AN OPTIMAL ALGORITHM FOR ATTITUDE AND TRAJECTORY STABILIZATION FOR THE REMOTELY-PILOTED VEHICLES

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Abstract

A system of automatic stabilization for motion parameters of the remotely-piloted vehicles based on an optimal control algorithm is discussed here. An optimal, in terms of square performance criteria, solution could be completed using dynamic programming method by solving of matrix algebraic Riccati Equation. The asymptotical stability of the control loop is characteristic of this method. The Riccati Equation is solved numerically using standard mathematical methods on the computer. The algorithm gains could be regulated to minimize the weight of the parameters, which could not be measured, in the control law. The described method could be used to create a control algorithms for the attitude and trajectory stabilizations of the remotely-piloted vehicles in longitudinal and lateral fast and slow motions

Introduction

Control and monitoring systems for the remotely-piloted vehicles have received considerable attention in recent years as a possible solution to the problem of minimizing the payment for objective operations without any lost of efficiency. Technology apart, the way of improving tracking and control algorithms makes a major contribution to the solving of the problem and has already been rather considered [1-4] and also by the author of this paper [5].

One more system of automatic stabilization for motion parameters of the remotely-piloted vehicles based on an optimal control algorithm is discussed here. The composition and the quantity of measuring sensors on board the vehicle could be different. It is necessary to provide a satisfactory characteristics of the transition processes in the loops of control and stability of the whole system irrespective of the dynamic model complication in longitudinal and lateral fast and slow motions of the vehicle.

The flight regime (horizontal flight, lift, descent etc.) change time is rather more than the transition time in the loops of control system under consideration, so the flight dynamic characteristics of the vehicle change insignificantly and the associated equation coefficients could be taken constant for the flight regime. Consequently, flight dynamics could be described by the linear system of differential equations with associated set of coefficients and an optimal closed-loop of control could be analytically constructed using the dynamic programming method[6] for every flight regime.

An optimal, in terms of square performance criteria, solution could be completed by solving of matrix algebraic Riccati Equation[7]. The asymptotical stability of the control loop is characteristic of this method. The Riccati Equation could be solved numerically using standard mathematical methods on the computer.

Motion models

It is assumed here that the regime for sample calculations is horizontal direct flight. Forces and moments increments are have been expanded into a series till the first-order derivatives. Only the most important partial derivatives were taken into account.

The "Δ" means variations relative to the undisturbed flight regime. Subscript "$_0$" means the values of the undisturbed flight regime. Super scripts mark partials.

Longitudinal aircraft dynamics

Longitudinal motion of a rigid aircraft in an undisturbed air could be described as[8]:

$$\begin{aligned}
\Delta\dot{V} &= \frac{[-P_0\sin(\alpha_0)-Q_0^{\alpha}]\Delta\alpha+[P_0^V\cos(\alpha_0)-Q_0^V]\Delta V}{m_0}-g\Delta\Theta\\
\Delta\dot{\Theta} &= \frac{[P_0\cos(\alpha_0)+Y_0^{\alpha}]\Delta\alpha+[P_0^V\sin(\alpha_0)+Y_0^V]\Delta V}{m_0V_0}\\
\Delta\dot{\omega}_Z &= (M_{Z0}^{\alpha}\Delta\alpha+M_{Z0}^{\delta a}\Delta\delta_a+M_{Z0}^{\omega z}\Delta\omega_Z+M_{Z0}^{V}\Delta V)/I_Z\\
\Delta\dot{\xi} &= \Delta V\\
\Delta\dot{\eta} &= V_0\Delta\Theta\\
\Delta\dot{\vartheta} &= \Delta\omega_Z\\
\Delta\alpha &= \Delta\vartheta-\Delta\Theta
\end{aligned} \tag{1}$$

where m is the mass of the aircraft, V is the velocity relative to the air, P is thrust, g is gravity acceleration, Q is air resistance, Y is lift, I_z is longitudinal moment of inertia, M_z is pitching moment, α is an angle of attack, Θ is slope of the flight path, ω_z is rate of pitch, ξ is the path moved, η is the altitude, ϑ is pitch angle, δ_a is control surface angle.

Lateral aircraft dynamics

Lateral motion of a rigid aircraft in an undisturbed air could be described as[8]:

$$
\begin{aligned}
&\Delta\dot{\Psi}_V = \{(P_0 - Q_0 - Z_0^{\beta})\Delta\beta - [P_0 \sin(\alpha_0) + Y_0]\Delta\gamma_V\} / (m_0 V_0) \\
&\Delta\dot{\omega}_x = (M_x^{\delta e}\Delta\delta_e + M_x^{\omega x}\Delta\omega_x + M_x^{\beta}\Delta\beta + M_x^{\omega y}\Delta\omega_y) / I_x \\
&\Delta\dot{\omega}_y = (M_y^{\delta d}\Delta\delta_d + M_y^{\omega y}\Delta\omega_y + M_y^{\beta}\Delta\beta + M_y^{\omega x}\Delta\omega_x) / I_y \\
&\Delta\dot{\zeta} = -V_0\Delta\Psi_V \\
&\Delta\dot{\Psi} = \Delta\omega_y \\
&\Delta\dot{\gamma} = \Delta\omega_x \\
&\Delta\beta = \Delta\Psi - \Delta\Psi_V, \quad \Delta\gamma_V = \Delta\gamma
\end{aligned}
\tag{2}
$$

where I_x, I_y are lateral and yawing moments of inertia, M_x, M_y are roll and yawing moments, Z is lateral force, Ψ_V is course, Ψ is heading, ω_x is roll rate, ω_y is yawing rate, γ is roll, γ_v is velocity roll, β is the bank angle, ζ is lateral move, δ_e, δ_d are control surface angles.

Control algorithm

Generally the equations (1,2) could be represented in matrix form as:

$$
\begin{aligned}
&\dot{\vec{\mathbf{X}}} = \mathbf{A}\vec{\mathbf{X}} + \mathbf{B}\vec{\mathbf{U}} \\
&\vec{\mathbf{X}}(t_0) = \vec{\mathbf{X}}^{(0)}, \quad t_0 = 0
\end{aligned}
$$

where **X** is the state vector of the system of n dimension, **U** is the vector of control variables of m dimension, **A** is a transition matrix of the system of n×n dimensions, **B** is a matrix of control coefficients of n×m dimensions, n is a quantity of state variables, m is a quantity of control variables.

The control low which must be found in general form could be depicted as:

$$\vec{\mathbf{U}} = \mathbf{C}^T\vec{\mathbf{X}}$$

where **C** is some matrix of m×n dimensions.

So as the functional

$$\mathbf{J} = \int_0^{\infty} (\vec{\mathbf{X}}^T\mathbf{Q}\vec{\mathbf{X}} + \vec{\mathbf{U}}\vec{\mathbf{U}}^T)dt$$

could get a minimum with any initial values of state variables **X**, the **C** matrix must be defined as:

$$\mathbf{C} = -\mathbf{PB}$$

where **Q** is a positive definite square matrix of n×n dimensions, **P** is a symmetric matrix of n×n dimensions, which could be found as a solution of algebraic matrix Riccati Equation:

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBB}^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{3}$$

The solution of the equation (3) could be calculated on the computer through using standard procedures such as Repin-Tretjakov method[7], for example.

Matrix **C** must be calculated only once for every flight regime.

The concrete form of vector and matrix for longitudinal and lateral motions are depicted below.

Longitudinal aircraft dynamics

$$
\vec{\mathbf{X}} = \begin{bmatrix} \Delta V \\ \Delta\Theta \\ \Delta\vartheta \\ \Delta\omega_Z \\ \Delta\xi \\ \Delta\eta \end{bmatrix}
\qquad
\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M_{Z0}^{\delta a} / I_Z \\ 0 \\ 0 \end{bmatrix}
\qquad
\vec{\mathbf{U}} = [\Delta\delta_a]
$$

$$
A[1,1] = \frac{P_0^V \cos(\alpha_0) - Q_0^V}{m_0}, \quad A[1,2] = \frac{P_0 \sin(\alpha_0) - Q_0^{\alpha}}{m_0} - g
$$

$$
A[1,3] = \frac{-P_0 \sin(\alpha_0) - Q_0^{\alpha}}{m_0}, \quad A[2,1] = \frac{P_0^V \sin(\alpha_0) + Y_0^V}{m_0 V_0}
$$

$$
A[2,2] = \frac{-P_0 \cos(\alpha_0) - Y_0^V}{m_0 V_0}, \quad A[2,3] = \frac{P_0 \cos(\alpha_0) + Y_0^V}{m_0 V_0}
$$

$$
A[3,4] = 1, \quad A[4,1] = \frac{M_{Z0}^V}{I_Z}, \quad A[4,2] = -\frac{M_{Z0}^{\alpha}}{I_Z}, \quad A[4,3] = \frac{M_{Z0}^{\alpha}}{I_Z}
$$

$$
A[5,1] = 1, \quad A[6,2] = V_0
$$

Lateral aircraft dynamics

$$
\vec{\mathbf{X}} = \begin{bmatrix} \Delta\Psi_V \\ \Delta\omega_x \\ \Delta\omega_y \\ \Delta\zeta \\ \Delta\Psi \\ \Delta\gamma \end{bmatrix}
\qquad
\mathbf{B} = \begin{bmatrix} 0 & 0 \\ M_{x0}^{\delta e} / I_x & 0 \\ 0 & M_{y0}^{\delta d} / I_y \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\qquad
\vec{\mathbf{U}} = \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_d \end{bmatrix}
$$

$$
A[1,1] = -\frac{P_0 - Q_0 - Z_0^{\beta}}{m_0 V_0}, \quad A[1,5] = \frac{P_0 - Q_0 - Z_0^{\beta}}{m_0 V_0},
$$

$$
A[1,6] = -\frac{P_0 \sin(\alpha_0) + Y_0}{m_0 V_0}, \quad A[2,1] = -\frac{M_{x0}^{\beta}}{I_x}, \quad A[2,2] = \frac{M_{x0}^{\omega x}}{I_x}
$$

$$A[2,3]=\frac{M_{x0}^{\omega y}}{I_x},\ A[2,5]=\frac{M_{x0}^{\beta}}{I_x},\ A[3,1]=-\frac{M_{y0}^{\beta}}{I_x},\ A[3,2]=\frac{M_{y0}^{\omega x}}{I_x}$$

$$A[3,3]=\frac{M_{y0}^{\omega y}}{I_x},\ A[3,5]=\frac{M_{y0}^{\beta}}{I_x},\ A[4,1]=-V_0,\ A[5,3]=1$$

$$A[6,2]=1$$

Simulation results

The simulation has been employed to demonstrate the ability to stabilize the motion parameters of flight vehicle in longitudinal and lateral slow and fast motions with the developed algorithms.

The control loop performance was tested through the simulation which represented the fully vehicle kinematic and dynamics using standard computer procedures based on several different integration methods.

Longitudinal aircraft dynamics

The values of coefficients for equations (1) are shown in Table 1, The Q matrix is:

$$\mathbf{Q}=\begin{bmatrix}0&0&0&0&0&0\\0&20&0&0&0&0\\0&0&20&0&0&0\\0&0&0&0.02&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0.01\end{bmatrix}$$

Table 1

V_0	45	m/s	M_{Z0}^{α}	−5.4	$kg\cdot m^2/s^2$
m_0	12.3	$kg\cdot s^2/m$	$M_{Z0}^{\omega z}$	−3.59	$kg\cdot m^2/s$
Q_0^{α}	94.2	$kg\cdot m/s^2$	$M_{Z0}^{\delta a}$	−65.2	$kg\cdot m^2/s^2$
Q_0^{V}	0.53	kg/s	M_{Z0}^{V}	−0.03	$kg\cdot m/s$
Y_0^{α}	1190	$kg\cdot m/s^2$	P_0^{V}	−0.26	kg/s
Y_0^{V}	7.1	kg/s			

The C matrix was calculated using Repin-Tretjakov method as:

$$\mathbf{C}^T=\begin{bmatrix}0.03156 & 2.90573 & 5.67612 & 1.04141 & 0 & 0.09991\end{bmatrix}$$

The disturbing factor which is reflected in pitch angle variations in fast and slow motions and the system reaction on the disturbing factor for the case of using of the developed algorithm and without it (the control surface is fixed) is depicted in Fig 1.

Lateral aircraft dynamics

The Q matrix is:

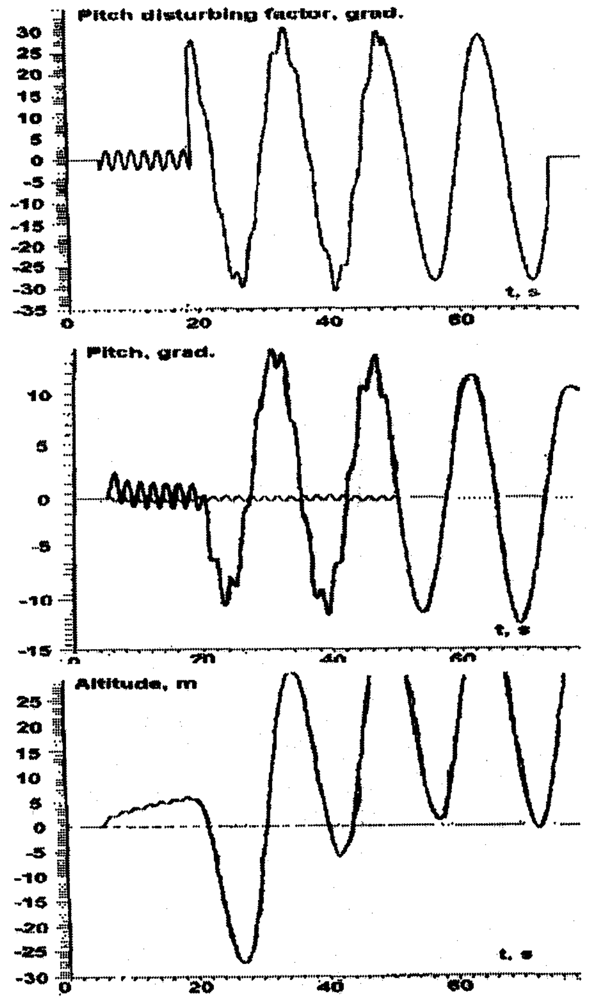

Fig. 1. Longitudinal aircraft dynamics.

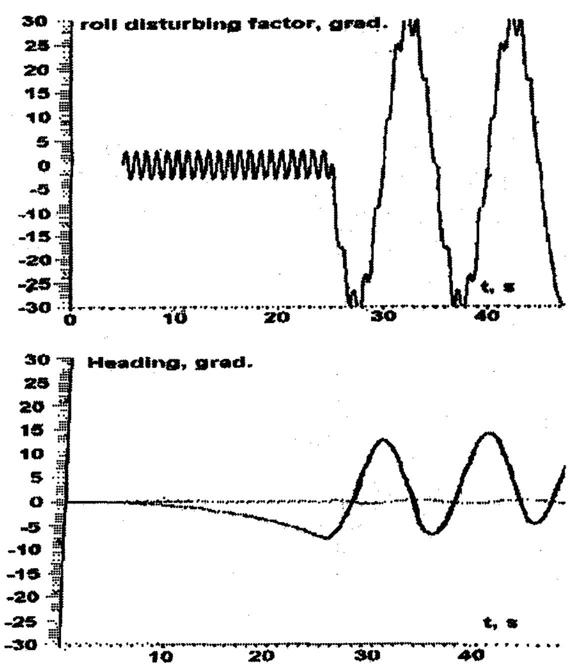

Fig. 2. Lateral aircraft dynamics.

$$
\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

The C matrix was calculated using Repin-Tretjakov method as:

$$
\mathbf{C} = \begin{bmatrix} -6.409 & 2.523 \\ 0.0739 & -0.0077 \\ -0{,}1152 & 0.3211 \\ 0.0955 & -0.0296 \\ -0.1727 & 0.4822 \\ 1.2173 & -0.0706 \end{bmatrix}
$$

The disturbing factor which is reflected in roll variations in fast and slow motions and the system reaction on the disturbing factor for the case of using of the developed algorithm and without it (the control surfaces were fixed) is depicted in Fig 2.

The eleron surface reaction for the case of using the developed algorithm is shown in Fig. 6.

It can be observed that the low frequency part of the disturbing factor, which reflects the slow motion of the vehicle, is decreased almost at all by the control system. The high frequency part of the disturbing factor, which is near the aerodynamic bandwidth of the aircraft, is also rather suppressed. However, there is no unstabilities in the control loop and transitions are relatively quick and almost without any oscillations.

The gains of the control algorithms can be regulated, without loosing an optimality, by choosing the elements of matrix Q. For example, the increasing of diagonal elements could result in high dynamic characteristics of the control system but the values of the control variables are restricted by the technology and the control law could become not optimal. It is also could be useful to less those gains corresponding the elements of the state vector which could be measured with difficulties or could not be measured at all.

The simulation results depicted above are only illustrative and simply strengthen the principal ability to create a control system using optimal regulator construction methods.

Simulation of the developed algorithms was made using personal computer IBM PC/AT 386 DX in the programming language TurboPascal 6.0 of Borland International. The software developed could be used as an instrument for analyzing the control quality of the algorithms of a given class.

Conclusions

The above described method was used to create a control algorithm for the attitude and trajectory stabilizations of the remotely-piloted vehicles in longitudinal and lateral fast and slow motions. The control laws developed combine the regulation of the fast motion as well as the slow motion at once and it is characteristic of this approach.

The method could also be used to construct a dynamic system control loops, which are not in the field of the professional interests of a designer, and as a cause the information on that loops is poor but it is necessary to provide the simulation and analyze of the whole system so as the model of the unknown loops could be rather adequate.

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