OPTIMIZATION OF BRANCHED TRAJECTORIES FOR AEROSPACE TRANSPORT SYSTEMS

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Abstract

The optimization of branched ascent trajectories is considered as applied to two cases: a discrete branch set and a continual one. Different types of limitations are taken into account including global ones, relating to several branches simultaneously, and those on extrema of the function of state variables on the continuum of branches.

The problems of the first type with global limitations arose, as a rule, in the consideration of requirements for safe staging, of aerospace transport systems (ATS). Those of the second type are characteristic of, e.g., the optimization of fail-safe injection trajectories from every point of which a recoverable vehicle can be returned.

A regular computer procedure for numerical solutions of the problems using the maximum principle is developed. Some examples of computations for different ATS classes are given.

Introduction

By branched trajectories are understood trajectories with a variable dimensiality of the state vector. In practice, the branched processes occur in the case when the objects recombine, i.e., divide into several components or, vice versa, group together forming a united object or, in the general case, k^j objects divide (group) into k^{j+1} objects at certain time moments $\{t^j, j = 1,\ldots,m\}$ with maintaining, as a rule, their quantity between the branch points. Methods of optimization of such systems are considered, e.g., in works[1-3]. Each object can move according to its own control program (objects may also be uncontrollable), its motion can be described by different equations, and the state vectors can differ both quantitatively and qualitatively.

One such example is a branched injection trajectory for an aerospace transport system (ATS) when it is necessary to account for limitations on reentry stages. In this case, the motion of distinct ATS components at different flight phases can be analyzed using diverse mathematical models. The limitations on control and trajectory may be global and relate to several branches simultaneously. For example, the consideration of requirements for safe staging restricts their relative position in the vicinity of the time moment when the stage linkages break down.

In more complicated problems, a set of the objects under consideration can form the continuum at some motion phases. For example, in article[4] the problem is formulated for fail-safe ATS injection trajectories from each point of which a recoverable vehicle (RV) can be returned to the Earth. In this problem, imaginary RV trajectories form a continuous sheet shedding from the ATS injection trajectory. Then $\Delta t_j = t_{j+1} - t_j \to 0$, and the condition of conservation of object quantity on the branch degenerates. In the general case, the control optimization in these problems requires the application of techniques used to optimize systems with distributed parameters[5].

The present report considers problems of both types. General optimality conditions based on indirect techniques and some examples of numerical solutions of several practical problems of optimal ascent of ATS with horizontal and vertical launches are given. The computations were performed using a modified computer program package ASTER[6] intended for automated (without user's intervention) solution of optimization problems based on the maximum principle.

2. Optimization of Trajectories With Discrete Branch Set.

2.1. General problem statement

Consider a branched trajectory. Branch points can be both on the stem and on the offshoots which in turn can also branch out. Number sequentially all branch points. Fictitious points can be introduced, if necessary. Combine all branch points with the same order number, that are reached, in the general case, at different time moments, in one node (Fig.1).

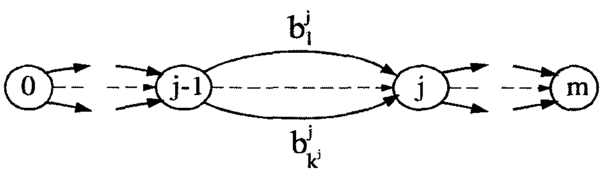

Figure1. *A scheme of a trajectory with a discrete branch set.*

Make use of the following notations: b_i^j is the i-th branch between the (j-1)-th and j-th branch nodes,

$$b^j = \{b_i^j,\ 1 \le i \le k^j\},\quad b = \{b^j,\ 1 \le j \le m\}.$$

Accordingly, x_i^j is the n_i^j-dimensional state vector on b_i^j; u_i^j is the p_i^j-dimensional control vector on b_i^j; $t_i^j \in [\tau_i^j, T_i^j]$ is the independent variable on b_i^j;

$$x_i^j(\tau) = x_i^j(\tau_i^j),\ x_i^j(T) = x_i^j(T_i^j),$$
$$x^j = \{x_i^j,\ 1 \le i \le k^j\},\ u^j = \{u_i^j,\ 1 \le i \le k^j\},$$
$$t^j = \{t_i^j,\ 1 \le i \le k^j\},\ x = \{x^j,\ 1 \le j \le m\},$$
$$u = \{u^j,\ 1 \le j \le m\},\ t = \{t^j,\ 1 \le j \le m\}.$$

Let the motion of the body system under discussion be imposed by the following conditions:

1) condition of the type of equality at branch points

$$Q(x(\tau), x(T), \tau, T) = 0,\quad Q = \{Q_i,\ 1 \le i \le q\}; \tag{1}$$

2) condition of the type of inequality on control of separate branches or group of them

$$W(x,u,t) \le 0,\ W = \{W_i,\ 1 \le i \le r\}; \tag{2}$$

3) differential or functional couplings

$$\left\{\varepsilon\frac{dx}{dt} - F(x,u,t) = 0,\ t \in [\tau, T]\right\}_i^j, \tag{3}$$

for branches b_i^j, where ε_i^j is the diagonal matrix $n_i^j \times n_i^j$ whose elements assume values from set $\{0,1\}$.

Formulate now a problem of choosing a control $u_{opt}(t)$ that, in view of couplings (1) to (3), gives minimum to the functional

$$f(x(\tau), x(T), \tau, T) \Rightarrow \min_{\{u\}}. \tag{4}$$

Note 1. Conditions (1), (2) make it possible to encompass cases of state limitations of the s-th order

$$[\Omega(x,t)]_i^j \le 0.$$

To do this, the point at which limitation $[\Omega(x,t)]_i^j \equiv 0$ is in force should be taken as the j-th branch point with imposed limitations of the type of (1)

$$[\Omega(x(T),T)]_i^j = 0, \ldots, \left[\frac{d^{s-1}\Omega(x(T),T)}{dt^{s-1}}\right]_i^j = 0,$$
$$x_i^j(T) - x_{i'}^{j+1}(\tau) = 0,\ T_i^j - \tau_{i'}^j = 0,$$

and limitations of the type of (2)

$$\frac{d^s\Omega}{dt^s} = \Omega^{(s)}(x,u,t)_{i'}^{j+1} \le 0$$

imposed on control $u_{i'}^{j+1}$.

Note 2. m nodes are applied only for convenience of geometric representation of a trajectory. To derive the optimality conditions and especially to construct a universal computational algorithm it suffices to have two nodes at which all left and all right ends of branches are grouped together.

1.2. Optimality conditions.

Consider an extended functional

$$\mathcal{L} = f + \nu^T Q + \sum_{\{i,j\}} \left\{ \int_{\tau}^{T} \left[\psi^T (F - \varepsilon \dot{x}) + \lambda^T W \right] dt \right\}_i^j , \quad (5)$$

where ν, ψ, λ are the Lagrangian multipliers, $[\ ]^T$ means transposition. It suffices to take account of term $\lambda_i W_i$ only in one of those integrals in (5) where it is essential; $\lambda_i = 0$, if $W_i < 0$.

The analysis of variations of extended functional (5) made in the same manner as in[7] yields equations for conjugate vectors ψ_i^j

$$\left[\varepsilon \frac{d\psi}{dt} + \left(\frac{\partial F}{\partial x} \right)^T \psi + \left(\frac{\partial W}{\partial x} \right)^T \lambda \right]_i^j = 0, \quad (6)$$

specific transversality conditions at branch points and control optimality conditions

$$\left. \begin{aligned} & \left[\varepsilon \psi(\tau) + \left(\frac{\partial Q}{\partial x(\tau)} \right)^T \nu + \frac{\partial f}{\partial x(\tau)} \right]_i^j = 0, \\ & \left[\varepsilon \psi(T) - \left(\frac{\partial Q}{\partial x(T)} \right)^T \nu - \frac{\partial f}{\partial x(T)} \right]_i^j = 0, \\ & \left[\mathcal{H}(\tau) - \left(\frac{\partial Q}{\partial \tau} \right)^T \nu - \frac{\partial f}{\partial \tau} \right]_i^j = 0, \\ & \left[\mathcal{H}(T) + \left(\frac{\partial Q}{\partial T} \right)^T \nu + \frac{\partial f}{\partial T} \right]_i^j = 0, \end{aligned} \right\} \quad (7)$$

where $\mathcal{H}_i^j = \psi_i^{jT} F_i^j$,

and control optimality conditions

$$\left\{ u_{opt} = arg \min_{\{u\}} (\mathcal{H} + \lambda^T W) \right\}_i^j . \quad (8)$$

In the general case, relations (6) to (8) reduce the initial problem to a closed boundary-value problem for $2n_\Sigma$ equations (3), (6), where $n_\Sigma = \sum_{i,j} n_i^j$, with $q + 2n_\Sigma$ boundary conditions (1), (7) containing q unknown Lagrangian multipliers (ν).

The boundary-value problem is solved numerically using program package ASTER[6] based on a modified Newton method of solving boundary-value problem and the method of uninterrupted parameter continuation of the solution which eliminates the need to choose a "good" initial approximation.

1.3. Comprehensive optimization of ATS ascent trajectory with accounting for requirements for safe staging.

As an example, consider the problem of branched ascent trajectory optimization for an aerospace transport system of the type of MAKS[8,9] or An-225/Interim Hotol[10]. The ATS consists of a subsonic aircraft-carrier (AC) from which an orbiter (OR) starts to inject payload into a given orbit.

Fig. 2 shows a schematic of the ascent trajectory: 0- initiation of AC/OR prestart maneuver; 1 - time moment of AC/OR linkage breakage; 2- termination of AC/OR separation; 3 - attainment of a given orbit by OR.

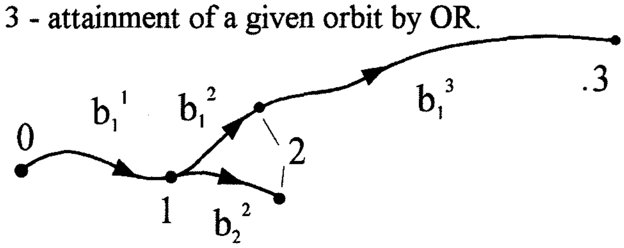

Figure 2. *ATS ascent trajectory including AC/OR separation section.*

Branch b_1^1 corresponds to a joint AC/OR flight, branches b_1^2, b_1^3 to an autonomous OR flight, and b_2^3 to an autonomous AC flight.

The separation trajectory section is of a comparatively small extent. Nevertheless, it has an appreciable influence not only on injected payload but also on the configuration of ATS as a whole. The basic causes of the influence are as follows:

1) requirements for safe separation impose strict limitations on AC and OR motions;
2) injection trajectory is very sensitive to variations in flight regimes at the initial section;
3) payload is governed by a difference of great closely-spaced numbers, therefore, small errors

of optimal trajectory simulation can change qualitatively real dependences of payload on parameters being studied (e.g., on orbiter engine firing time).

The comprehensive ascent trajectory optimization with simultaneous consideration of all flight phases in view of a common functional (injected payload) enables one to obtain an objective estimation of the functional and optimal control program, as well as the influence of different parameters on them. The last point renders the application of strict optimization methods and complete motion models indispensable starting with initial designing stages. The more so as the experience of servicing program packages based on strict methods shows that they reduce considerably scope and time of computing compared to approximate techniques since a more effective purposeful search for optimal solution is used (it can be automated) and the results obtained are more informative.

Assumptions. The ATS motion is described by full equations[6] accounting for three-dimensional nature of motion (for section b_1^3), central position of field of gravity, aerodynamic forces, and dependences of thrust delivered by AC and OR on flight regimes.

Because of small length, section b_1^1 of prestart AC/OR maneuver and sections $b_{1,2}^2$ of AC/OR separation will be considered in a vertical plane neglecting mass decrease due to operation of AC air-breathing engines.

Consider the ATS motion in the coordinate system fixed to the launch point. Introduce the following notations: $\vec{r}$ is the radius-vector from the start point to the vehicle center-of-mass, $\vec{v}$ is the speed; m is the vehicle mass, $\bar{x} = \{\vec{r}, \vec{v}\}$, $x = \{\bar{x}, m\}$. Because of assumptions used, the dimensiality of state vectors is 5 in sections b_1^1 and $b_{1,2}^2$, and 7 in section b_1^3. Control u is a unit vector $\vec{e}_\tau$ directed along the longitudinal ATS axis. Thrust vector $\vec{T}$ has a fixed application point and directed to the vehicle mass center.

General limitations. Each branch has own limitations on control and state vector in terms of:

- angle of attack α, $\cos\alpha = (\vec{e}_\tau, \vec{e}_v)$, $\vec{e}_v = \vec{v}/v$,

$$\left[\alpha_{min}(x) \le \alpha \le \alpha_{max}(x)\right]_i^j, \qquad (9)$$

- normal and longitudinal accelerations

$$\left[a_{z\,min} \le a_z \le a_{z\,max}\right]_i^j, \left[a_{x\,min} \le a_x \le a_{x\,max}\right]_i^j; \qquad (10)$$

- dynamic pressure $q = \frac{\rho v^2}{2}$, where ρ is the atmosphere density

$$\left[q \le q^{adm}\right]_i^j; \qquad (11)$$

- Mach number (for b_1^1 and b_2^2)

$$\left[M \le M^{adm}\right]_i^j. \qquad (12)$$

Limitations on safe separation. The requirements for safe AC/OR separation involve the conditions of impactless relative motions of AC and OR, and maintenance of admissible levels of thermal, aerodynamic and acoustic effects of orbiter engine (provided they are fired before or during separation) on the AC characteristics.

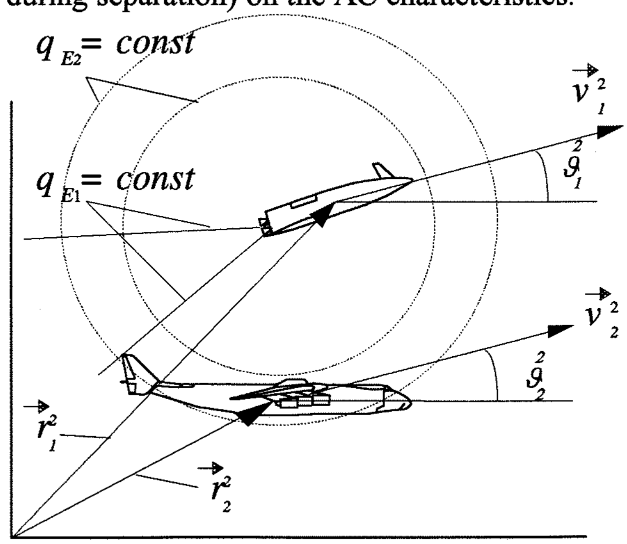

Figure 3. *The scheme of AC/OR separation with fired OR engine.*

It is assumed that the distributions of heat fluxes q_{E1}, aerodynamic q_{E2} and acoustic q_{E3} disturbances caused by orbiter engines do not depend on the AC position (Fig. 3): $q_E = q_E(\vec{r}\,')$, where $\vec{r}\,'$ is the radius-vector in the OR-fixed

coordinate system.
Let

$\vec{r}' = \vec{l}_1(p_1),\ \vec{r}'' = \vec{l}_2(p_2),\ p_1 \in \mathcal{P}_1,\ p_2 \in \mathcal{P}_2,$

be parametric equations of OR and AC loops in the coordinate systems fixed to these vehicles, $q_E^{adm} = q_E^{adm}(p_2)$ be the known distribution of admissible loads from orbiter engines along the AC loop.

Then the limitation on admissible relative positions of OR and AC is governed by the conditions

$$W_1 \equiv q_E\left(A(-\vartheta_1^2)(\vec{r}_* - \vec{r}_1^2)\right) - q_E^{adm}(p_{2*}) \le 0, \qquad (13)$$

where p_{2*}:

$$\frac{\partial W_1(p_{2*})}{\partial p_2} \equiv \frac{\partial q_E}{\partial \vec{r}'} A(\vartheta_2^2 - \vartheta_1^2)\frac{d\vec{l}_2(p_{2*})}{dp_2} - \frac{dq_E^{adm}(p_{2*})}{dp_2} = 0, \quad (14)$$

$$\vec{r}_* = \vec{r}_2^2 + A(\vartheta_2^2)\vec{l}_2(p_{2*}),\ A(\vartheta) = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix},$$

$$\vec{e}_{\tau i}^{\,j} = (\cos\vartheta_i^j, \sin\vartheta_i^j),\ j = 1,2.$$

If critical point $\vec{l}_2(p_{2*})$ on AC is fixed, contact condition (14) is not imposed.

Maximum OR pitch angle ϑ_1^2 derived from the condition of impactless OR separation from AC depends on tangency conditions for OR and AC surface contours

$$\vec{r}_1^2 - \vec{r}_2^2 + A(\vartheta_1^2)\vec{l}_1(p_{1*}) - A(\vartheta_2^2)\vec{l}_2(p_{2*}) = 0, \quad (15)$$

$$\left(\frac{d\vec{l}_2(p_{2*})}{dp_2}\right)^T A\left(\frac{\pi}{2} + \vartheta_1^2 - \vartheta_2^2\right)\frac{d\vec{l}_1(p_{1*})}{dp_1} = 0. \quad (16)$$

If critical point $\vec{l}_2(p_{2*})$ (or $\vec{l}_1(p_{1*})$) is fixed, tangency condition (16) is not imposed.

Note. Relations (15), (16) can also be used to determine maximum OR pitch angle followed from the conditions of OR engine effect. In this case, $\vec{l}_1(p_1)$ should be understood as a level line

$$q_E(\vec{l}_1(p_1)) - q_E^{adm}(p_{2*}) = 0,$$

or an external contour combining the orbiter contour and the level line.

The specific character of limitations (13) to (16) is that they relate controls $u_{1,2}^2$ and state vectors $x_{1,2}^2$ on both branches b_1^2 and b_2^2 simultaneously.

Boundary conditions. Initial state vector $\bar{x}_1^1(\tau)$ belongs to boundary Γ of possible regimes of a stationary level flight of the AC/OR system:

$$\bar{x}_1^1(\tau) \in \Gamma.$$

Assume that Γ is the piecewise smooth curve represented parametrically

$$\Gamma = \left\{\bar{x}_1^1 = \vec{L}(p),\ p \in \bigcup_i \mathcal{P}_i\right\},$$

where $\mathcal{P}_i$ is the smoothness sections Γ, then

$$Q^1 \equiv \bar{x}_1^1(\tau) - \vec{L}(p) = 0,\ Q^1 = \{Q_{1-4}\}. \qquad (17)$$

Initial AC/OR mass is specified:

$$Q_5 \equiv m_1^1(\tau) - m_{AC/OR} = 0. \qquad (18)$$

At branch point 1 vector $\bar{x}$ remains to be continuous

$$\left.\begin{aligned} Q^2 &\equiv \bar{x}_1^1(T) - \bar{x}_1^2(\tau) = 0,\ Q^2 = \{Q_{6-8}\}, \\ Q^3 &\equiv \bar{x}_1^1(T) - \bar{x}_2^2(\tau) = 0,\ Q^3 = \{Q_{9-13}\}, \end{aligned}\right\} \quad (19)$$

and the mass conserves

$$\begin{aligned} Q_{14} &\equiv m_1^1(T) - m_1^2(\tau) - m_2^2(\tau) = 0, \\ Q_{15} &\equiv m_2^2(\tau) - m_{AC} = 0, \end{aligned} \quad (20)$$

Branch point 2 corresponds to the termination of the separation, i.e., to the time moment when limitations (13) to (16) on the OR orientation cease to be essential. Therefore, at point 2 the OR mass conserves

$$Q_{16} \equiv m_1^2(T) - m_1^3(\tau) = 0, \qquad (21)$$

and remaining state variables are scaled uninterruptedly from the two-dimensional coordinate system to a three-dimensional one, e.g.,. as follows:

$$\left.\begin{aligned} Q^4 &\equiv (\vec{r}_1^2(T), 0)^T - \vec{r}_1^3(\tau) = 0,\ Q^4 = \{Q_{17-19}\}, \\ Q^5 &\equiv (\vec{v}_1^2(T), 0)^T - \vec{v}_1^3(\tau) = 0,\ Q^5 = \{Q_{20-22}\}. \end{aligned}\right\} \quad (22)$$

AC state vector $x_2^2(T)$ may be free provided flight limitations (e.g., on Mach number) are not violated in a subsequent section of the AC transition to the cruising return flight.

At final point 3, conditions[6] $Q_{23-26}(\bar{x}_1^3(T))$ of passing perigee of a given elliptic orbit are specified.

Functional. It is necessary to evaluate control $\vec{e}_{\tau opt}(t)$ and ATS flight regime $\bar{x}_1^1(\tau, p_{opt})$ at the beginning of the prestart maneuver to provide minimum OR fuel mass for the injection section:

$$f \equiv m_1^1(\tau) - m_2^2(\tau) - m_1^3(\tau) \Rightarrow \min_{\{u\}} \qquad (23)$$

provided limitations (13) to (22) are satisfied.

Transversality conditions. According to (6)-(8), the specific character of branched trajectories, as compared, e.g., to[6], consists, basically, in transversality conditions for branch points 1 and 2.

Denote conjugate vector, corresponding to $x = (\vec{r}, \vec{v}, m)$, by $\psi = (\vec{P}, \vec{S}, P_m)$ as done in[6], $\overline{\psi} = (\vec{P}, \vec{S})$. Using (7) for conditions (17) to (23) and excluding, where possible, Lagrangian multipliers ν yield the following simple transversality conditions

at point 1:

$$\left.\begin{aligned} &\overline{\psi}_1^1 = \overline{\psi}_1^2 + \overline{\psi}_2^2,\ P_{m1}^1 = P_{m1}^2,\ P_{m2}^2 = 0, \\ &\mathcal{H}_1^1 = \mathcal{H}_1^2 + \mathcal{H}_1^3, \end{aligned}\right\} \qquad (24)$$

at point 2:

$$\left.\begin{aligned} &\vec{P}_1^3 = (\vec{P}_1^2, \nu_1)^T,\ \vec{S}_1^3 = (\vec{S}_1^2, \nu_2)^T, \\ &P_{m1}^2 = P_{m1}^3,\ \mathcal{H}_1^2 = \mathcal{H}_1^3, \\ &\psi_2^3 = 0, \end{aligned}\right\} \qquad (25)$$

where ν_1, ν_2 are the free parameters.

As it is obvious from (25), the maximum principle condition[11] for nontriviality of conjugate vector may be not satisfied on several branches. In this case, $\psi_2^3(T) = 0$, since the subsequent AC motion does not influence the functional.

Example of numerical solution. The boundary-value problem was solved numerically using a modified program package ASTER[6]. By virtue of a small staging section length it is convenient to divide the general boundary-value problem into an external problem and an internal problem. The external problem is related to the boundary conditions for branches b_1^1 and b_1^3, and the internal one for $b_{1,2}^2$. The internal boundary-value problem is solved in the calculations of each trajectory during the iterative process of the external cycle. In this case, taking account of branched nature of the ascent trajectory increases slightly (by 1-3%) the problem solution time.

The above technique and program package ASTER for optimization of branched ascent trajectories were applied to extensive parametric investigations of the aerospace transport system MAKS[8,9] designed at the scientific production association "Molnia" and of the Soviet-British project An-225/Interim Hotol[10].

As an example, Fig.4 shows optimal ascent trajectory (initial part of branch b_1^1) and control program for one of the configuration versions of the Soviet-British ATS project (the calculations are performed by O.Yanova).

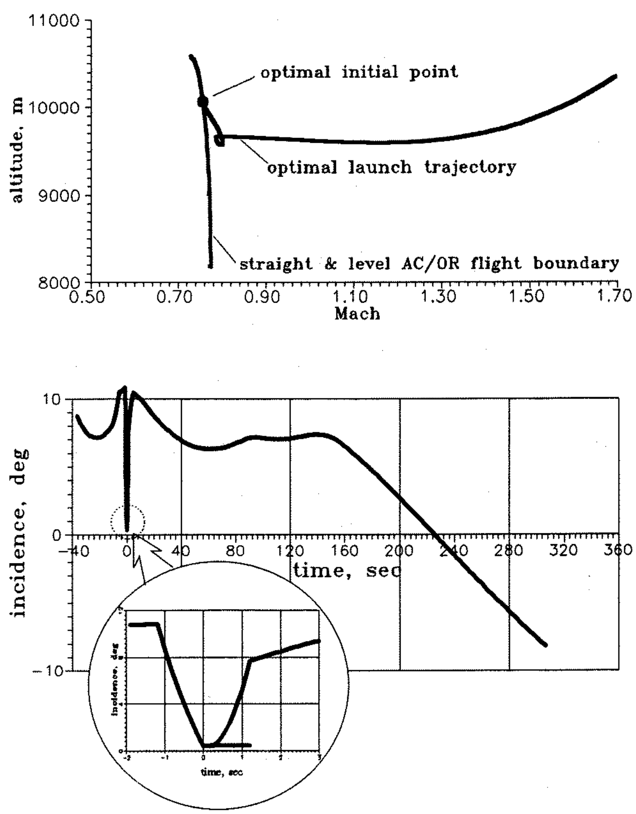

Figure 4. *Optimal ascent trajectory (initial part of branch b_1^1) and optimal incidence program for ATS of the type of An-225/Interim Hotol.*

The effective procedure of the automated search applied in ASTER to the boundary-value problem solution by using the parameter solution continuation made it possible to analyze the influence of main configuration parameters and OR engine cyclogram on the functional and optimal flight conditions with minimum computation time. The mean iteration number (for the external boundary-value problem) in these computations did not exceed 2 to 3, relative accuracy of meeting the boundary conditions being of about $\sim 10^{-6}$.

3. Optimization of injection trajectories with branch continuum.

3.1. Problem statement.

Consider trajectory b^1 of the ATS center-of-mass described by the equation of the type of (3)

$$\frac{dx^1}{dt^1} - F^1\left(x^1, u^1, t^1\right) = 0,\ t^1 \in \left[\tau^1, T^1\right]. \quad (26)$$

For definiteness, assume initial state vector $x^1(\tau)$ is fixed and the conditions[6] of motion along a given orbit must be satisfied at the right end $t^1 = T^1$.

In the "standard" problem of the injection trajectory optimization (see, e.g.,[6]), optimal control $u_{opt}(t)$ must minimize the functional of the form (4)

$$f \equiv f\left(x^1(\tau), x^1(T), \tau^1, T^1\right) \Rightarrow \min_{\{u\}} \quad (27)$$

with taking account of possible limitations of the form (2) on control u and may be limitations of the type (1) on the state vector at internal trajectory points. In[6] and in examples considered below, the injection fuel mass

$$f \equiv m^1(\tau) - m^1(T) \Rightarrow min \quad (28)$$

is taken as a functional.

In addition to the "standard" formulation, let require that the injection trajectory satisfy the fail-safe requirements. By fail-safe injection trajectory is understood here the injection trajectory at any point of which it is possible to interrupt the ATS flight program, to separate the recoverable vehicle (RV) and return it to the Earth with given limitations being satisfied. In practice, both a special vehicle (a capsule, a detachable cabin, etc.) and an orbiter, a carrier or an aerospace transport system as a whole can serve as a RV.

Let the motion of the RV center-of-mass be described by a vector equation of the form (3)

$$\frac{dx^2}{dt^2} - F^2\left(x^2, u^2, t^2\right) = 0,\ t^2 \in \left[\tau^2, T^2\right], \quad (29)$$

at initial condition

$$x^2(\tau) - x^1(t) = 0. \quad (30)$$

As proved[12], during atmospheric reentry with initial speeds lesser than local circular speed v_{circ}, limitations on dynamic loads q (dynamic pressure, accelerations, hinge moments, etc.) gain in importance. In this respect, the RV reentry trajectories from the mean section of the ATS injection trajectory with speeds of $v/v_{circ} \le \frac{1}{\sqrt{2}}$ are critical[12] (the higher is the RV L/D-ratio, the lesser is the critical speed).

In the general case, the above limitation is written in the form of an inequality for time-dependent maximum of a function of the state vector

$$\Omega \equiv \max_{\{t^2\}} q\left(x^2\right) - q^{adm} \le 0 \quad (31)$$

(It can be shown that the limitations on aerodynamic heating and the condition of RV landing at a given Earth's region can also be reduced to the limitations of this kind).

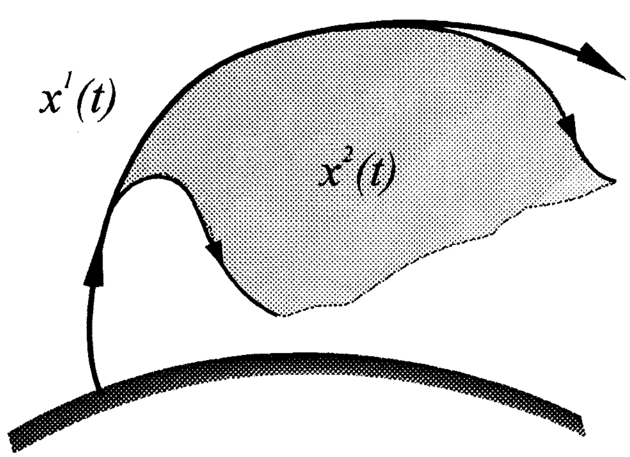

Figure 5. *Fail-safe injection trajectory with branch continuum.*

By virtue of (30), the RV trajectories depend on state vector $x^1(t)$ on the injection trajectory. Hence, if condition (31) cannot be met only due to RV control $u^2(t)$, inequalities (31) limit the admissible state vector $x^1(t)$ on the ATS injection trajectory.

Thus, we finally arrive at an optimization problem for branched trajectories (Fig.5) in which functional (28) is determined at the right end of the stem (injection trajectory), while state limitations (3) on the offshoots (imaginary RV rescue trajectories).

The peculiarity of the problem in question compared to that considered in Sec. 2 is that the offshoots form a continuous set (continuum). In the general case, its solution requires the application of optimization techniques for systems with distributed parameters(5).

3.2. Reduction to the standard Mayer problem.

The optimization problem for a branched trajectory with state limitations at internal points of the branch continuum is reduced to the standard Mayer statement by transmitting the above limitations to the branch point, i.e., to the stem, namely to the injection trajectory. To do this, it is necessary to know functions $\frac{\partial q_{max}}{\partial x^2(\tau)}$ of the influence of initial state vector $x^2(\tau)$ on $q_{max} = \min_{\{u^2(t)\}} \max_{\{t^2\}} q(x^2)$

Really, let $[\tau_q^1, T_q^1]$ be the time interval when limitation (31) is essential for the injection trajectory, i.e.:

$$\begin{aligned} &q_{max}\left(x^2(\tau) = x^1(t^1),\ t^1 \in [\tau_q^1, T_q^1]\right) - q^{adm} \equiv 0, \\ &q_{max}\left(x^2(\tau) = x^1(t^1),\ t^1 \notin [\tau_q^1, T_q^1]\right) - q^{adm} < 0, \end{aligned} \quad (32)$$

Denote this trajectory section by

$$\Gamma = \left\{x^1(t),\ t^1 \in [\tau_q^1, T_q^1]\right\}.$$

Then, from condition (32) for stationarity q_{max} on section Γ, motion equations (26) and boundary condition (30) it is inferred that

$$\begin{aligned} \left.\frac{dq_{max}}{dt^1}\right|_\Gamma &= \frac{\partial q_{max}}{\partial x^2(\tau)} \cdot \frac{dx^2(\tau)}{dt^1} = \left.\frac{\partial q_{max}}{\partial x^2(\tau)} \cdot \frac{dx^1}{dt^1}\right|_\Gamma = \\ &= \left.\frac{\partial q_{max}}{\partial x^2(\tau)}\right|_{x^2(\tau)\in\Gamma} \cdot \left.F^1(x^1, u^1, t^1)\right|_{x^1(\tau)\in\Gamma} = 0. \end{aligned} \quad (33)$$

From (33) it follows that state vector $x^1(t)$ and control $u^1(t)$ on section Γ must ensure the orthogonality of vector $F^1(x^1, u^1, t^1)$ and vector $\left(\frac{\partial q_{max}}{\partial x^2(\tau)}\right)^T$, i.e., the influence function:

$x^1(t) \in \Gamma$:

$$W(x^1, u^1, t^1) \equiv \left.\frac{\partial q_{max}}{\partial x^2(\tau)}\right|_{x^2(\tau)=x^1(t)} \cdot F^1(x^1, u^1, t^1) = 0. \quad (34)$$

If influence function $\frac{\partial q_{max}}{\partial x^2(\tau)}$ is known the initial problem with branch continuum is reduced to the standard Mayer problem for equation set (26) with functional (27) and limitation (34).

Consider conjugate vector $\psi^2(t)$ for equation set (29) with the following boundary condition

$$\psi^2(T) = \left(\frac{\partial q}{\partial x^2}\right)^T. \quad (35)$$

If the control is $u^2(t) = \arg \min_{\{u^2\}} \max_{\{t\}} q$ (or $u^2(t) \equiv const$) then, by virtue by invariance of the scalar production of the conjugate vector and state vector variation(13)

$$(\psi^2, \delta x^2)_{\tau^2} = (\psi^2, \delta x^2)_{T^2} = \delta q(T) = \delta q_{max}:$$

we have

$$\left(\frac{\partial q_{max}}{\partial x^2(\tau)}\right)^T = \psi^2(\tau). \quad (36)$$

Thus, the influence functions entering into limitation (34) coincide with the initial conjugate vector value in the solution of a separate problem of the RV reentry trajectory optimization according to the criterion

$$\max_{\{t^2\}} q \Rightarrow \min_{\{u^2\}}. \quad (37)$$

Optimal control $u^1_{opt}(t)$ is found from condition (8) where, by virtue of (34) and (36), we have

$$W(x^1,u^1,t^1) \equiv \left(\psi^2(x^1)\right)^T \cdot F^1(x^1,u^1,t^1). \quad (38)$$

3.3. Approximate synthesis of optimal control on fail-safe injection sections

The optimal ATS control on dangerous trajectory section Γ can be synthesized by using approximate analytical formulas for conjugate vector ψ^2 taken from[12].

Assume that section Γ lies in the region where the influence of aerodynamic forces is small as compared to gravity forces (in practice, this condition is always satisfied, see[12]). Then we have, according to[12]:

$$\left.\begin{aligned}&\psi_v \equiv \frac{\partial q_{max}}{\partial v} = A\frac{Bv^2-1}{v}; \quad \psi_\gamma \equiv \frac{\partial q_{max}}{\partial \gamma} = A\,tg\gamma;\\ &\psi_h \equiv \frac{\partial q_{max}}{\partial h} = A\frac{B-R}{R^2};\end{aligned}\right\} (39)$$

Here γ is the angle of inclination of the speed vector with respect to a local horizontal plane, v is the speed related to the first cosmic speed, R is the distance from the Earth's center to the vehicle center-of-mass related to the Earth's radius, h is the altitude related to the Earth's radius, A, B are the known[12] functions of $x^2(\tau)$. Limitation (38) in the case of (39) assume the form

$$W \equiv \left[\frac{Bv^2-1}{v}\cdot\frac{dv}{dt} + tg\gamma\cdot\frac{d\gamma}{dt} + \frac{B-R}{R^2}\cdot v\,sin\gamma\right]^1_\Gamma = 0. \quad (40)$$

If the ATS motion is considered in a vertical plane then condition (40) governs unambiguously ATS control $\vec{e}_\tau$ at boundary Γ in the form of the function of a current state vector

$$tg\,\varphi = \left(1 - Bv^2\right) ctg\,\gamma, \quad (41)$$

where φ is the angle between $\vec{e}_\tau$ and $\vec{v}$.

3.4. Numerical examples.

Numerical solutions of the optimization problems for fail-safe injection trajectories of ATS of the type of Energia-Buran were obtained for an approximate (40) and an exact form of representing limitations (34). Only the flight section of the second stage was optimized, while the injection trajectory of the first stage was fixed. The calculations were performed using package ASTER[6].

Fig.6 compares two optimal injection trajectories with and without consideration of the approximate fail-safety condition.

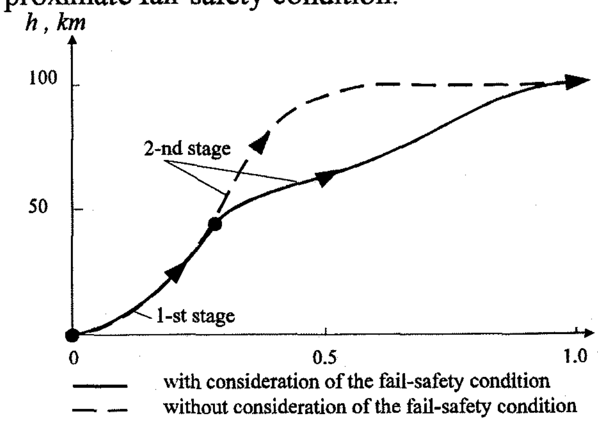

Figure 6. *Optimal ATS injection trajectories with and without consideration of the fail-safety condition (40).*

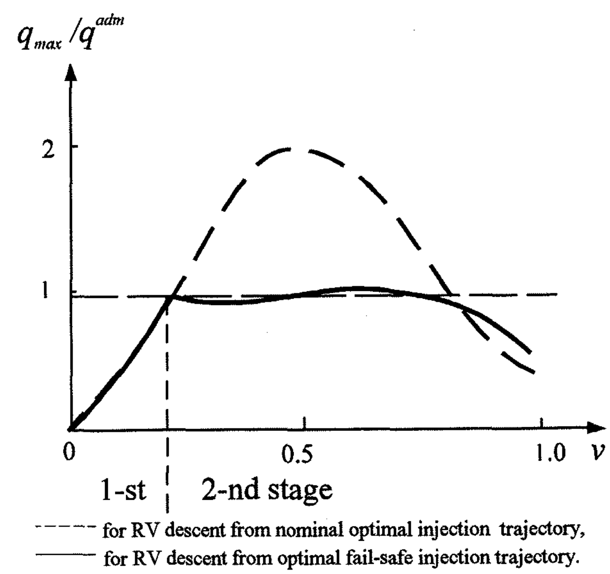

Figure 7. *Maximum dynamic pressure during RV reentry after emergency injection interruption at speed v related to admissible one.*

Fig.7 shows maximum dynamic pressures during the reentry of a hypothetical RV, having the characteristics close to the orbiter Buran, after emergency interrupting the injection at the time moment when relative velocity v $(v < 1)$ is attained. It is seen that approximate control law (41) enables one to maintain rather precisely fail-safety condition (31).

Real limitations (31) can be taken into account in a more rigorous manner without applying approximate solutions (39) by means of interpolating numerically values of $q_{max}(x^2(\tau))$ calculated for a set of discrete values of initial state vector $x^2(\tau)$. As shown(12), it suffices to determine dependence q_{max} only on two parameters: speed v_0 and altitude h_0 at apogee $(\gamma = 0)$ of the RV reentry trajectory: $q_{max}(v_0, h_0)$. Fig.8 illustrates the obtained form of limitations on parameters at apogee of the trajectory of a recoverable vehicle of the type of "Buran" because of limitations on the admissible dynamic pressure (boundary Γ_1) and hinge moments (boundary Γ_2). Fig.8 shows also the optimal fail-safe injection trajectory of the ATS second stage (flight program of the first stage was fixed: gravity turn).

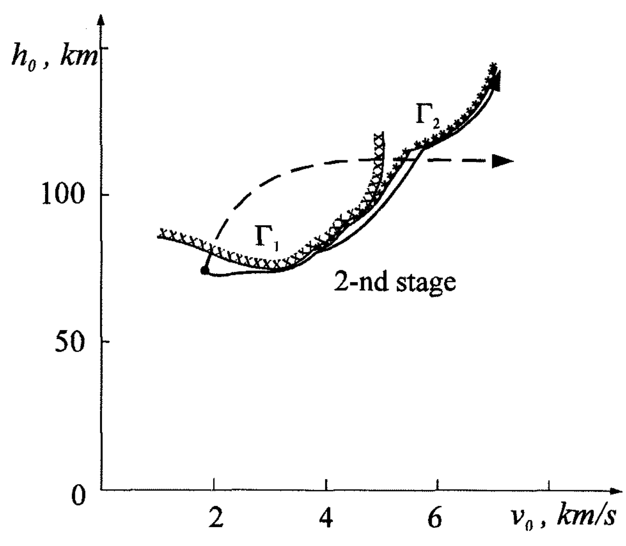

Figure 8. *Optimal fail-safe ATS injection trajectory.*

Γ_1, Γ_2 -boundaries of regions for initial admissible RV reentry conditions due to limitations on dynamic pressure and hinge moment.

As follows from the above examples, the duration of dangerous section Γ of the optimal injection trajectory is often more than 50% of the injection trajectory. The optimal trajectory modification using the technique suggested provides the fail-safety of the whole trajectory with a relatively small penalty of the mass injected.

Conclusion

The techniques and computer program package developed for the optimization of branched trajectories based on indirect methods of optimization make it possible to analyze efficiently the capabilities of advanced ATS by applying a comprehensive consideration of all flight phases according to the common criterion and to account for all required limitations including those on flight safety.

The application of strict optimization techniques enables one to estimate reliably not only payload mass and nominal parameters of the optimal trajectory and the control program but also the dependences of criteria under study on parameters. Therefore, the program package developed is a convenient tool not only at final stages of designing aerospace transport systems or for the definition of intellectual systems of optimal ATS guidance but also at initial design stages.

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