EVALUATION OF OPTIMAL SEGMENTS IN AERIAL COMBAT SIMULATIONS

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Abstract

Optimisation of manoeuvring in combat promises significant theoretical benefit. However, in order to determine if this benefit is real, the optimal rules must be assessed in a realistic scenario. Optimal trajectories have been defined for fighter aircraft engaged in Beyond Visual Range (BVR) combat with medium range missiles. This has been done for pre-launch and post-launch / evasive phases of the combat. The first problem is a 2-D energy management optimisation. The second is a more complex two-stage, 3-D manoeuvre. Control laws have been developed to match the optimal solutions. These have been adapted and implemented in a combat model, using realistic information. A single missile exchange between two combatants is analysed. The results from this model have been analysed to provide measures of effectiveness to allow assessment of the relative merits of different optimal strategies in a realistic scenario.

1. Introduction

Air to air combat tactics and manoeuvring can be very complex. Considerable analysis and experimentation can be required to derive suitable tactics to maximise the effectiveness of the aircraft and weapon system. Optimisation techniques can be applied to some elements of the problem. Such optimised trajectories can help to simplify the analysis of air combat engagements in combat models by reducing the number of tactical parameter variations. This allows analysis to be concentrated on the weapon system parameters and their effects.

Associated with this optimisation process is an information problem. There is little point in attempting to optimise the entire tactical process as one problem, since the future development of the fight is unknown. Feedback is required as the fight develops, and any optimal solution must adapt as new information is received. Following this approach allows us to consider limited segments. The opponent is not considered as a player, simplifying the analysis to a one-player game. If the opponent is considered as a player, then this would require a differential game analysis, which is complicated by the information problem.

Reference 1 identified segments of air combat tactics which could be optimised and applied in combat model simulations. The paper concentrated on the pre-launch attack phase. Evaluation was limited to largely qualitative consideration of the potential benefits of optimal manoeuvring. Significant potential gains were identified, but these were shown to be strongly dependent on the tactical logic of both combatants, in particular the launch decision. This paper describes the continuation of the work of optimising further segments of the combat, and seeks to quantify the benefits by way of exchange ratios and other measures of effectiveness. A different combat model and analysis approach is needed in order to isolate the effect of the optimisation from that of the firing decision.

The modelling technique used has some similarity with that described in reference 2. The study in reference 2 was constrained to two dimensions, and suggested the use of artificial intelligence techniques, combined with differential game methodology to determine optimal launch timing and evasion cues.

The methodology used for this study uses a simple combat simulation model, operating in three dimensions. It is based on a simple statistical assessment across a range of launch times or ranges for each side. This approach removes the need to optimise the launch decision process. For the purposes of comparing weapon systems and tactics, the results from this methodology have been shown to correlate well with other, more complex, combat simulations.

The purpose of the paper is to demonstrate potential improvements in combat effectiveness by the use of optimised tactics. In doing so, it is also demonstrates that it is possible to implement the optimal guidance in a practical way which might be applicable to provide on-board, real-time, tactical guidance to aircrew.

2. Models Used

Aircraft Model

The aircraft are modelled as three degree of freedom point masses for both the optimisation and combat modelling. The point mass model is :

$$\dot{x} = v \cos(\gamma) \cos(\chi), \quad x(t_0) = x_0 \tag{1}$$

$$\dot{y} = v \cos(\gamma) \sin(\chi), \quad y(t_0) = y_0 \tag{2}$$

$$\dot{z} = v \sin(\gamma), \quad z(t_0) = z_0 \tag{3}$$

$$\dot{\chi} = g_0 u_h / (v \cos(\gamma)), \quad \chi(t_0) = \chi_0 \tag{4}$$

$$\dot{\gamma} = g_0 (u_v - \cos(\gamma)) / v, \quad \gamma(t_0) = \gamma_0 \tag{5}$$

$$\dot{v} = (\text{Thrust}(z,v) - D(z,v,n)) / \text{mass} - g_0 \sin(\gamma), \quad v(t_0) = v_0 \tag{6}$$

$$\dot{\text{mass}} = - B(z,v), \quad \text{mass}(t_0) = \text{mass}_0 \tag{7}$$

where x and y are the horizontal co-ordinates, z is altitude, χ is heading relative to x axis, γ is the climb angle, v is the velocity, mass is the total mass of the aircraft and g_0 is gravitational acceleration. The load factor is divided into a vertical component (u_v) and a horizontal component (u_h). These are used as the controls for the aircraft. Thrust control has been ignored for this study, since the optimal setting has been found to be mostly maximum.

The aircraft model used is a generic supersonic fighter. The aircraft used in reference 1 were high performance fighters. In this case, a lower performance level has been chosen, as this is felt to provide more scope for optimisation. The aircraft is assumed to have a datum operational mass of 10000 kg. Figure 1 shows the 1g Specific Excess Power (SEP) for this aircraft.

The manoeuvres of the aircraft are constrained within realistic limits, as follows :

$$z \geq z_{min} \tag{8}$$

$$-30° \leq \gamma \leq 30° \tag{9}$$

$$q \leq q_{max} \tag{10}$$

$$n = \sqrt{(u_v^2 + u_h^2)} \leq n_{max} \tag{11}$$

where q is the dynamic pressure, and n is total load factor. The load factor limit, n_{max}, is the the lesser of the stall limit or structural limit.

In addition, the heading, χ, must be restrained during the post launch manoeuvre to allow continued command update of the missile for a time t_{track}.

$$|\chi| \leq f(\text{radar gimbal, geometry, } t_{track}) \tag{12}$$

Missile Model

The missile used in this study is a generic medium range missile. The kinematic performance of the missile is the same as that described in reference 1. The missile is assumed to require command guidance until 10 km range. At this point the missile is able to use its own autonomous radar seeker.

A simplified function representing missile speed, integrated for range, is used in the optimisation models. The function is of the form :

$$\text{Range} = R(\text{Alt}_L, \text{Mach}_L, \text{Alt}_T) \tag{13}$$

where Alt is the altitude, Mach is the Mach number. Subscript L represents the launcher state, subscript T represents the target state at the estimated impact point.

This function was derived from the results of a flyout model. This flyout model also forms an integral part of the combat model used for the evaluations.

Scenario

The scenario chosen for this study is a one versus one BVR missile exchange. The tactical sequence followed by the aircraft in this type of engagement is shown in figure 2.

The pre-launch phase is triggered by detection of the target. This normally involves a climb and acceleration on a collision course with the target.

A pre-fire pointing manoeuvre may be carried out a few seconds prior to launch. This involves pointing on a missile collision course to minimise missile manoeuvre during flight. There may also be benefit to be gained from lofting the missile. Control laws for lofting have not been developed as part of this study. In order to simplify the evaluation process, the elevation demanded for launch is $\gamma = 0°$.

Post launch manoeuvring involves a turn away from the target. This will initially be limited to the gimbal limits of the radar, as the aircraft must command guide the missile until it acquires the target. This turn is coupled with a dive. This takes the aircraft into denser air, reducing the effective range of any threat missile. Acceleration during the dive is used to generate a good speed for out-running the threat missile.

A final turn and dive to out-run the threat is cued by either :
- detection of threat missile seeker, or
- own missile autonomy.

The elements examined in this study are :
- Pre-launch and pre-fire elevation control,
- Post launch and evasion elevation and azimuth control.

3. Optimised Elements

The work in reference 1 concentrated on optimisation of the pre-launch phase. Further analysis of this phase is included in this study. Much of the work for this study has been concerned with the post launch and evasion manoeuvres.

The optimisation problems are formulated as a cost function of the form :

$$V(X(t_o) ; t_o) = F(X(t_f) ; t_f) \quad (14)$$

where X entirely describes the state as set out in equations (1-7) and V is the function to be minimised. The goal at time t_f is described modelled by F. Subscript 'o' describes the time at the beginning of the optimised segment and subscript 'f' the end of it. The Optimisation method used is a modified version of Differential Dynamic Programming. This method has been in use at Saab-Scania for many years and is adapted to aerospace problems [3].

Pre-Launch Optimisation

Four criteria are chosen for the optimal pre-launch manoeuvres, giving the following forms of equation (14) :

1. Maximise energy,

$$V(X(t_o);t_o) = -z(t_f) - \tfrac{1}{2}v^2(t_f)/g_o \quad (15)$$

2. Earliest launch,

$$V(X(t_o);t_o) = -x(t_f) - R(z(t_f),Mach_L,Alt_T) \quad (16)$$

where $Mach_L$ corresponds to $v(t_f)$ and Alt_T is an estimate of target altitude at impact.

3. Maximise launch range,

$$V(X(t_o);t_o) = -R(z(t_f),Mach_L,Alt_T) \quad (17)$$

4. Maximise launch range advantage.

$$V(X(t_o);t_o) = R(Alt_T, Mach_T, Alt_{eva}) - R(z(t_f),Mach_L,Alt_T) \quad (18)$$

where Alt_{eva} is own altitude following evasion. This is a function of $z(t_f)$ and $Mach_L$. The dominating term in equation (18) is found to be the second term. This equation therefore degenerates to be close to equation (17). This solution was therefore not considered for further analysis.

Functions 2, 3 and 4 were chosen based on experience from reference 1, and optimisation of profiles against passive targets.

Realistic constraints are applied to these profiles. Most notably, the final climb angle, γ, is restricted to 0°. This is achieved by adding a penalty term to the cost functions (15-18). Although the work in ref. 1 showed a considerable benefit for launch range from lofting the missile using high climb angles at launch, for the purposes of this study the additional variability introduced by lofting was removed in order to clarify the analysis of the optimal manoeuvres.

As in reference 1., 'Master Curves' have been generated to meet these criteria. These curves are defined as v(t), z(t) and γ(t). The term in γ is necessary to bring the aircraft smoothly to the desired trajectory. The master curves are shown in figure 3, along with the approximation used in the GAMBIT baseline tactics.

The master curves involving missile launch range show a characteristic zoom prior to launch. This is because increased altitude has a strong influence on the range of the missile.

Control laws have been developed to bring the aircraft smoothly onto the curve, matching closely optimal solutions from any state at $t=t_o$.

Additional control laws were developed to control the pre-launch zoom (where applicable) and return to 0° for launch.

The master curves have been developed for a straight flying aircraft. It is possible to make an allowance for manoeuvre. Figure 4 shows the effect of allowing for a constant horizontal manoeuvre component of 1 g or 2 g during a maximum energy climb.

Post Launch and Evasion

The trajectories for post launch and missile evasion were developed in three dimensions.

A simple, single master curve solution does not exist for this problem. The optimal trajectories are matched using control laws.

In order to simplify the problem for implementation, start points for the optimisation are chosen in the region of the flight envelope indicated by typical pre-launch trajectories.

The problem requires an initial turn away, restricted to the radar gimbal limit, as given by equation (12). After some time, t_{track}, the aircraft is threatened by an incoming missile. A further turn is made to attempt to out-run the missile. The final turn is made through an angle θ so as to place the threat directly on the tail. The goal is to fly as far away from the missile as possible, and also to exhaust it. This requires a high final velocity. The cost function for this manoeuvre is given by :

$$V(X(t_o);t_o) = -x(t_f)\cos(\theta) - y(t_f)\sin(\theta) + R(Alt_T, Mach_T, z(t_f)) \quad (19)$$

The stop condition is given by :

$$t_f = \arg_t\{v(t) = v_{missile}(t) - v_{hit}\} \quad (20)$$

where v_{hit} is the minimum closing speed for the missile.

The nature of the post launch and evasive manoeuvre is shown to depend on :
- the time before evasion, t_{track},
- the radar gimbal limit,
- the final evasive turn required, θ.

Figure 5 shows two very different geometries of the evasive turn, depending on variations in t_{track} and θ. Figure 6 shows the speed and altitude variations for these cases. In these figures, the times given refer to time from the start of the manoeuvre (own missile launch). Tlaunch is the time of the opponent's missile launch, Topen is the seeker switch-on time corresponding to t_{track} and Tend corresponds to t_f.

Given a very long evasion, such as that shown in figure 6, it can be seen that the aircraft requires a control law to limit speed as it approaches the q_{max} limit. This has been derived and incorporated in the modelling.

4. Combat Modelling

Optimal trajectories for pre-launch, post launch and evasive manoeuvring have been implemented in the BAe GAMBIT combat model.

GAMBIT

The GAMBIT combat model simulates a 1-v-1 BVR duel with one shot only per side. In order to simplify the engagement as much as possible, the launch time for each side is fixed. This removes the effect of tactical variation for this decision, allowing an assessment which is independent of launch range tactics.

By varying the launch timing on each side, a matrix of possible engagements is built up. Analysis of this matrix forms the basis of the evaluation used in this study.

Each possible engagement in the matrix has a number of possible outcomes.

- Blue dead, Red survives (Blue win)
- Red dead, Blue survives (Red win)
- Blue dead, Red dead (Mutual kill)
- Blue survives, Red survives (Null result)

There is one other result, which stops the engagement if the aircraft get too close. If the aircraft pass within 5 km, it is assumed that this would no longer be a BVR combat. Since GAMBIT has no close range tactics, the engagement is declared invalid.

The resulting matrix is limited by the following criteria :

- Maximum extent is set just before the 'aircraft too close' point, on the diagonal.
- Earliest fire times are selected as those which give a scoring result.

Figures 7 to 12 show typical matrices derived for this study. It may be possible to perform in-depth analysis of these matrices, to arrive at an optimal launch solution for each side. Unfortunately, this almost always yields a null result, which is of little help in assessing the effectiveness of tactics. This type of analysis is pursued further in reference 2. For the purposes of this study, overall evaluation measures are based on the relative sizes of 'blue' and 'red' zones.

This simple measure has been shown to correlate well with more complex modelling when comparing the effectiveness of different options.

There is a caveat on interpretation of the scores from this type of analysis. The score is not the result of definitive engagements, and relies strongly on the actions of the opponent. It should not be treated as an absolute result. It is preferred to consider the resulting score as indicative of potential performance, or a margin for tactical error.

Qualitative assessment is also possible by examining and comparing other matrix characteristics.

Implementation of Optimal Solutions

The control laws and master curves developed to match the optimal trajectories have been implemented into the GAMBIT combat model with no difficulty.

In order to integrate the control laws, assumptions regarding the controlling parameters must be made. This is because the optimisation model effectively assumes perfect information regarding the key points in the engagement, such as launch time, t_{track}, evasion heading etc. Obviously, in a realistic scenario, this information will not be known.

In GAMBIT, the launch time is pre-set, so this value is available to the control laws for precisely controlling the zoom (where necessary), and the push down to $\gamma = 0°$ for launch. In other modelling applications, an estimate of launch timing may be adequate. Experience suggests that this would be the case, provided the estimate converges smoothly to the actual launch time.

For the post launch manoeuvring, the degree of the turns and the value of t_{track} will depend on the development of the engagement. In order to implement this, the following assumptions are made :

- The degree of initial turn to radar gimbal limit is calculated dynamically, depending on current relative geometry.
- The degree of the final turn for evasion is calculated dynamically, based on threat missile position. In the absence of information on the threat missile (no launch or not yet active), the target aircraft position is used.
- In place of a predicted value of t_{track}, the parameter used is actual time spent in the manoeuvre plus some 'time to go'. This 'time to go' could be used to control the urgency of the manoeuvre, although a single value has been found to be acceptable for the purposes of this study.

Evaluation

Identical aircraft and missiles are used on each side for the evaluation. Both players are assumed to start at equal energy states, subsonic at medium altitude. The only difference between the players is the tactics used.

Base-line tactics used for the evaluation are standard GAMBIT tactics. These consist of the following :

Pre-launch :
An approximation to an 'optimal' energy climb using a constant Mach number climb for the subsonic element, a level acceleration to transition to the supersonic climb, and a supersonic climb at constant equivalent air speed (constant q). The values used are chosen by analysis of SEP, shown in figure 1. Azimuth control is simple collision course, as described in section 2.

Post-launch :
A simple constant dive angle is used for the entire post launch and evasion manoeuvre. A value of -25° has been chosen as an approximation to a typical mean value generated by the optimal methods. Further analysis has demonstrated very little gain from decreasing this angle to -35°, more than that used in the optimal solutions. Azimuth steering demands are as described in section 2.

The following options have been evaluated against the baseline tactics :

- Optimal energy gain, optimal evasion,
- Optimal energy gain, GAMBIT evasion,
- Optimal early shot, optimal evasion,
- Optimal early shot, GAMBIT evasion,
- Optimal long range shot, optimal evasion,
- Optimal long range shot, GAMBIT evasion.

The resulting matrices are shown in figures 7 to 12.

Relative effectiveness is shown in figure 13 in terms of an 'exchange ratio'. This is simply defined as (own hits / opposition hits), including mutual hits.

Analysis

The effectiveness results shown in figure 13 show a significant benefit to be gained from the use of optimal tactics. This can lead to an improvement in exchange ratio from 1 to 1.3 in the best case. When interpreting these results, the reader is referred to the caveat in the section describing the GAMBIT modelling methodology.

However, it can also be seen that when the optimal pre-launch tactics are coupled with the simple GAMBIT evasion, these gains disappear.

The results show that, of the pre-launch options considered here, the energy criteria is the best for this scenario. The other optimal options may pay off only in scenarios where the opponent is not expected to return fire, or there is already strong dominance due to other factors.

The optimal pre-launch profiles which include missile range in the criteria show considerably poorer scores. In the case of the 'Early Launch' option, the profile favours speed over altitude, leading to rapid closure on the threat. This has the consequence of allowing the threat early counterfire opportunities. This is seen by the increased GAMBIT kill zone for early launch times in figure 10, compared to those in figures 8 and 9. However, emphasis on altitude as in the 'Maximum Launch Range' option, although having a strong pay-off for missile range, places the aircraft in a poorer position for subsequent evasion.

The results for optimal energy option and the coarse GAMBIT approximation show very little difference. This may indicate that, provided there is not excessive closure (speed emphasis) or climb (missile range emphasis), the profile is relatively insensitive. The energy option, or something very close to it, provides a good compromise between offensive and defensive requirements.

It is the optimal post-launch and evasion which pays off most in improving effectiveness. Analysis of the matrices in figures 7 - 12 show that gains in effectiveness are made both by negating opposition hit opportunities and adding own hit opportunities. Both these effects occur at early fire times for the optimal aircraft. The additional hits gained are due to a secondary effect of the more effective evasion. The more effective turn forces the opposition aircraft to follow a more direct track, flying into the missile in flight. This effect is strongest for early own launches coupled with late opposition launches. All this has the effect of increasing the firing window in which the aircraft can launch with a possible effective shot, but without risk from counter-fire. This is important, as it allows the aircraft to take the initiative early in the engagement.

Further analysis has been carried out using a lower initial energy for both sides. The same characteristics are present in the results from these cases, increasing confidence in the robustness of the analysis.

5. Conclusions and Discussion

The study has demonstrated the feasibility of implementing optimal controls in a realistic combat scenario. The implementation of the 2-D pre-launch energy management was also shown in reference 1. More significantly, the more complex two stage, three dimensional problem of the post launch and evasive manoeuvre has been optimised and successfully implemented in the scenario.

Evaluation using combat modelling has shown that energy gain is a good criteria for the pre-launch phase. It has also been shown that this profile is relatively insensitive to coarse approximation. This profile is the best compromise between maximising the offensive performance of the weapon system and subsequent defensive performance against counter-fire. Other profiles emphasise the offensive performance, and may have value for use against non-aggressive opponents. A more detailed cost function may yield further improvement. Such a function should include factors for :

- Closure with target,
- Missile range,
- Energy state for post-launch and evasion,
- Early or first shot.

Considerable potential benefit is shown for managing the post-launch and evasive manoeuvres optimally.

The work described in this study has looked at optimising one cycle of an engagement. In order to close the cycle, further work can be considered on the re-engagement manoeuvre. This will be a 3-D turn to re-establish an optimal pre-launch profile. Whereas it may be possible to compromise pre-launch performance for evasive performance, as shown in this paper, it is unlikely that there will be any compromise in evasive performance for re-engagement potential. In order to re-engage, survival must be guaranteed. An optimal evasion will also allow the earliest possible re-engagement.

Continuing with the 1 versus 1 scenario, it is possible to formulate the problem as a differential game. In reference 4, such a duel problem was solved with perfect information. A differential game will need to be formulated using realistic combat and information processes in order to obtain practical, usable solutions. This optimisation process should be integrated with the evaluation methodology to provide realistic assessment of weapon systems and tactics. Reference 5 moves towards this ideal integrated methodology, although confined to combat within visual range, without the information problems associated with BVR combat.

6. References

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5) Lynch, U., "Optimal Tactics for Agile Aircraft Technology - A Differential Game Methodology", Eidetics International Limited, AGARD FMP Symposium, October 1993, Maryland, USA.

7. Figures

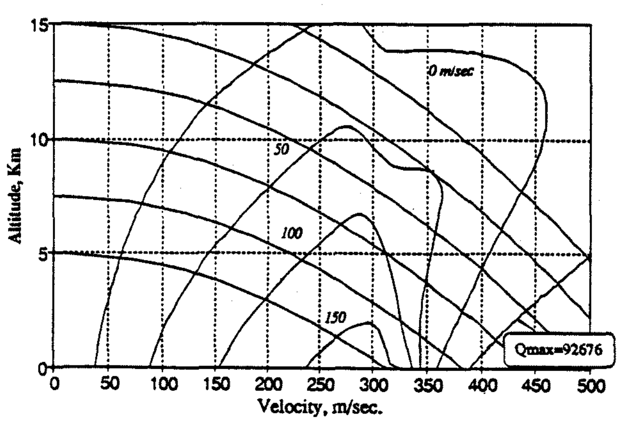

Figure 1. **Generic Aircraft Model, Specific Excess Power**

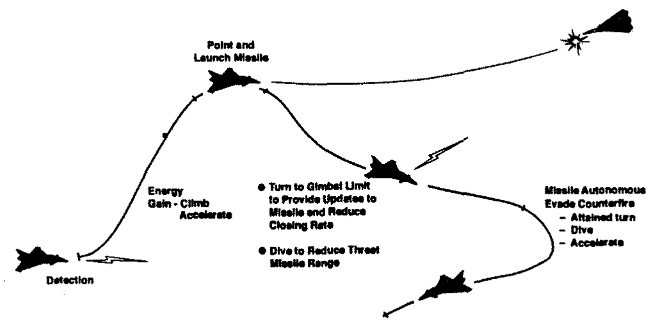

Figure 2. **Tactical Elements in BVR Combat.**

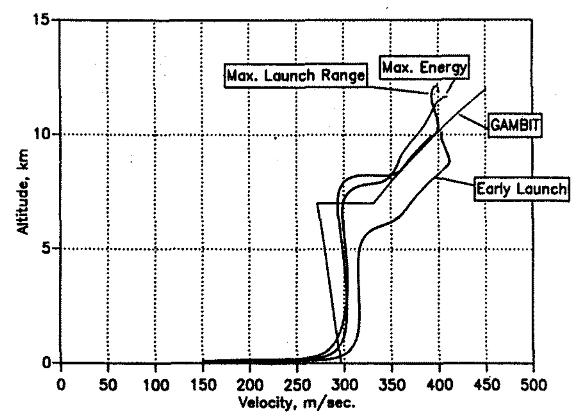

Figure 3. **Pre-Launch Master Curves**

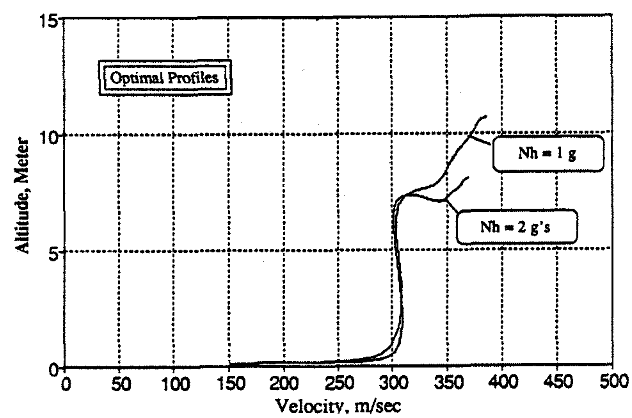

Figure 4. **Effect of Horizontal Manoeuvre** Component, 1g and 2 g

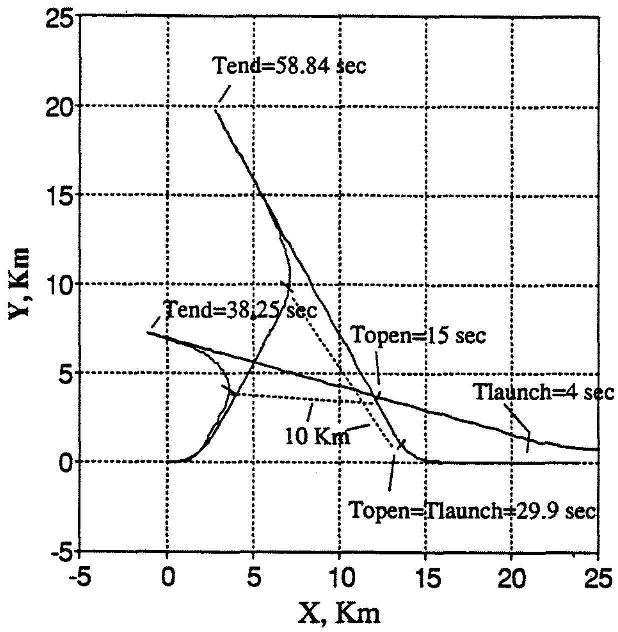

Figure 5. **Post-Launch, Evasive Trajectories, θ = 120°, 165°**

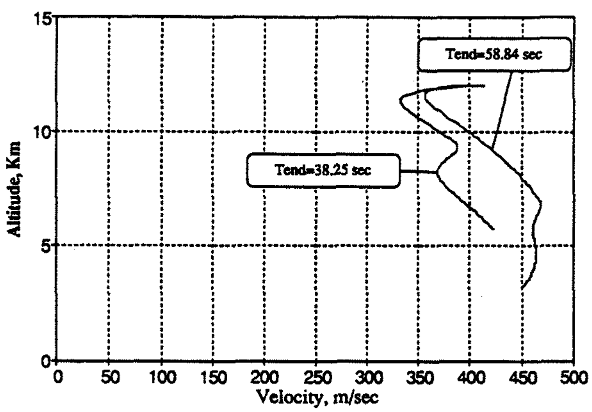

Figure 6. **Speed & Altitude Trajectories for Post Launch, Evasive Manoeuvre**

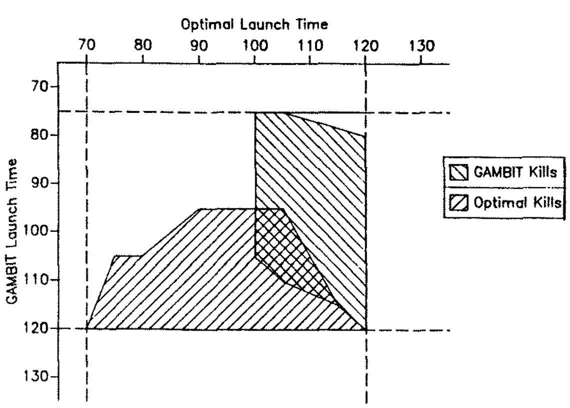

Figure 7. **Launch Time Matrix - GAMBIT vs. Optimal Energy, Optimal Evasion**

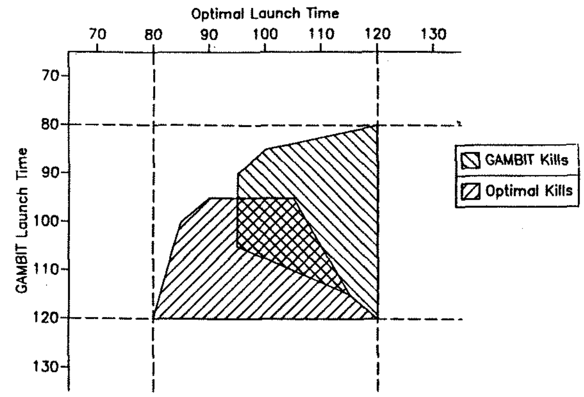

Figure 8. **Launch Time Matrix - GAMBIT vs. Optimal Energy, GAMBIT Evasion**

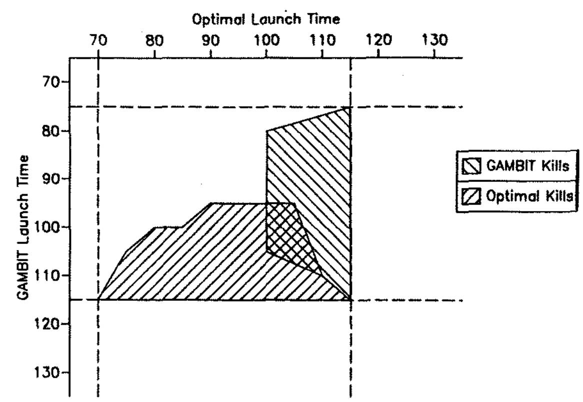

Figure 9. **Launch Time Matrix - GAMBIT vs. Optimal Early Launch, Optimal Evasion**

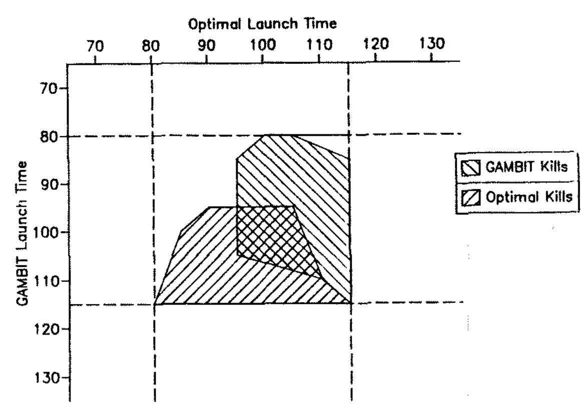

Figure 10. **Launch Time Matrix - GAMBIT vs. Optimal Early Launch, GAMBIT Evasion**

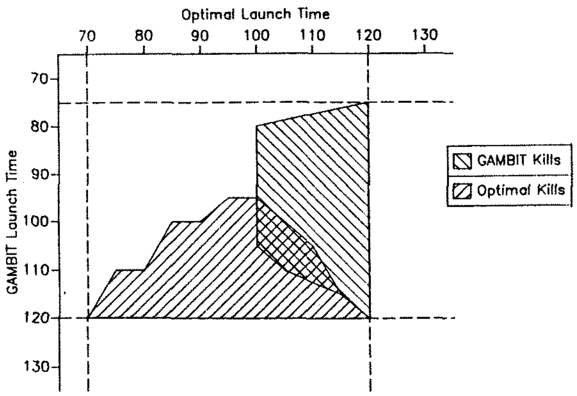

Figure 11. **Launch Time Matrix - GAMBIT vs. Optimal Long Range Launch, Optimal Evasion**

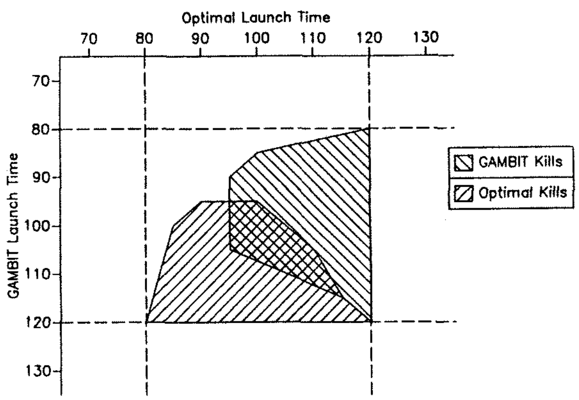

Figure 12. **Launch Time Matrix - GAMBIT vs. Optimal Long Range Launch, GAMBIT Evasion**

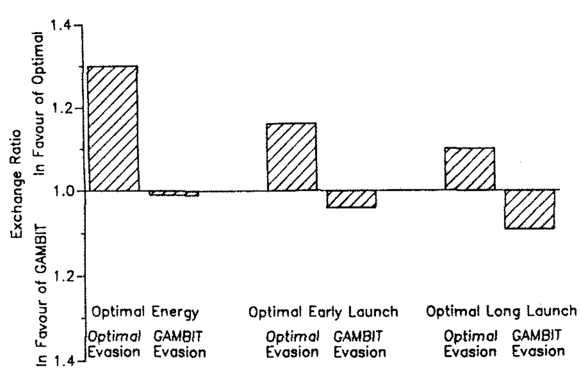

Figure 13. **Relative Effectiveness, Exchange Ratio**