Riccati equation. Procedure guaranties determination of one local minimum or local extreme. In a connection with other statements expressed in this paper, main problem is how to cross over local extreme toward local minimum, which is of interest only. Procedure leads to one of the possible local extremes or local minimums from one assumed initial point.

For determination of reduced form of the initial dynamic model of internal flexible system dynamics it is of interest to determine one of possible real local minimums. If we determine a local extreme instead local minimum, direct search technics is the possible way only for determination another initial point for further computational procedure.

Approximation model.

Presented results corresponds to the following simplified example of rotation of flexible disk with lumped six equal masses on both sided beams fixed to the base body. Approximation model is presented on the figure 1.
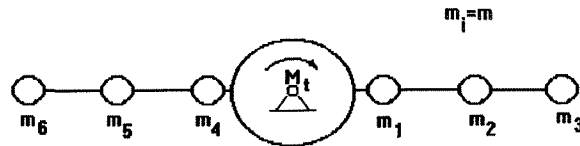
Fig. 1.

Corresponding parameters and constants are:
-body mass moment of inertia: J=20.0 kgm2;
-distance between lumped masses and base body: l=0.1 m;
-relative natural frequency: ratio EI/ml3=1000.0 s-2, were all lumped masses are equal;
-coefficients of proportional closed loop: k0=400.
-coefficients of differential closed loop: k1=300.
-coefficients of integral closed loop: k2=0.

Corresponding shape modes are shown on the fig. 2.

There are presented five cases of possible iterations for reducing mathematical model and solving corresponding matrix Riccati equation of given example. On the following graphs are shown convergence velocity and corresponding accuracy of numerical solution.

Iteration 1.
  index of norm type: INOR=1
  value of error matrix norm: ENOR=0.5495
  factor of steps: TET=0.9
  factor of steps for gradient calculations: D=.00001
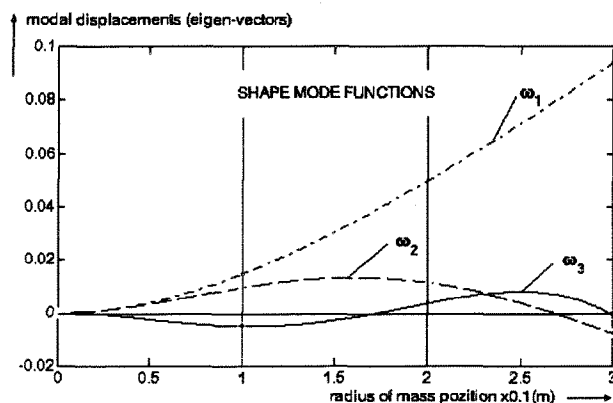 total number of iterations: NIT=201

Fig. 2.

initial values of matrix L0 and final values of matrix L:

$$
L_0 = \begin{vmatrix} 1.2 & 0.0 \\ -0.8 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{vmatrix} \quad L = \begin{vmatrix} 0.9671 & 0.0 \\ -0.7075 & 0.0 \\ 0.0 & 0.7510 \\ 0.0 & -0.2438 \end{vmatrix}
$$

Eigen-values of nontransformed model are:

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ω1=9.25 Hz ω2=60.57Hz ω3=162,72Hz

Eigen-values of transformed model are:

ω1=9.45 Hz ω2=55.07Hz ω3=163.61Hz

Difference of frequencies for complete and reduced approximation model is relatively small (not more than· 10%). For practical purposes this approximation can be used as initial mathematical formulation of the given model. Corresponding graphs are shown on the following figures.
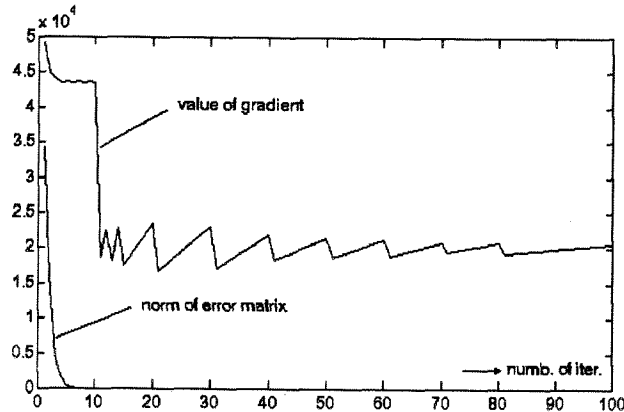

Fig. 3

On fig. 3 are shown norm of error matrix ENOR and gradient value depending of number of iterations NIT. Convergence is relatively fast because starting point is close to the calculated minimum, which is not a solution (ENOR is to different than zero). Evaluated gradient value shows that the calculated minimum is not a local extreme, but accuracy of minimum calculation is satisfied because norm of step matrix is less than .0001. (fig. 4.).
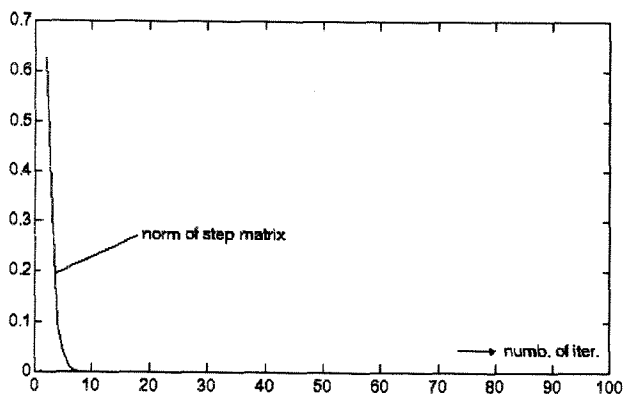
Fig. 4.

On fig. 5. is indicated that calculation procedure oscillate around local minimum, because the generalized scalar product of last two step matrices is closely to (-1.)

Iteration 2.

INOR=1 ; TET=.9 ; D=.00001 ; NIT=201 ; ENOR=.5697
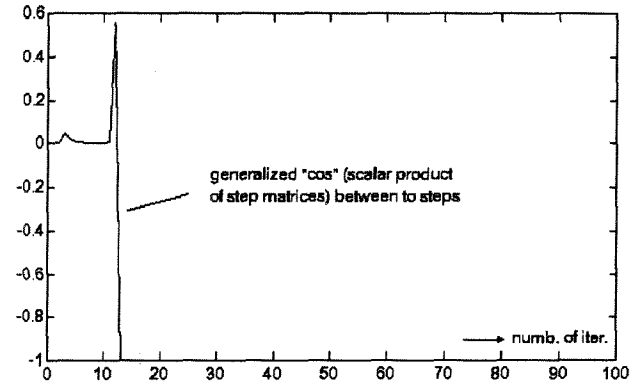
Fig. 5.

$$L_0 = \begin{vmatrix} 0.7 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{vmatrix} \quad L = \begin{vmatrix} .4397 & .0 \\ .2969 & .0 \\ .0 & .6269 \\ .0 & -.1754 \end{vmatrix}$$

Iteration 3.

INOR=1 ; TET=.1 ; D=.000001 ; NIT=1001 ; ENOR=.4858

$$L_0 = \begin{vmatrix} 0.7 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{vmatrix} \quad L = \begin{vmatrix} .4959 & .0 \\ .2837 & .0 \\ .0 & .6366 \\ .0 & -.1805 \end{vmatrix}$$

On fig. 6 is shown the difference of convergence velocities and calculated minimum between gradient step factor values TET=.9 and TET=.1. Norm of step matrix for these different gradient factors are shown on a fig. 7.
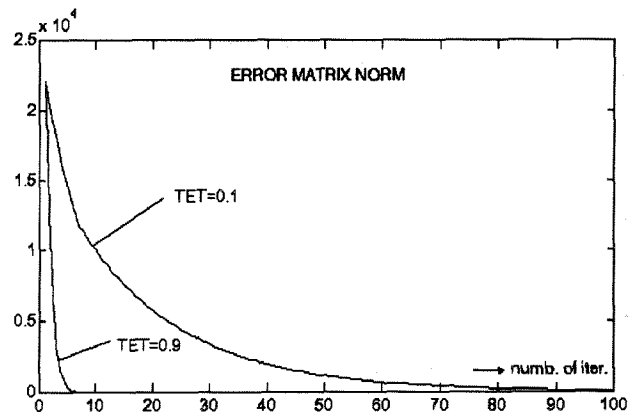
Fig. 6.

Figures 8 and 9 corresponds to the value of gradient factor TET=.1 It is interesting that the convergence is monotone because the corresponded scalar product of step matrices is always positive (fig. 9) and indicates iterations around local minimum (no local extreme) which is indicated on fig. 8.
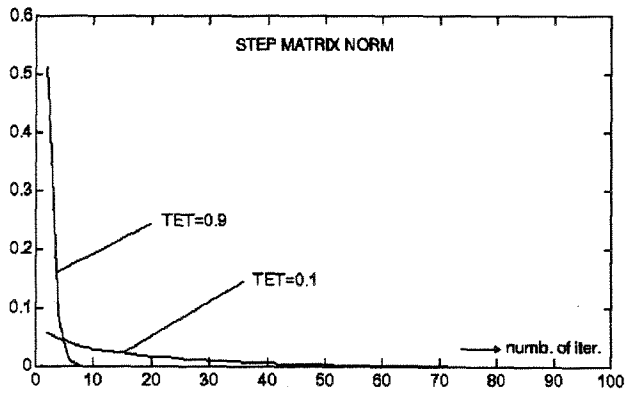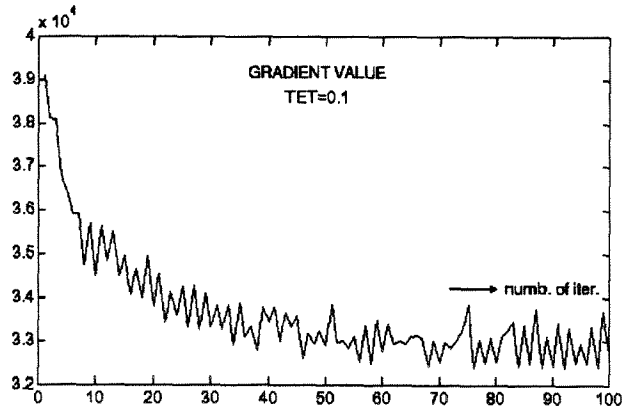
Fig. 7.

Fig. 8.

Iteration 4.

NOR=1 ; TET=.9 ; D=.00001 ; NIT=201 ;ENOR=.4453

$$L_0 = \begin{vmatrix} 0.7 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.7 \\ 0.0 & 0.2 \end{vmatrix} \quad L = \begin{vmatrix} .6212 & .0 \\ .2291 & .0 \\ .0 & .6583 \\ .0 & -.1921 \end{vmatrix}$$
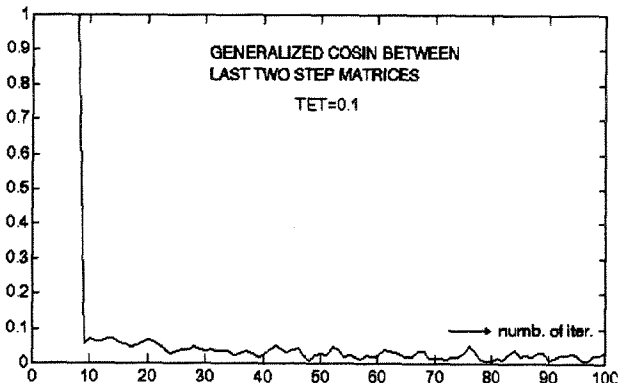

Fig. 9.

Iteration 5.

INOR=1 ; TET=.1 ; D=.000001 ;NIT=1001 ; ENOR=.418

$$L_0 = \begin{vmatrix} 0.7 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.7 \\ 0.0 & 0.2 \end{vmatrix} \quad L = \begin{vmatrix} .6441 & .0 \\ .2236 & .0 \\ .0 & .6617 \\ .0 & -.1939 \end{vmatrix}$$

On the figures 10 and 11 are presented convergence procedure (norm of error matrix and norm of step matrix), as the preprocedure before final calculation of corresponding solution. As in the case before, this solution is not exact because the error matrix norm is not too close to zero. But, calculated eigen-values of the reduced system form indicates that the given solution can be used for practical purposes in system control synthesis.
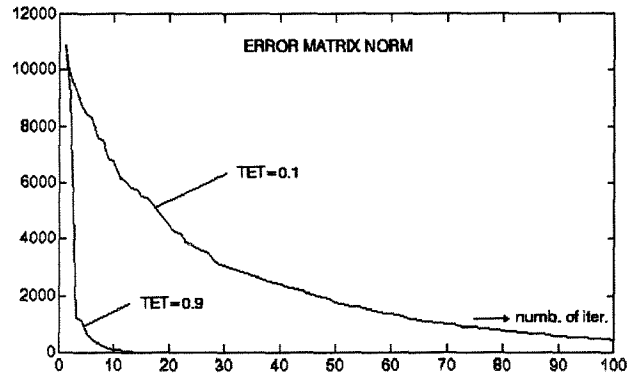
Fig. 10.

Final results of model reduction.

Corresponding frequencies of the reduced model are:

ω1=157.1 Hz;   ω2=64.239 Hz;   ω3=9.02 Hz.

Accuracy of final iteration is not too better than for the first one (about 10% also). Simulation results of complete and reduced model indicates accuracy of given reducing approximation.
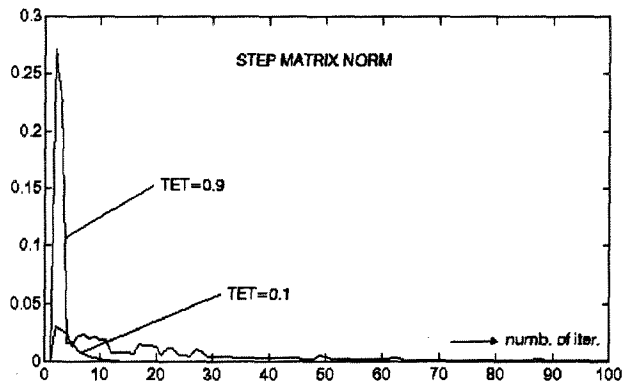
Fig. 11.

On figures 12 to 15 are shown characteristic computational performances for the final solution of system reduction. For that purpose it is calculated the corresponding solution of second of the matrix Riccati

equation of relations (10). Corresponding transformation matrix N as its solution is presented in following matrix form:

$$N = \begin{vmatrix} -0.3770 & 0.0803 & 0 & 0 \\ 0 & 0 & 0.3770 & -0.0803 \end{vmatrix}$$

Complete transformation matrix corresponding to the relations (11) can be presented in a following form:

$$T = \begin{vmatrix} 1.0 & 0 & 0.3770 & -0.0803 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & -0.3770 & 0.0803 \\ -0.6441 & 0 & 0.7572 & 0.0517 & 0 & 0 \\ -0.2236 & 0 & -0.0843 & 1.0180 & 0 & 0 \\ 0 & -0.6617 & 0 & 0 & 1.2495 & -0.0531 \\ 0 & 0.1939 & 0 & 0 & -0.0731 & 1.0156 \end{vmatrix}$$

If we neglect subvector W1 of fast generalized coordinates in equation (11) we can define relations between basic and dependent coordinates in a form:

$$T_z = \begin{vmatrix} 0.4864 & -0.1036 & 0 & 0 \\ 0 & 0 & -0.2980 & 0.0635 \end{vmatrix}$$
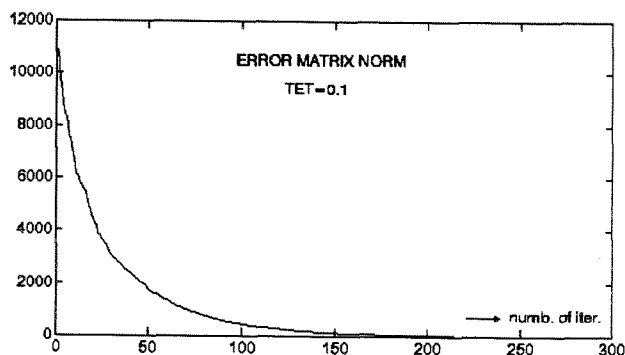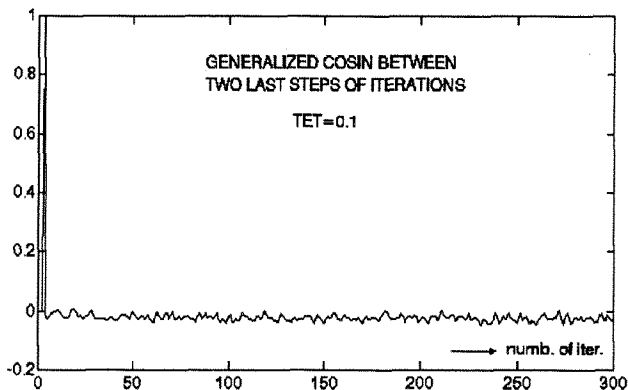

Fig. 12.
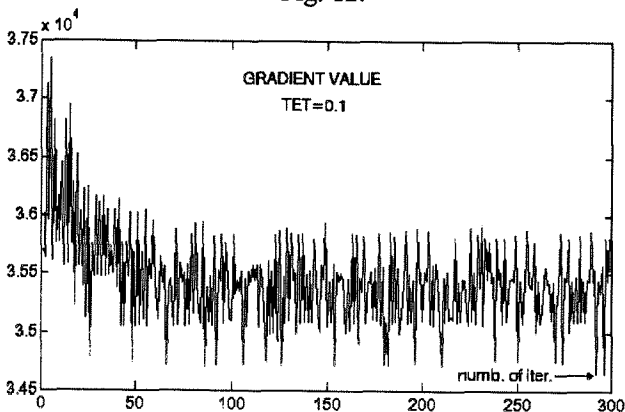
Fig. 13.
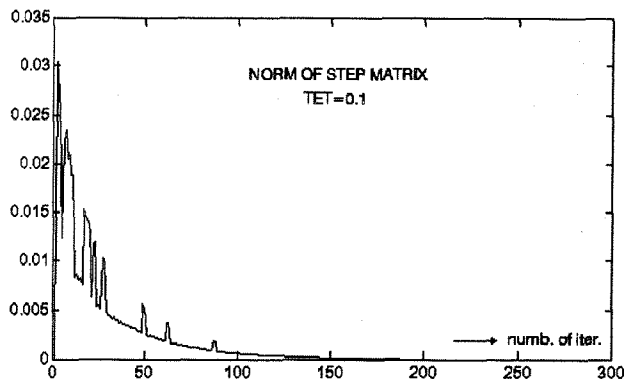
Fig. 14.

Fig. 15.

Method of system control synthesis based on the results of flexible system model reduction.

In the paper the PID control system design is presented in accordance to the example in a paper [2]. The difference is in change of torsion beam with radial flexible beam with lumped masses only. All other parameters are estimated to make any difference as small as possible. Block-diagram of corresponding controlled system is presented on the fig. 16. Corresponding constants are approximated [2] to give equivalent proportional and differential gain as k0 and k1, defined as input constants of the given example. Applied torsion moment as generalized control force is defined in a matrix form as:

$$M_t = k_0 (\theta_{ref} - \theta) + k_1 (\dot{\theta}_{ref} - \dot{\theta}) \qquad (46)$$

Corresponding desire state of nominal system motion is: θref=π/2 and (dθ/dt)ref=0. Given results are presented on the following figures.
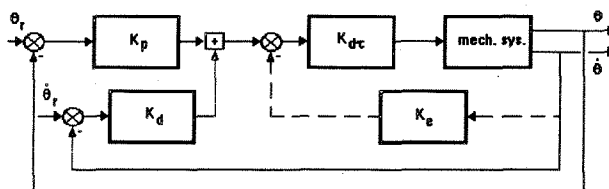

Fig. 16.

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On fig. 17. are presented base body system motion (angle θ) in the cases of rigid body, reduced model and full model motion. Control law is designed as a rigid one only, without direct compensation of internal flexible dynamics. On figures 18 and 19. are presented the top mass motion in a cases of reduced and full dynamic model.
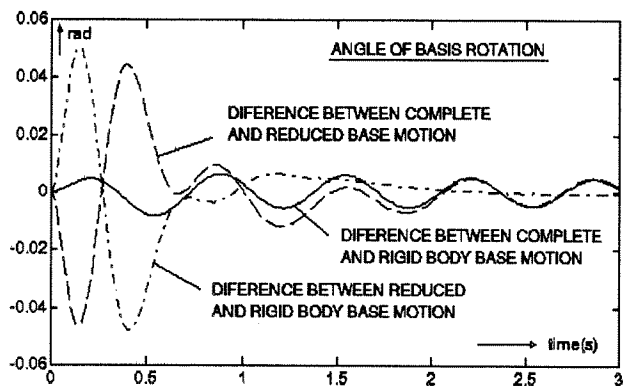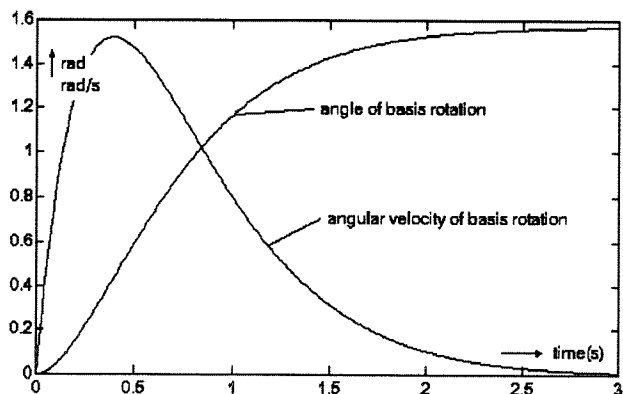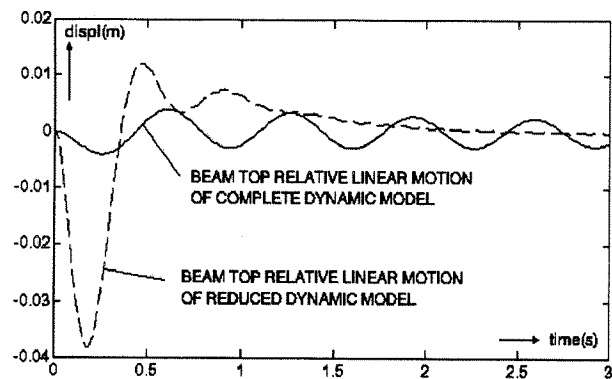

Fig. 17.

Fig. 18.

Fig. 19.

At last, it is subjected advanced active control system structure. Active control compensation of flexible system dynamics can be presented as a closed multiloop system with equilibrium state which is equal zero (negative closed loop). Suggested structure of adaptive control of flexible systems is shown on a fig. 20. This concept of control design can exist if modified control law of nominal system dynamics can not satisfy expected performances. X is

vector of nominal system state, Y1 and Y2 are vectors of reduced and complement flexible dynamic state, Uy1 control vector of reduced flexible model, Yoz1 and Yoz2 desired states of reduced and complement flexible model.
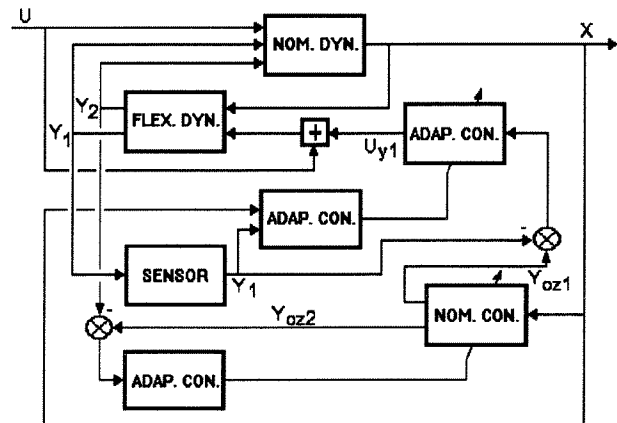

Fig. 20.

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