SIMPLENO — A NEW COMPUTATIONAL PROCEDURE FOR SUBSONIC, TRANSONIC AND SUPERSONIC FLOWS

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by

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Abstract

The present paper introduces a new methodology for computation of all speed flows in complex geometry. The proposed method follows the semi-implicit pressure correction concept but incorporates for the first time in this framework the so called ENO schemes, approximate Riemann solvers and characteristic interpolation practices. The method also uses the deferred correction technique for the stabilization of the iterative procedure and the Strongly Implicit Method for matrix inversion of the segregate discretized equations. Simulation of flows in different regimes: incompressible, compressible subsonic, transonic, and supersonic proved the method SIMPLENO to be robust, accurate and self adaptive to local flow characteristics.

Introduction

In recent years, considerable efforts have been made towards the unification of numerical methods developed for incompressible and compressible flows(1-4). The main aim consists in the development of methods for computation of viscous flows at all Mach numbers that are as good as the current compressible flows solvers that employ density as a primary variable and those for incompressible flows based in a pressure-velocity formulation. Yet to fully attain these goals some puzzle in the formalism of these methods have to be settled, e.g. how to account for the spreading of acoustic waves and the Riemann problems in the framework of the pressure-correction ? These problems translate themselves into insufficient shock capturing properties and have so far prevented its wider use in applications.

In the present work a first positive answer to these questions is provided by means of a new method — called SIMPLENO for SIMPLE-Essentially Non-Oscillatory. This method incorporates for the first time in a pressure ,correction framework the ENO technique for the reconstruction of the characteristic variables, together with the use of a Riemann solver for computation of the fluxes at the interfaces. The main steps for its derivation are presented in turn bellow (for the sake of clarity the Cartesian coordinates are considered)

Governing Equations

The continuum steady flow at thermodynamic and chemical equilibrium is considered. The flow is described by the Conservation Laws of mass, momentum (linear and angular), and energy. The resulting system of equations are a summarized in the Navier-Stokes equations, which in the integral form reads,

$$\oint_A (M-R)dy - \oint_A (N-S)dx = 0 \qquad (1)$$

Herein is,

$$M=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ \rho uH\end{pmatrix}\quad N=\begin{pmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ \rho vH\end{pmatrix}$$
$$R=\begin{pmatrix}0\\ \tau_{xx}\\ \tau_{xy}\\ q_x+u\tau_{xx}+v\tau_{yx}\end{pmatrix}\quad S=\begin{pmatrix}0\\ \tau_{yx}\\ \tau_{yy}\\ q_y+u\tau_{xy}+v\tau_{yy}\end{pmatrix} \qquad (2)$$

where the stress terms and the heat flux are expressed by Stokes and Newtons assumption, and the Fourrier Law, respectively, i.e.,

$$\begin{aligned}\tau_{xx} &= 2\mu\partial_x u - \frac{2}{3}\mu\nabla.\mathrm{v}\\ \tau_{xy} &= \tau_{yx} = \mu\left(\partial_y u + \partial_x v\right)\\ \tau_{yy} &= 2\mu\partial_y v - \frac{2}{3}\mu\nabla.\mathrm{v}\end{aligned} \qquad (3)$$

$$\begin{aligned}q_x &= k\partial_x T\\ q_y &= k\partial_y T\end{aligned} \qquad (4)$$

Discretization Procedure

The finite-volume method is employed for the numerical approximation of the governing equations. In this method the surface integrals in (1) are approximated by their average values over all faces of the control volume. The discrete form in 2D reads,

$$\begin{aligned}&\left((M+R)_{i+1/2}-(M+R)_{i-1/2}\right)\Delta y\\ &+\left((N+S)_{j+1/2}-(N+S)_{j-1/2}\right)\Delta x = 0\end{aligned} \qquad (5)$$

Evaluation of the fluxes at the interfaces are performed selectively: i) the viscous terms, R and S, are evaluated by the 2nd order centered-difference scheme, ii) the inviscid terms, M and N, are computed by an upwind technique.

For the latter two (M and N) distinct procedures are employed depending on the local Mach number:

a) if Ma < 0.3

Then the mass flux is obtained by PWIM formula of Rhie and Chow[5], and reconstruction operates over the primitive variables ρ, u, v, h. Their interface value is obtained by upwind with convective velocity, e.g. for i+1/2,

$$\phi_{i+1/2} = F^+_{i+1/2}\phi^+_{i+1/2} + F^-_{i+1/2}\phi^-_{i+1/2} \qquad (6)$$

where $\phi = (\rho, u, v, h)$; F gives the flow direction at the interface, i.e.,

$$F^{\pm}_{i+1/2} = \frac{(u)_{i+1/2} \pm \left|(u)_{i+1/2}\right|}{2(u)_{i+1/2}} \qquad (7)$$

$$\begin{aligned}\phi^-_{i+1/2} &= R\left(x^-_{i+1/2};\phi\right)\\ \phi^+_{i-1/2} &= R\left(x^+_{i-1/2};\phi\right)\end{aligned} \qquad (8)$$

where R is the reconstruction polynomial. On the other hand the value of the pressure at the interfaces are interpolated by the non-oscillatory polynomial H_2 [6] i.e.,

$$H_2(x;\phi) = \phi_i + \frac{s_{i+1/2}}{h}(x - x_i) + \frac{d_{i+1/2}}{2h^2}(x - x_i)(x - x_{i+1}), \quad \text{for } x \in (x_i, x_{i+1}) \tag{9}$$

and, $p_{i+1/2} = H_2(x_{i+1/2};p)$. This technique avoids creation of new extrema in the solution domain and in addition it is of third order where the pressure is smooth

b) if $Ma \geq 0.3$

In this case reconstruction operates over the characteristic variables and a Riemann solver is used for the evaluation of the interface flux, i.e.,

$$M_{i+1/2} = \frac{1}{2}(M_L + M_R) - \frac{1}{2}\sum_k \alpha_k |\lambda_k| e_k \tag{10}$$

where α, λ, and e are the characteristic variable strengths, and eigenvalue and the eigenvector of the linearized Jacobian $\tilde{A}$, respectively. The values of the left and right states, M_L and M_R, respectively, are evaluated by the reconstruction on the characteristics, i.e.,

$$R(x;\phi) = \phi_i + \frac{(x - x_i)}{h}\sum_k \sigma_{k,i}\tilde{e}_k, \quad \text{for } x \in (x_{i-1/2}, x_{i+1/2}) \tag{11}$$

where $\tilde{e}$ is computed with the variables averaged as proposed by Roe[7], and σ is defined as,

$$\begin{aligned} \sigma_i &= M(\sigma_i^+, \sigma_i^-) \\ \sigma_i^{\pm} &= \tilde{\alpha}_{i\pm 1/2} \mp \frac{\tau_{i\pm 1/2}}{2} \\ \tau_{i\pm 1/2} &= M(\tau_i, \tau_{i\pm 1}) \\ \tau_i &= \tilde{\alpha}_{i+1/2} - \tilde{\alpha}_{i-1/2} \end{aligned} \tag{12}$$

Similar expressions follow for the other cell faces (note that the index j of the second direction was omitted for simplicity).

Substitution of these fluxes into the discrete equations gives rise to algebraic equations for each dependent variable. These equations are highly non-linear and strongly coupled. Decoupling of these equations is achieved by the segregate approach, characteristic of the pressure-correction schemes, and retained in the SIMPLENO algorithm (see below). Linearization is performed by the method proposed by Orzag[8] in conjunction with implicit Spectral methods, i.e.,

$$L_{ap}\phi^{n+1} = L_{ap}\phi^n + \gamma\left(L\phi^n - b\right) \tag{13}$$

where the expression under parenthesis denotes the original higher order system of equations and the operator L_{ap} is a lower-order, robust approximation of L (in the present work the first order upwind scheme was employed).

SIMPLENO Algorithm

The SIMPLENO algorithm is a predictor-corrector scheme, in which at the end of each outer iteration the variable fields satisfy some discrete form of the governing equations. A brief description of these steps is presented bellow.

Predictor Step

In this step the velocity and enthalpy are calculated by using the available values of the dependent variables for the evaluation of the coefficient matrix. The predicted values, u^*, v^* and h^* are computed as follow,

$$
\begin{aligned}
u^*_{i,j} &= A^u\left(u^*_{i,j}\right) + Q^u_{i,j}p + S_u \\
v^*_{i,j} &= A^v\left(v^*_{i,j}\right) + Q^v_{i,j}p + S_v \\
h^*_{i,j} &= A^h\left(h^*_{i,j}\right) + S_h
\end{aligned}
\qquad (14)
$$

where, e.g. A^u is defined as,

$$
A^u\left(u^*_{i,j}\right) = \frac{\sum_{nb \in I} a^u_{nb} u_{nb}}{a^u_{i,j}} \qquad (15)
$$

in which, I is the index set relative to the neighboring points involved in the interpolation and a^u_m is the coefficient multiplying u_m; S stands for all other terms that are explicitly computed (source terms, known values, etc.) and e.g. $Q^u_{i,j}$ is defined by,

$$
Q^u_{i,j}p = \frac{\delta_i p}{a^u_{i,j}} \Delta y \qquad (16)
$$

In this equation, δ denotes the second order central differencing operator.

Similar definitions follows for the operators in Eq. (14).

Corrector Step

Usually, these values of the velocity and density do not satisfy the mass balance, i.e.,

$$
\begin{aligned}
S_m &= u^*_{i+1/2,j} - u^*_{i-1/2,j} + v^*_{i,j+1/2} - v^*_{i,j-1/2} \\
S_m &\neq 0
\end{aligned}
\qquad (17)
$$

Correction of these values is performed so that the corrected velocity and density fields satisfy the discrete mass balance equation and some linearized form of the momentum equation, in symbols:

$$
w^{n+1} = L_C\left(w^*\right) \qquad (18)
$$

where w = (ρ, u, v, h, p) and L_C the abstract corrector operator defined by:

$$
L_C = \begin{cases} SIMPLE, \quad if\ Ma < 0.3 \\ SIMPLE + \text{Projection of the corrections} \\ \text{on the vector fluxes space,} \quad if\ Ma \geq 0.3 \end{cases}
$$

The projection step is essential: it avoids a new computation of the approximate the Riemann solver within the same outer iteration for Mach greater or equal 0.3 (with the consequent excessive increase in the computational effort) while not affecting the accuracy of the converged solution (since at convergence all correction values are zero). In this procedure the inviscid fluxes are corrected, e.g. for $M^{n+1}_{i+1/2}$,

$$
M^{n+1}_{i+1/2} = M^*_{i+1/2} + \sum_k \alpha_k \lambda_k e_k \qquad (20)
$$

where the arithmetic mean is used between the corrected and the predicted values of the variables, and first order upwind (with the convective velocity) is employed for determination of the corrections at the interfaces. Similar expressions follow for the other fluxes.

Results

Arc Bump Flow

This is an inviscid flow test case with strong interactions of oblique shocks: shock-shock interaction and shock reflection. The configuration correspond to a wall mounted circular arc bump with an approaching Mach number of 1.65 (details can be found in Eidelman et al.[9]. Fig. 1 shows the grid used in the computations comprising 80x30 control volumes.

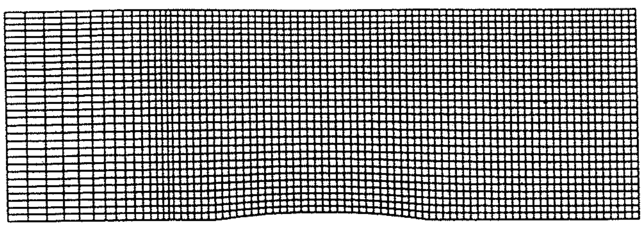

Figure 1

Fig. 2 depicts the results obtained using the standard SIMPLE method together with a third order interpolation for u, v and h and the first order upwind scheme for density. Apart form the interpolation of primitive or conservative variables this is the standard approach with pressure correction[1-4]. This figure clearly indicates the inadequacy of this methodology for solving shocks: the shock are too smeared or not resolved at all. This situation can be improved by using a second order non-oscillatory procedure for all variables, including density, as was done by the authors in Ref. 10. Fig. 3 reproduced from the later, shows the results obtained with the improved technique and the Minmod reconstruction. In this case it was possible to ameliorate the shock resolution. However, the use of a non-conservative discretization and of the convective velocity as upwind velocity still smear the shocks and creates spurious oscillations.

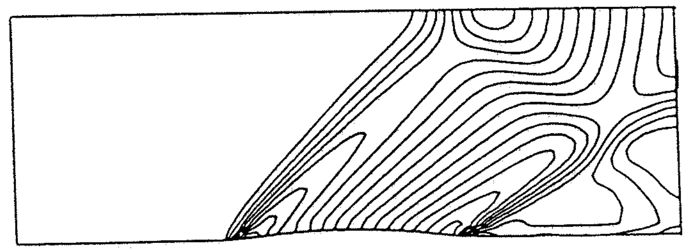

Figure 2

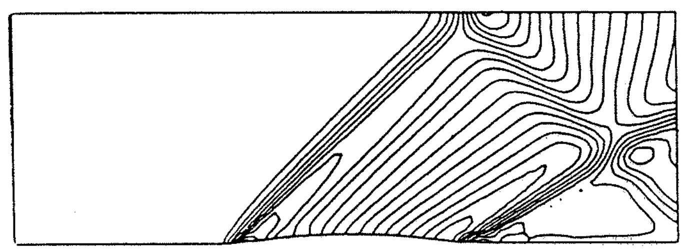

Figure 3

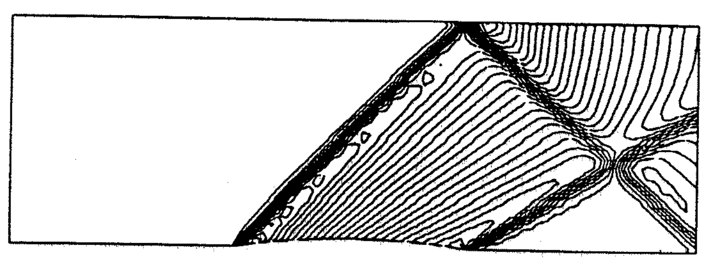

Figure 4

Fig. 4 shows the results obtained with the new methodology SIMPLENO (using the UNO2[6] reconstruction). As can be seen from this figure all shock structures are very well resolved: their representation is crispy and non-oscillatory The solution field has no spurious oscillations while the high accuracy of the ENO technique is evident. This figure let no doubt about the capability of the SIMPLENO method to resolve shock waves or their interactions.

Double Throat Nozzle

This flow problem has been the subject of a workshop[11] on numerical computation of compressible viscous flows. The workshop produced very accurate data that can serve as reference values.

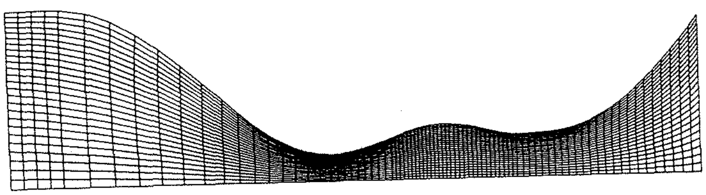

Figure 5

Fig. 5 shows the geometry and the grid comprising 80x20 control volumes used in the simulation. Fig. 6 depicts the results obtained with SIMPLENO (using UNO2[6] reconstruction). Again the new methodology predicts a very sharp shock profile and in general captures all flow features: shock wave, shear layer, viscous/inviscid interaction, separation. In particular it can provide an answer to the natural question on the effect of the flux limiters on the viscous layer, where strong, yet continuos gradients arise. This is provided in Fig. 7 which shows the comparison of the present results with reference values (the later was obtained in a much finer grid). It is evident from this figure that the flux limiters do not prevent the high accuracy of the method in viscous flows neither in the shock region nor in the shear layers. This quantitative comparison corroborates the qualitative findings above.

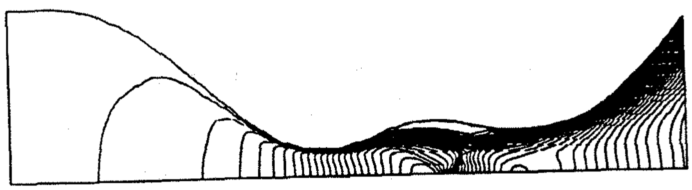

Figure 6

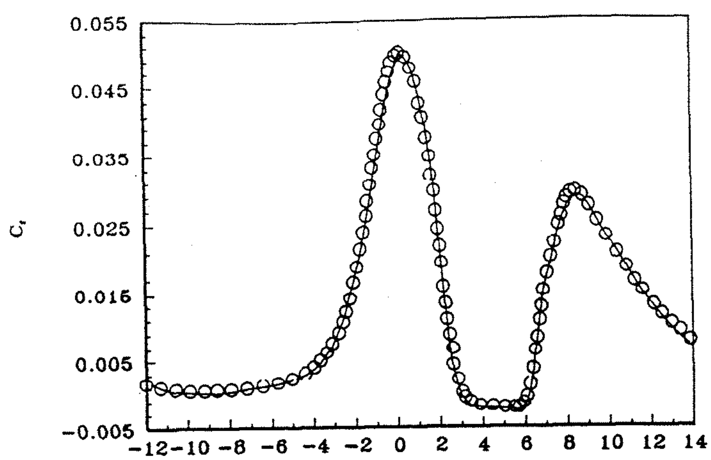

Figure 7

Lid Driven Cavity Flow

To complete the presentation of the results in this sub-section the standard test case of the incompressible flow induced by a moving wall is considered. A mammoth number of data concerning this flow exists. The reference values selected are those of Ghia et al.[12] who used very fine grids and a second order accurate scheme.

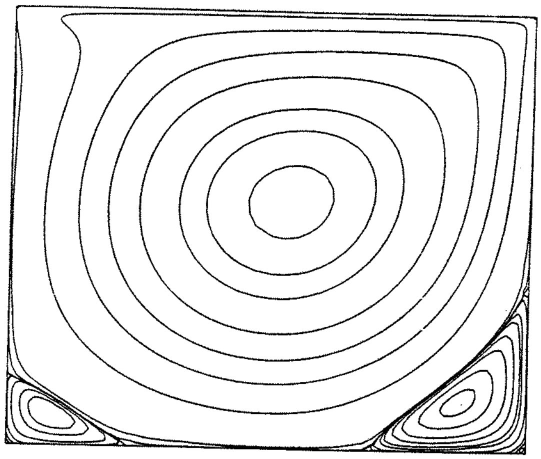

Figure 8

Fig. 8 depicts the streamlines obtained using SIMPLENO (using SONIC-Q[13] reconstruction) for Re=1000. The stretched grid used comprises 60x60 points, roughly a quarter of the points used by Ghia et al. Despite this fact the results display very good qualitative agreement with the reference values: good resolution of the separated zones, both in its location and its extension.

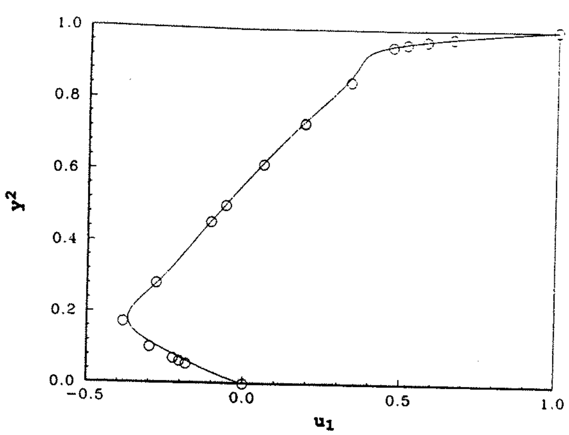

Figure 9

The quantitative comparison is provided in Fig. 9 where the present results are plotted together with the reference values of Ghia et al.. They only confirm the above assertive.

Conclusions

The SIMPLENO algorithm introduces an array of new features, the implementation of the characteristic interpolation through the approximation of the non-linear operator, the use of an approximate Riemann solver for the computation of the cell face fluxes and the projection of the correction on the fluxes space in the framework of the segregated pressure-correction algorithm. Simulation of flows under severe conditions (shock/shock interaction, shock reflection, shock/viscous layer interaction, etc) has proven the method to be as accurate as its counterparts which use density as a primary dependent variable in the hyperbolic regions of the flows while extending its range of applications to the limit of very low (zero) Mach number flows.

References

1 - ISSA, R.I. and LOCKWOOD, F.C. (1977) "On the Predictions of Two-Dimensional Supersonic Viscous Interactions Near Walls", *AIAA Journal*, Vol. 15, pp. 182-188.

2 - KARKI, K.C. and PATANKAR, S.V. (1989) "Pressure Based Calculation Procedure for Viscous Flows at All Speeds in Arbitrary Configurations", *AIAA Journal*, Vol. 27, pp. 1167-1174.

3 - MCGUIRK, J. and PAGE, G.J. (1990) "Shock Capturing Using a Pressure-Correction Method", *AIAA Journal*, Vol. 28, pp. 1751-1757.

4 - LIEN, F.S. and LESCHZINER, M.A. (1992) "Modeling Shock/Turbulent-Boundary-Layer Interaction with Second-Moment Closure within a Pressure-Velocity Strategy, *Proc. 13th Int. Conf. on Numerical Methods and Fluid Dynamics*, Rome 6-10 July, Italy.

5 - RHIE, C.M. and CHOW, W.L. (1983) "Numerical Study of the Turbulent Flow Past an Airfoil with Trailing Edge Separation", *AIAA J.*, Vol. 21, Nº 11, pp. 1525-1532.

6 - HARTEN, A. and OSHER, S. "Uniformly High-Order Accurate Nonoscillatory Schemes. I", *SIAM J. Numer. Anal.*, Vol. 24, pp 279-309.

7 - ROE, P.L. (1981) "Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes", *J. Comp. Phys.*, Vol. 43, pp. 357-372.

8 - ORZAG, S.A., (1980) "Spectral Methods for Problems in Complex Geometries", *J. Comput. Phys.*, Vol. 37, pp. 70-92.

9 - EIDELMAN, S., COLELLA, and P. e SHREEVE, R.P. (1984) "Application of the Godunov Method and Its Second-Order Extension to Cascade Flow Modeling", AIAA *Journal*, Vol. 22, pp. 1609-1615.

10 - KOBAYASHI, M.H. and PEREIRA, J.C.F. (1992) "Predictions of Compressible Viscous Flows at All Mach Number Using Pressure Correction Collocated Primitive Variables and Non-Orthogonal Meshes", AIAA paper 92 -0426, Reno, N.V..

11 - BRISTEAU, M.O., GLOVINSK, R., PERIAUX, J. and VIVIAND, H. (1987) "Numerical Simulation of Compressible Navier-Stokes Flows", GAMM - Workshop, Vol. 18, ; Friedrich Vieweg & Sohn, Wiesbaden.

12 - GHIA, U., GHIA, K. N. and SHIN, C.T. (1982) "High-Re Solutions for Incompressible Flow Using the Navier-Stokes Equations and a Multigrid Method", *J. Comp. Phys.*, Vol. 48, pp. 387.

13 - KOBAYASHI, M.H. and PEREIRA, J.C.F (1994) "The Application of High Order Non-Oscillatory Schemes in Incompressible Flow Computations", to appear.