PISTON THEORY REVISITED and FURTHER APPLICATIONS

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ABSTRACT

A new lifting surface method utilizing the concept of Piston Theory has been developed which could account for wing thickness/incidence effect in supersonic flow. From various cases studied it is concluded that the present method makes a substantial improvement over the conventional lifting surface theory (Linear Theory) and Piston Theory in terms of unsteady pressures, stability derivatives and flutter speeds. Among other theories it also predicts the most conservative flutter boundary in that it confirms the supersonic thickness effect is to reduce the flutter speed. Furthermore, when the third order Piston Theory is replaced by a composite uniformly-valid series, the applicability of the present method is readily extended to a unified supersonic/hypersonic domain.

INTRODUCTION

Hayes-Lighthill's Piston Theory[1, 2] has been one of the most commonly practiced methods in supersonic aeroelastic applications. Because of its simplicity and its inclusion of nonlinear thickness/incidence effect. Its ease of application and its acceptable accuracy render the theory an effective tool for many aeroelastic problems[3].

However, two inherent undesirable features of Lighthill's Piston Theory (hereafter Piston Theory) invalidate its capability in general aeroelastic applications. First, the theory is a strictly one-dimensional, quasi-steady theory, whereby no upstream influence nor flow history could be accounted for. Second, its domain of application usually covers a restrictive range of Mach numbers, depending on the thickness and frequency parameters given.

Within the last decade, exact three dimensional linear theory has been sufficiently developed for treatments of lifting surfaces in unsteady supersonic flow[4]. Nevertheless, supersonic lifting surface methods, such as the Harmonic Gradient Method (HGM or known as the ZONA51 code)[4], are confined to planforms of very thin sections, whereby no thickness effect is accounted for. On the other hand, it has been known for sometime that the supersonic thickness effect could render a forward shift in the aerodynamic center, thereby reducing the flutter speed.

In view of the recent development of the NASP and HSCT, a general supersonic flutter method that could account for the effect of wing sections would be very desirable. One is therefore led to the possibility of developing a hybrid method which could possibly carry the better features of Piston Theory and the supersonic lifting surface methods.

The objective of this paper is to present our recent development of such a hybrid supersonic lifting surface method which can include the nonlinear effect of wing thickness or incidence.

PISTON THEORY

Subsequent to the original publication of Lighthill[2], Ashley and Zartarian[3] first proposed the application of Piston Theory for flutter analysis and other aeroelastic applications. They found that the nonlinear thickness effect provided by the theory indeed results in a more conservative flutter boundary, which was validated by measured data. Based on a criterion that if any one of the conditions holds, namely

$$M^2 >> 1, \quad kM^2 >> 1 \quad \text{or} \quad k^2M^2 >> 1 \qquad (1)$$

Landahl, Ashley and Mello-Christiansen[5] further established a consistent linearized Piston Theory. With this theory, they obtained an explicit flutter solution for a typical two dimensional wing section. The flutter speed according to their theory approaches those predicted by the exact linear theory[6] as the Mach number increases, whereas they tend to depart from the latter as the Mach number decreases toward unity.

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Originally, Lighthill's Piston Theory accounts for the effect of nonlinear thickness in the high Mach number range such that $M^2 >> 1$. It imposes the condition that the magnitude of the piston velocity never exceeds the speed of sound in the undisturbed fluid. The aerodynamics of this analogy is to state that

$$M\delta < 1 \quad \text{and} \quad kM\delta < 1 \tag{2}$$

where δ is the thickness or oscillatory amplitude of the airfoil, whichever is the larger; and k is the reduced frequency defined as $k = \omega c/U_\infty$.

According to his large-Mach-number expansion theory, Landahl[7] pointed out that Piston Theory amounts to ignoring a second order term in his linear amplitude sequence. Hence, the valid range of Mach number for Piston Theory can be defined by the criterion

$$\delta^{-1/3} < M < \delta^{-1} \quad \text{and} \quad M^2 >> 1 \tag{3}$$

In terms of Tsien's Hypersonic Similarity parameter[8] K, where $K = M\,\delta$, Eq (3) reads

$$\delta^{2/3} < K < 1 \tag{4}$$

For a wedge of semi-angle $\sigma = 10^\circ$, K falls in the range of $0.31 < K < 1.0$. Inspection of results obtained in Figure 1 shows that the valid lower bound of the Mach numbers is really more restrictive than the above criterion so indicated, whereas the upper bound $K < 1$ is less restrictive.

While the condition $K < 1$ for Piston Theory may be somewhat relaxed to include regions near $K \approx 1.0$, the condition $k\,K < 1$ of Eq (2) is a rather stringent one. For unsteady hypersonic flow, if $K \approx O(1)$ then the reduced frequency k must be kept very small. The failure of Piston Theory in the moderate to high range of k is evidenced by the Panel Flutter results presented in the work of Chavez and Liu[9].

THE Cp FORMULAS

Following the suggestion of Morgen, Herchel and Runyan[10], Rodden and Farkes[11] have arrived at a generalized expression for the pressure coefficients, i.e.

$$C_P = \frac{2}{M^2}\left[c_1\left(\frac{w}{a_0}\right) + c_2\left(\frac{w}{a_0}\right)^2 + c_3\left(\frac{w}{a_0}\right)^3\right] \tag{5}$$

where w represents the piston upwash.

For Piston Theory[2]

$$c_1 = 1, \quad c_2 = \frac{\gamma+1}{4}, \quad c_3 = \frac{\gamma+1}{12} \tag{6}$$

For Van Dyke's 2nd order theory[12]

$$c_1 = \frac{M}{m}, \quad c_2 = \frac{M^4(\gamma+1) - 4\,m^2}{4\,m^4} \tag{7}$$

where $m^2 = M^2 - 1$, and γ is the ratio of specific heats.

A modified Piston Theory is recommended[10] to replace c_1 and c_2 of Eq Set (6) by that of Eq Set (7) rendering an extension to the lower Mach number region.

The c_1 and c_2 of Van Dyke in fact were first obtained by Busemann[13], in which he also included a third-order term based on a consistent expansion of the simple wave theory, i.e.

$$c_3 = \frac{1}{6\,m^7}\left\{a_0 M^8 + b_0 M^6 + c_0 M^4 + d_0 M^2 + e_0\right\}$$

where (8)

$$a_0 = \gamma+1, \quad b_0 = 2\gamma^2 - 7\gamma - 5, \quad c_0 = 10\,(\gamma+1)$$

$$d_0 = -12, \quad e_0 = 8$$

Following Busemann, Donov[14] further developed a comprehensive theory in which he obtains series expansion solution up to the fourth-order term accounting separately for the isentropic part and the rotational part due to simple wave and shock wave respectively. Here, Donov's third-order term including shock wave, also derived independently by Carafoli[15], reads

$$c_3 = \frac{1}{6\,M\,m^7}\left\{a\,M^8 + b\,M^6 + c\,M^4 + d\,M^2 + e\right\}$$

where (9)

$$a = 3\left(\frac{\gamma+1}{4}\right)^2, \quad b = \frac{3\,\gamma^2 - 12\,\gamma - 7}{4}, \quad c = \frac{9\,(\gamma+1)}{2}$$

$$d = -6, \quad e = 4$$

In passing, it is noted that through a different approach Kahane and Lees[16] have obtained a correction term to c_3 of Eq (8) resulting in essentially the same c_3 as that of Eq (9).

Therefore, a consistent choice of C_P would be to adopt Donov's series and Busemann's series for flow compression and expansion respectively.

In the analysis that follows, we remain to adopt Lighthill's Piston Theory, Eq (6), in order to simplify the present approach.

For unsteady flow applications, Eq (6) is recast into the form of pressure differential of the upper and the lower wing surfaces, i.e. $\Delta C_P = C_{P_{lower}} - C_{P_{upper}}$, and the piston velocity w/U_∞ is represented by two terms, i.e. $w/U_\infty = w_0 + w_1$, where w_0 denotes the thickness distribution of the wing and w_1 the downwash. Thus, the total pressure differential $\Delta\overline{C}_P$ can be expressed as

$$\Delta\overline{C}_P = \Delta C_{P_0} + \Delta C_P \tag{10}$$

and up to the third-order term

$$\Delta C_{P_0} \cong \frac{2}{M^2} \sum_{n=1}^{3} c_n M^n \left(\Delta w_o\right)^{(n)} \tag{10a}$$

and

$$\Delta C_P \cong \frac{4\,w_1}{M^2} \sum_{n=1}^{3} n\, c_n M^n \left(\Delta \overline{w}_o\right)^{(n-1)} + \left(6\, w_1{}^2 \Delta w_0{}^{(1)} + 4\, w_1{}^3\right) c_3\, M \tag{10b}$$

where

$$\left(\Delta w_0\right)^{(n)} \equiv w_{0_{lower}}{}^n - w_{0_{upper}}{}^n$$
$$\left(\Delta \overline{w}_0\right)^{(n)} \equiv \frac{w_{0_{lower}}{}^n + w_{0_{upper}}{}^n}{2} \tag{10c}$$

For non lifting airfoil sections, where $(w_0)_l = (w_0)_u$, Eq (10b) reduces to the expression

$$\Delta C_P = \frac{4}{M}\left\{\left[c_1 + 2\,c_2\, M\, w_0 + 3\, c_3\, M^2\, w_0^2\right] w_1 + \left[c_3\, M^2\right] w_1{}^3\right\} \tag{11}$$

Substituting Eq (6) into Eq (11) and dropping the higher order terms in w_1 yields the linear amplitude version of Piston Theory, i.e.

$$\Delta C_P = \left[\frac{4}{M} + 2\,(\gamma+1)\, w_0 + (\gamma+1)\, M\, w_0^2\right] w_1 \tag{12}$$

In this work, we shall use the above expression to develop the Hybrid Lifting Surface Method.

HYPERSONIC SIMILITUDE

A classical Hypersonic Similarity[17] can be expressed as

$$C_p = \frac{2}{M^2} fn(K, \gamma) \tag{13}$$

where

$$fn = \frac{1}{\gamma}\left\{\left[1 + \frac{\gamma - 1}{2} K\right]^{\frac{2\gamma}{\gamma - 1}} - 1\right\} \tag{13a}$$

is the universal function due to the Prandtl-Meyer expansion, and

$$fn = K^2 \left\{\left[\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{K^2}\right]^{\frac{1}{2}} + \frac{\gamma+1}{4}\right\} \tag{13b}$$

is the universal function due to oblique shock waves, where $K = M\,\delta$ or $M\,\tau$.

Clearly, Eq (13a) is the basis of Lighthill's Piston Theory and hence of Eq (5). Eq (13b) was established by Tsien[8] and Linnell[18]. When Eq (13b) is expanded up to the third-order term, the coefficient c_3 corresponding to Eq (6) reads

$$c_3 = \frac{(\gamma+1)^2}{32} \tag{14}$$

This is to say that the departure between Eqs (13a) and (13b) starts from the third-order term and the difference of which amounts to $\Delta\, c_3 = \dfrac{\left(3\,\gamma^2 - 2\,\gamma - 5\right)}{96} < 0$, representing the difference in rotationality due to shock wave.

It is desirable to extend the previous third-order theories into the hypersonic flow regime where $K \geq O(1)$. Close examination of them reveals that the C_P's of these third-order theories diverge drastically as K increases toward the Newtonian limit (see Fig 1).

Second-order theories, on the other hand, usually result in one half the value of Newtonian pressure, whereas C_P of Linear theory vanishes at the Newtonian limit.

It is clear that Piston Theory does not yield the correct limit in the low supersonic end, nor does it approach the Newtonian limit in the hypersonic end. Figure 1 shows that Piston Theory has a limited valid range of $\ln K$ between roughly say -1

to at most *0.5* (*0.368 < K < 1.05*), for a wedge of semi-angle equal to 10 degrees.

To establish a uniformly-valid solution in the unified supersonic/hypersonic domain, therefore, requires a composite function, which could be properly recast into a "pseudo-similar" form as

$$\frac{C_P}{\tau^2} = f_c\left(K, \gamma\ ; c_1, c_2, c_3\right) \tag{15}$$

provided that the coefficients c_1, c_2 and c_3 could be suitably chosen from the appropriate third-order theories.

HYBRID LIFTING SURFACE METHOD

The pressure-downwash expression of Eqs (10), (11) and (12) allows the integration of Piston Theory (and other third order theories) with the classical lifting surface theory[19]. Recall the typical Lifting Surface formulation in which the so-called "Downwash equation" is solved on the surface, i.e.

$$[D_l]\ \left\{\Delta C_P\right\} = \{w\} \tag{16}$$

where $D_l = D_l(M, k)$ is the downwash matrix due to the Kernel integral and w is the given mode shapes. Linear forms of Eqs (10) and (11) can be recast into a downwash equation in the same manner,

$$[D_P]\ \left\{\Delta C_P\right\} = \{w\} \tag{17}$$

where $D_P = D_P(M, k\ ; w_0, \delta, \gamma)$ is a diagonal matrix whose elements are strictly self-influenced in their aerodynamic characteristics.

Superposition of Eqs (16) and (17) yields

$$[D]\ \left\{\Delta C_P\right\} = \{w\} \tag{18}$$

where the hybrid downwash matrix reads

$$D = f^* D_l + g^* D_P \tag{19}$$

and f^* and g^* are two generic operators defined simply according to Piston Theory as

$$f^* = [I] \quad and \quad g^* = [I] - \left(\frac{M}{4}\right)[D_P]^{-1} \tag{20}$$

APPLICATIONS

The Hybrid Lifting Surface Method (the present method) has been fully developed into a computer program now known as the ZONA51T code (T stands for thickness). In what follows, the present method, hence ZONA51T, is applied to several typical cases for aeroelastic applications. These include rigid wedge in pitching motions, leading-edge flap oscillation, panel flutter and flutter analysis for wing planforms. Unsteady pressures, stability derivatives, generalized forces and flutter boundaries are presented for these cases. Comparison with results of Hui's Exact Theory[20], Van Dyke's 2nd Order Theory[12], Perturbed Euler Characteristics (PEC) Method[9], Piston Theory[2, 3, 10, 11], Linear Theory (or ZONA51, Ref 4) and available measured data are shown whenever appropriate.

The stiffness and damping moments are defined respectively as

$$C_{m_\theta} = Re\ (C_m) \qquad C_{m_{\dot\theta}} = \frac{Im\ (C_m)}{k} \tag{21}$$

where

$$C_m = \int_0^c \Delta\, C_P\, (x - h)\, dx$$

The generalized aerodynamic forces are defined as

$$Q_{ij} = \int_0^c \Delta\, C_{P_i}\, z_j\, dx \tag{22}$$

where C_{P_i} is the unsteady pressure due to the z_ith mode and z_j is the j'th mode shape.

A rectangular wing model, with wedge profile containing two-dimensional flow at the inboard sections is shown in Figure 2. Computed results of ZONA51T from the root chord strip is used to verify with those provided by other two-dimensional theories (Fig 3 through Fig 16).

Oscillating Wedge/Diamond Profiles

- *Effect of Pitching Axis Location*

In Fig 3 and Fig 4, results of ZONA51T in damping-in-pitch derivative are compared with those of Linear Theory (ZONA51) and test data[21, 22], for a wedge profile and a diamond profile respectively. It is seen that ZONA51T predicts a closer trend to the test data than do Piston Theory and Linear Theory.

- *Effect of Reduced Frequency*

Fig 5 and 6 present the variations of generalized aerodynamic forces Q_{12} and Q_{22} with reduced frequency k for an oscillating wedge. Results of ZONA51T are compared with those of Linear Theory and PEC method. Good correlation is found between results of ZONA51T and PEC at M = 1.5 and M = 3.0 for a wedge of semi-angle $\sigma = 10°$, whereas substantial departures are found between that of Linear Theory and ZONA51T. Clearly, these departures represent the additional nonlinear thickness effect to the results of Linear Theory.

- *Effect of Mach Number*

Fig 7 presents the variations of stiffness and damping-in-pitch moments with freestream Mach number for a diamond profile of thickness ratio, $\tau = \tan 15°$. Here, two other versions of ZONA51T are included for comparisons: ZONA51Td adopts the third-order theory of Donov, i.e. Eq (9), whereas ZONA51Tu ("u" stands for unified) adopts the composite function format of Eq (15), which renders ZONA51T uniformly valid in the unified supersonic and hypersonic domain. It is seen that results of the stiffness moment of ZONA51T, Td, Tu follow the resulting trend of Hui's Exact theory throughout the Mach range, while those of Piston Theory and Linear Theory remain independent of Mach number. Considerable departures between various results are found for the damping moment beyond M = 4.0. Most third-order theories such as Piston Theory, ZONA51T, Td yield diverged results at the hypersonic end, whereas the results of ZONA51Tu is in good agreement with Hui's Exact theory throughout the Mach number range. In fact, only the latter two theories will approach the proper Newtonian limit.

- *Effect of Thickness*

Figs 8, 9 and 10 present the variations of Stability derivative for a Diamond profile with profile thickness at Mach number M = 2.0, 5.0 and 10.0 respectively. Fig 8 shows that both Linear Theory and Piston Theory underpredict the stiffness and damping moments, whereas ZONA51T overpredicts them. Good agreement is found between ZONA51Td, Tu and Hui's Exact theory up to $\sigma = 15°$. When Mach number is increased to M = 5.0 and 10.0, all third-order theories overpredict the damping moment. ZONA51Tu, however, yields results in close agreement with Hui's Exact theory up to $\sigma = 15°$ for all Mach numbers considered. Thus, given pitching axis location at half chord, an increase in thickness results in an increase in damping moment. The sudden increase in damping moment of Hui's Exact theory at M = 2.0 depicts the shock detachment occurring around $\sigma = 23.5°$. Such trend is beyond the capability of all versions of ZONA51T. Similar to Piston Theory, ZONA51T's nonlinear thickness effect is accounted for only by a quasi-steady approach, which ignores the flow history. Hui's Exact theory accounts for mildly unsteady flow, hence the unsteady shock and Mach wave interaction which includes the effect due to shock detachment.

Leading-Edge Flap Oscillation

Fig 11 shows an oscillating leading edge flap of thin wedge profile ($\sigma = 2°$) with a hinge line located at quarter chord. Figs 12 and 13 show the magnitude and phase angle of unsteady pressures at M = 2.0, k = 2.0 and at M = 5.0, k = 0.5 respectively. The parameter $M\tau k$ is bounded by 0.1 for both cases. In Fig 12, it is seen that not only Piston Theory underestimates the pressure magnitude but it predicts zero pressure downstream of the hinge line. By contrast, all other solutions show significant upstream influence due to flap motion. Small differences are found between pressures of Linear Theory and ZONA51T at this Mach number. The difference between results of PEC and ZONA51T lies in the inadequacy of the latter in accounting for the effect of unsteady shock/Mach wave interaction, thus the effects of rotationality and flow history.

In Fig 13, significant improvement over the Linear Theory by ZONA51T is found in the pressure magnitude on the flap. However, no improvement is found in the phase angle between them. Notice that the waviness of C_P in Fig 12 disappear in Fig 13. Physically, the waviness in C_P is created by the unsteady Mach wave reflection from the Shock wave. In the case of M = 5.0, the Mach wave is reflected farther downstream of the profile trailing edge.

Panel Flutter

Shown in Fig 14 are two flexible panels (membranes) mounted on both surfaces of a wedge ($\sigma = 2°$). The panels are performing oscillatory mode as depicted by

$$z = \varepsilon \sin\left(\frac{N\pi}{L}x\right)e^{ikt} \tag{23}$$

where N = 2, and ε is the amplitude of vibration.

Figs 15 and 16 present the effect of reduced frequency on Generalized Aerodynamic Forces (Q_{ij})

for these vibrating panels at M = 1.5 and M = 5.0 respectively. Results in Q_{12} and Q_{21} of Linear Theory, Piston Theory and ZONA51T are compared with that of PEC. Similar to the earlier observation in the case of Flap Oscillation, ZONA51T in Figs 15 and 16 substantially improves the pressure magnitude over that of Linear Theory but practically improves little in the phase angle. This is clear, for the nonlinear thickness term of Eq (19) is based on a quasi-steady approach, hence no additional phase change can be expected in the present method.

Wing Flutter

Two wing planforms are selected for performing flutter analysis using ZONA51T: A 70-Degree Delta Wing and a 15-Degree Swept Untapered Wing.

- *70-Degree Delta Wing*

Figs 17 and 18 present flutter boundaries for a 70 Degree Delta Wing with 6% thick Biconvex Airfoil and Diamond Airfoil sections respectively.

The flutter experiment was carried out at NASA Langley/LaRC by Hanson and Levey[23]. The wing model used was essentially a flat-plate. According to Ref 23, four measured modes are used in the present flutter analysis. Half of the delta planform is subdivided into 10 x 10 panels. The flutter boundary consists of the flutter points obtained for six Mach numbers (M = 1.19, 1.30, 1.64, 2.0, 2.25 and 3.0) using ZONA51 and ZONA51T. Flutter results computed by Piston Theory, ZONA51 and 51T are compared with the measured data[23]. Several observations on the performance of ZONA51T can be put forth:

- ZONA51T and the third-order Piston Theory predict more conservative flutter boundaries than that of ZONA51 and the first-order Piston Theory respectively, indicating that the thickness effect indeed reduces flutter speed in supersonic flight.

- ZONA51T predicts the most conservative flutter boundary of all methods considered.

- Similar flutter trends are found between results of Figs 17 and 18 showing that the shape of airfoil section makes relatively insignificant impact on the flutter boundary.

- *15-Degree Swept Untapered Wing*

Table 1 presents two computed flutter points for a 15-Degree Swept Untapered Wing of aspect ratio AR = 5.35 at M = 1.3 and M = 3.0.

The flutter experiment was carried out at NACA Langley Field by Tuovila and McCarty[24]. The wing model used is a cantilever wing with a 2% thick hexagonal airfoil section (see Fig 19). According to Ref 24, eight modes generated by MSC/NASTRAN are used in the present flutter analysis. Half of the wing planform is subdivided into 10 x 10 panels.

In Table 1, computed results of ZONA51, Rodden's method[25] (employing ZONA51), and ZONA51T are compared with test data of Tuovila and McCarty[24]. Note that while Rodden's method adopts Eq (7) plus c_3 of Eq (6), ZONA51T uses coefficients strictly from the third-order Piston Theory, i.e. Eq (6). As expected, Rodden's method yields closer agreement with test data than does ZONA51T. However, further flutter analysis using other versions of ZONA51T is pending whereby new sets of coefficients will be adopted by ZONA51T from those defined in other third-order theories. Overall, the listed results confirms once again the impact of thickness on flutter speed. Linear Theory, as computed by ZONA51, yields non-conservative flutter points at both Mach numbers.

CONCLUSIONS

A Hybrid Lifting Surface Method has been developed whereby it combines the classical lifting surface theory with a nonlinear thickness correction term based on Piston Theory. Accordingly, a computer program known as ZONA51T has been fully developed which represents a generalized ZONA51 code and could account for nonlinear thickness distribution or mean incidence of all lifting surfaces.

By adopting appropriately other third-order theories, such as those stated in Eqs (7), (8), (9), (14) and (15), two improved versions of ZONA51T are further developed. While ZONA51Td could account for flow rotationality due to supersonic shock wave, ZONA51Tu further extends the former's applicability to a unified supersonic/hypersonic domain. Close examination of all third-order theories including Piston Theory reveals that they all fail to yield the Newtonian limit in the hypersonic end, whereas Piston Theory is known to yield a different limit from the Ackeret limit in the sonic end. By adopting a uniformly-valid series in the nonlinear correction term, ZONA51Tu could yield the Newtonian limit and the Ackeret limit as would Hui's Exact theory. In passing, we remark that Hui's theory offers an exact solution of a perturbed Euler formulation, it is nevertheless a low frequency, two-dimensional theory and is rather restrictive in its aeroelastic applicability.

The nonlinear correction term introduced in ZONA51T is a self-influenced function, which carries the inherent quasi-steady nature of the Piston Theory. This means that the thickness effect is corrected locally and no related flow history can be accounted for by the present model. Hence, ZONA51T and all its improved versions could improve substantially the pressure magnitude over that of Linear Theory, but only yield minor changes in the phase angle.

For aeroelastic applications, ZONA51T is applied to four typical cases with different configurations. These include oscillating wedge and diamond profiles, oscillating leading-edge flap, panel flutter and wing flutter. Considerable effort has been directed toward verification of the present result with that of other existing theories. It is found that ZONA51T could substantially improve the results of Linear Theory in terms of pressures, stability derivatives and flutter speeds. In most cases, results of ZONA51T appear to be in better agreement than those of other theories with test data and results of Perturbed Euler solutions[9,20].

For flutter analysis of two selected wing planforms, the results of ZONA51 and 51T confirm that the supersonic thickness effect is to reduce the flutter speed, as expected. Such a reduction increases with increasing Mach number. For the case of a 70-Degree Delta Wing, ZONA51T yields the most conservative flutter boundary, whereas Linear Theory represented by ZONA51 yields the next conservative one. Piston Theory, by contrast, yields the least conservative boundary in comparison with the test data. The shapes of wing section are of less impact than the wing thickness on the flutter boundary.

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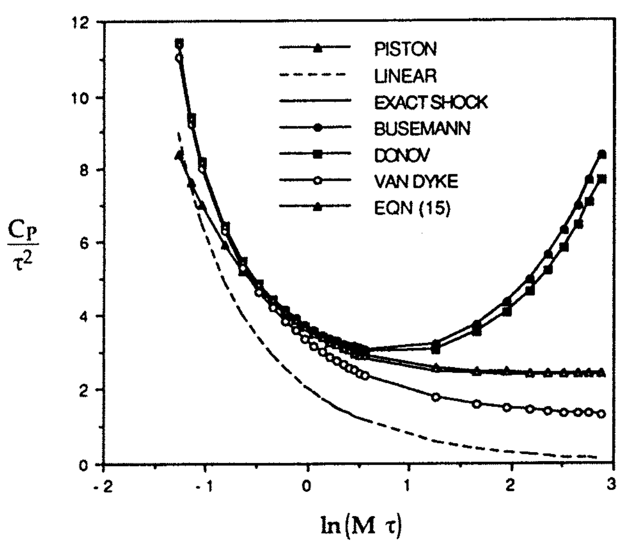

Fig. 1. Surface Pressure of a Wedge According to Various Supersonic/Hypersonic Models: $\tau = \tan 10^\circ, \gamma = 1.4$.

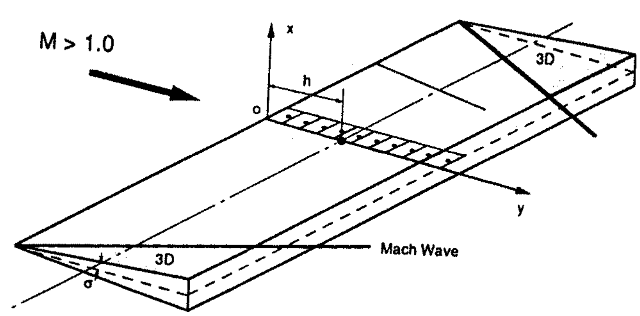

Fig. 2. Rectangular Wing Model with Wedge Profile in Supersonic Flow Showing a Two Dimensional Chordwise Strip.

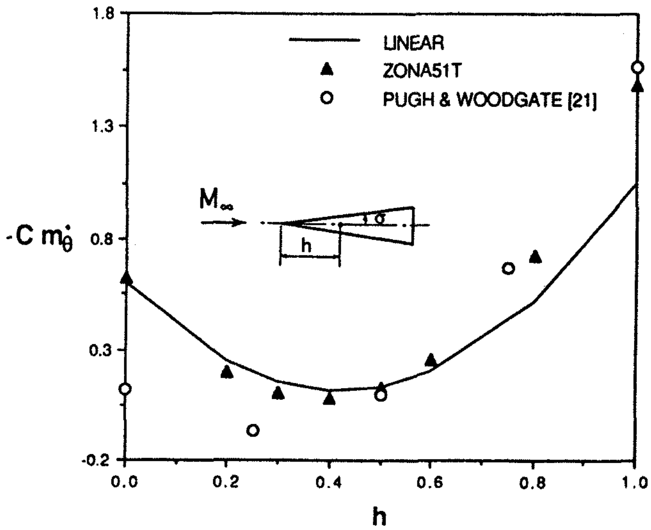

Fig. 3. Damping-in-Pitch Derivative for a Wedge Profile versus Pitching Axis Location: M = 1.75, σ = 6.85°.

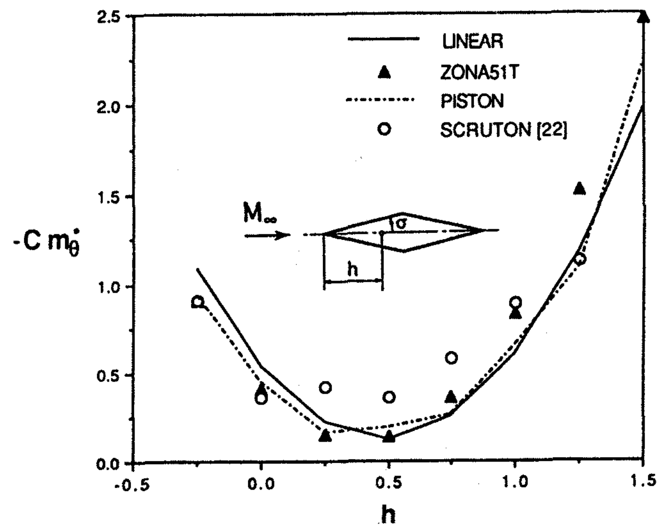

Fig. 4. Damping-in-Pitch Derivative for a Wedge Profile versus Pitching Axis Location: M = 2.43, σ = 6.85°.

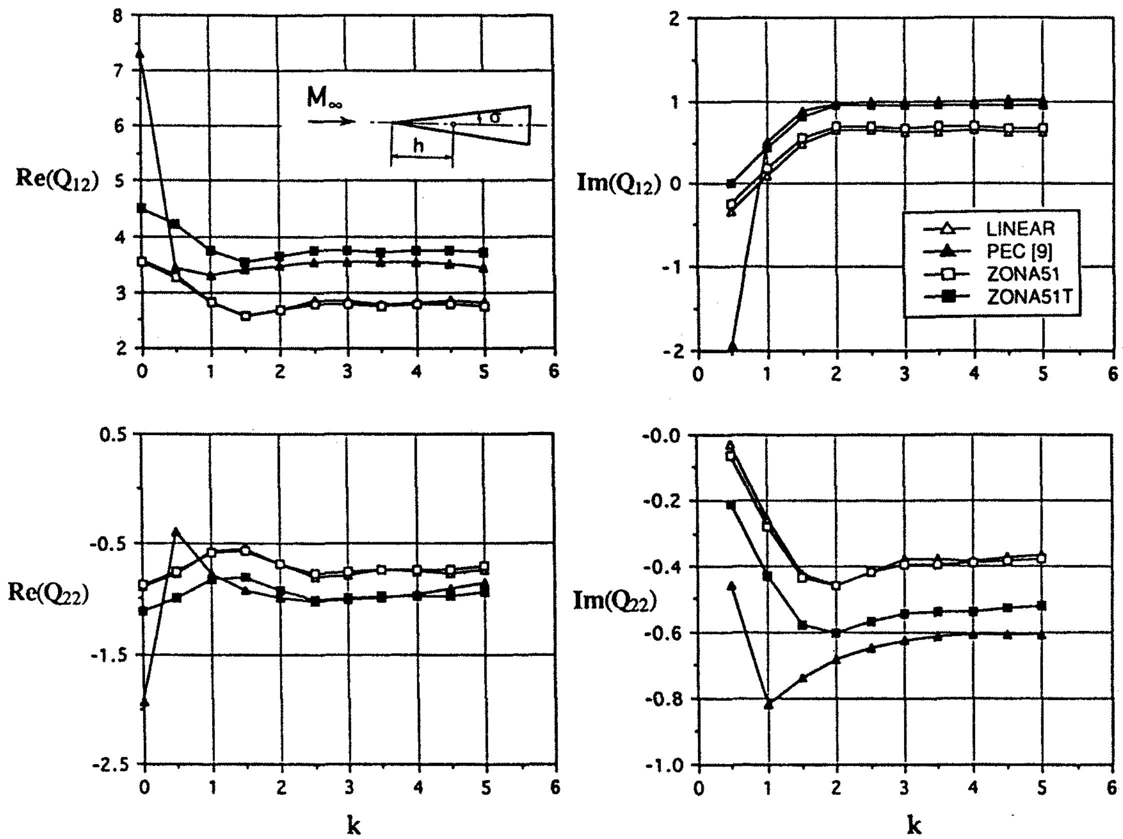

Fig. 5. Generalized Aerodynamic Forces for an Oscillating Wedge versus Reduced Frequency: M = 1.5, h = 0.25c, σ = 10°.

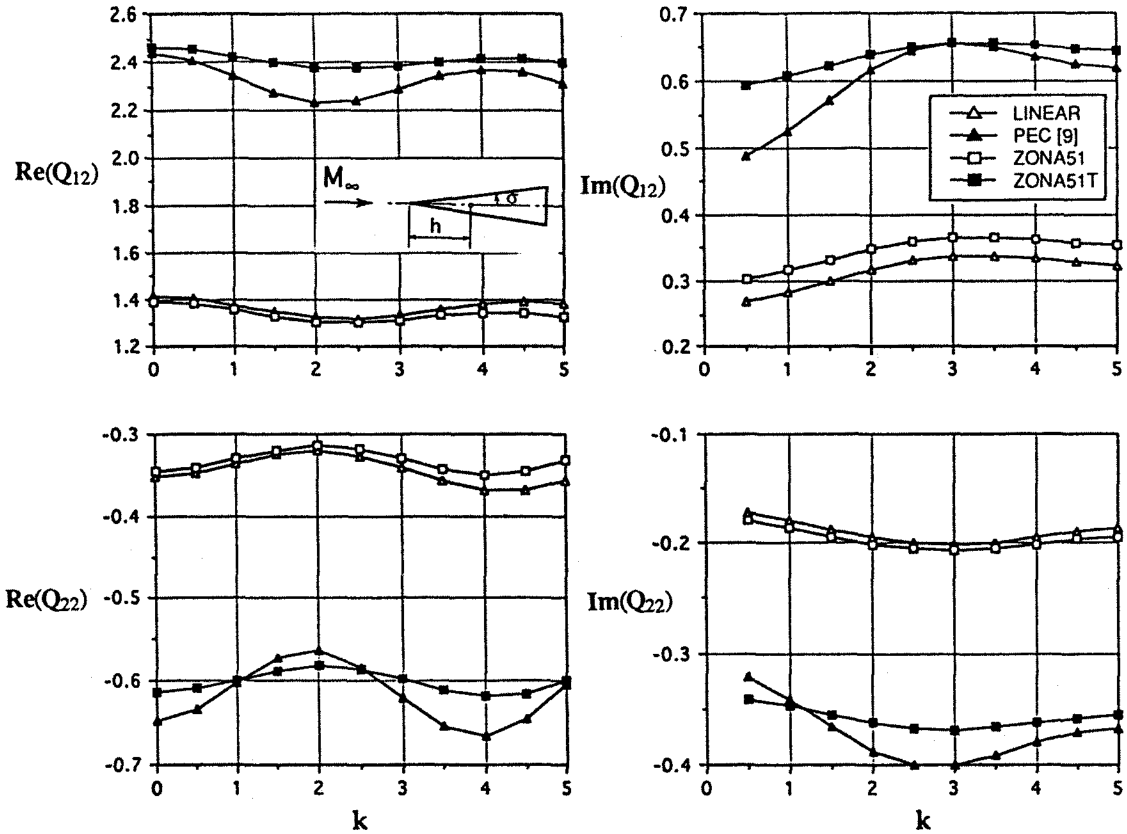

Fig. 6. Generalized Aerodynamic Forces for an Oscillating Wedge versus Reduced Frequency: $M = 3.0$, $h = 0.25c$, $\sigma = 10°$.

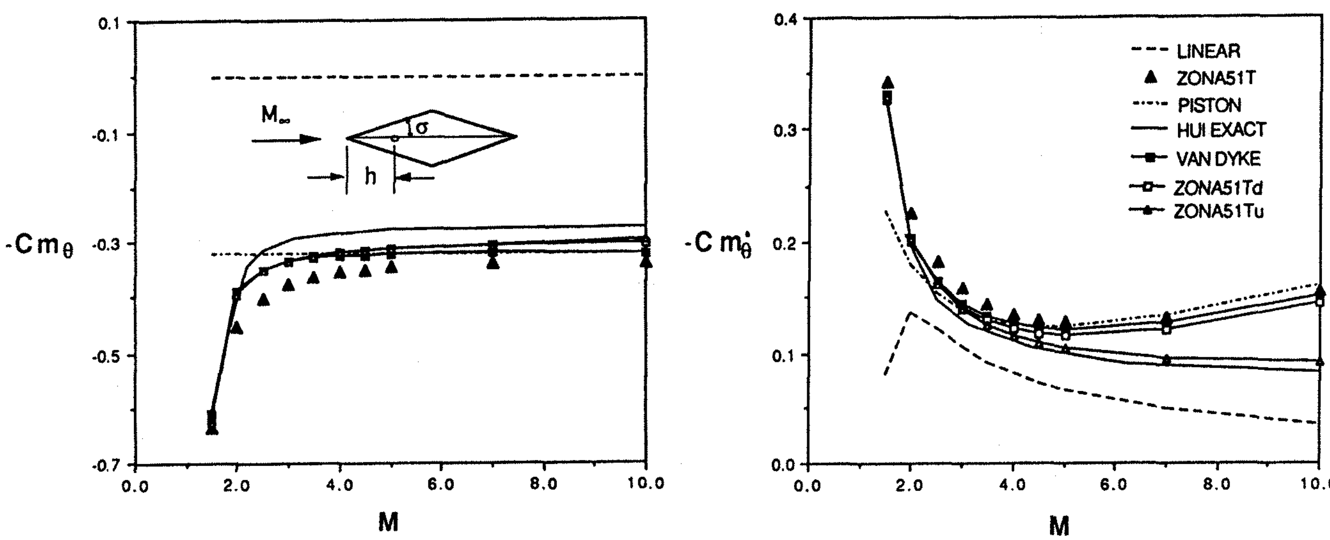

Fig. 7. Stiffness and Damping-in-Pitch Derivatives for a Diamond Profile versus Mach Number: $h = 0.5c$, $\sigma = 15°$.

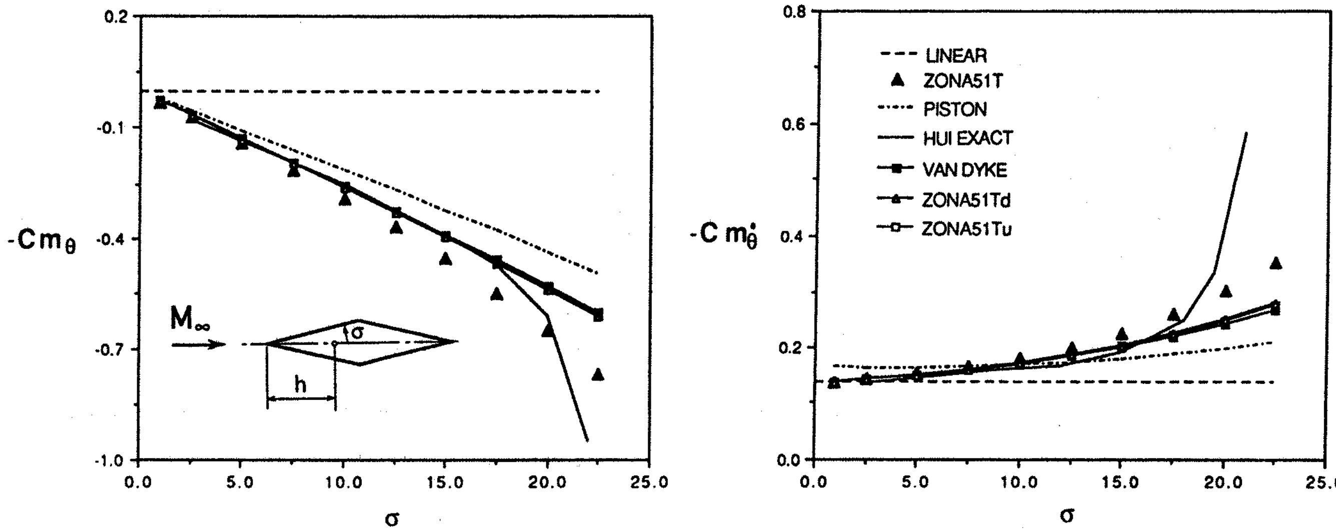

Fig. 8. Stiffness and Damping-in-Pitch Derivatives versus Semi-Wedge Angle: M = 2.0, h = 0.5c.

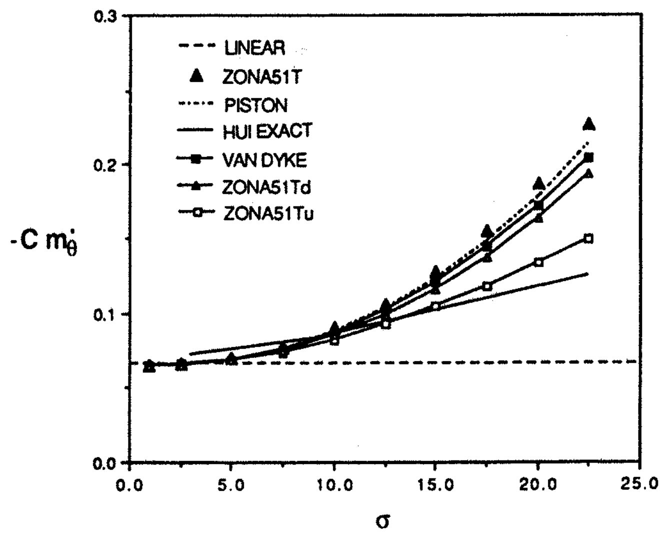

Fig. 9. Damping-in-Pitch Derivative versus Semi-Wedge Angle: M = 5.0, h = 0.5c.

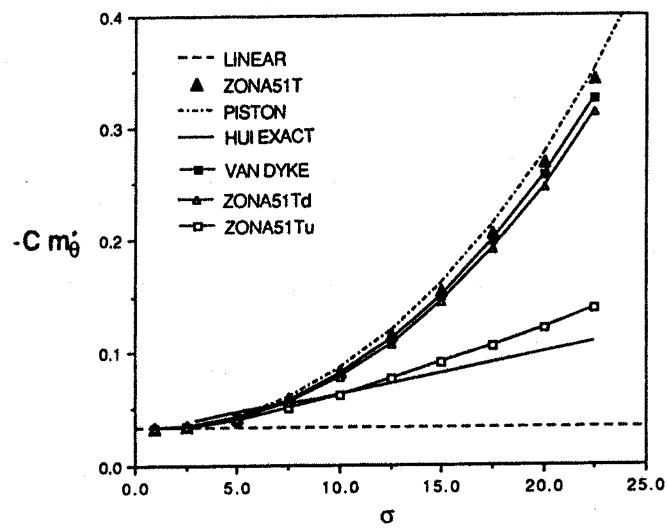

Fig. 10. Damping-in-Pitch Derivative versus Semi-Wedge Angle: M = 10.0, h = 0.5c.

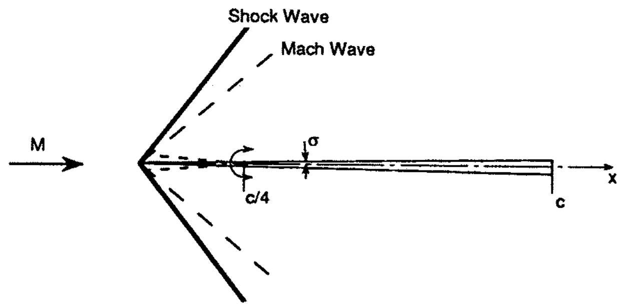

Fig. 11. Oscillating Leading-Edge Flap of a Thin-Wedge Airfoil: σ = 2°.

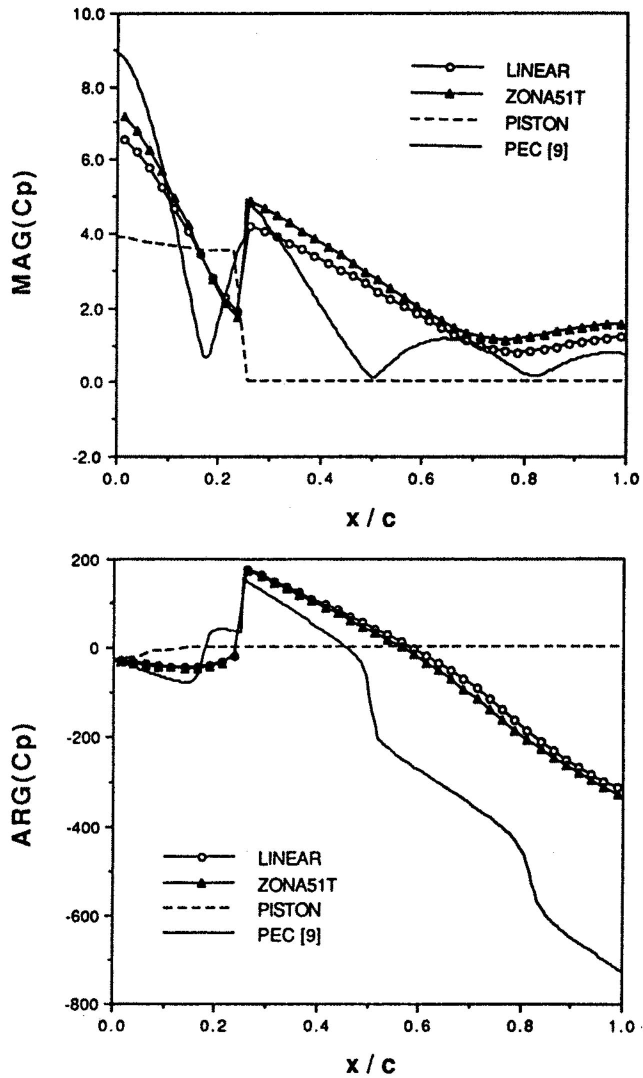

Fig. 12. Unsteady Pressure Distributions for an Oscillatng Leading Edge Flap with Hinge Line at Quarter-Chord: M= 1.2, k = 0.5, σ = 2°.

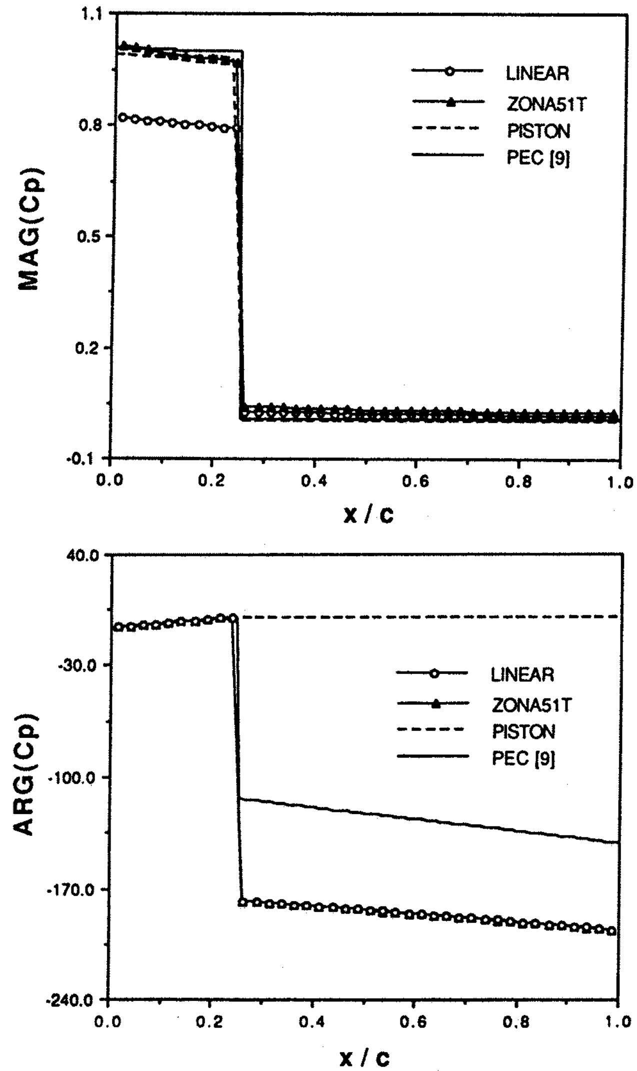

Fig. 13. Unsteady Pressure Distributions for an Oscillatng Leading Edge Flap with Hinge Line at Quarter-Chord: M= 5.0, k = 0.5, σ = 2°.

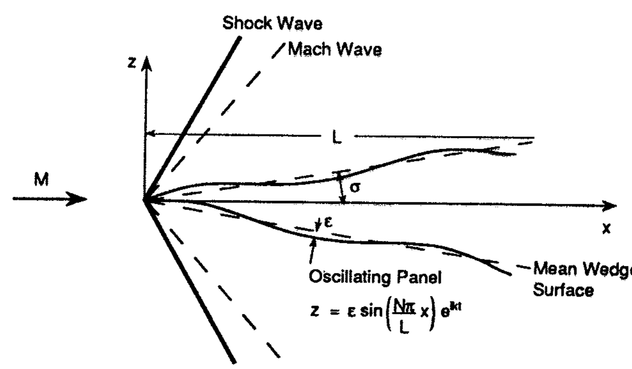

Fig. 14. Oscillating Panels Mounted on A Wedge with Semi-Wedge Angle σ = 2°.

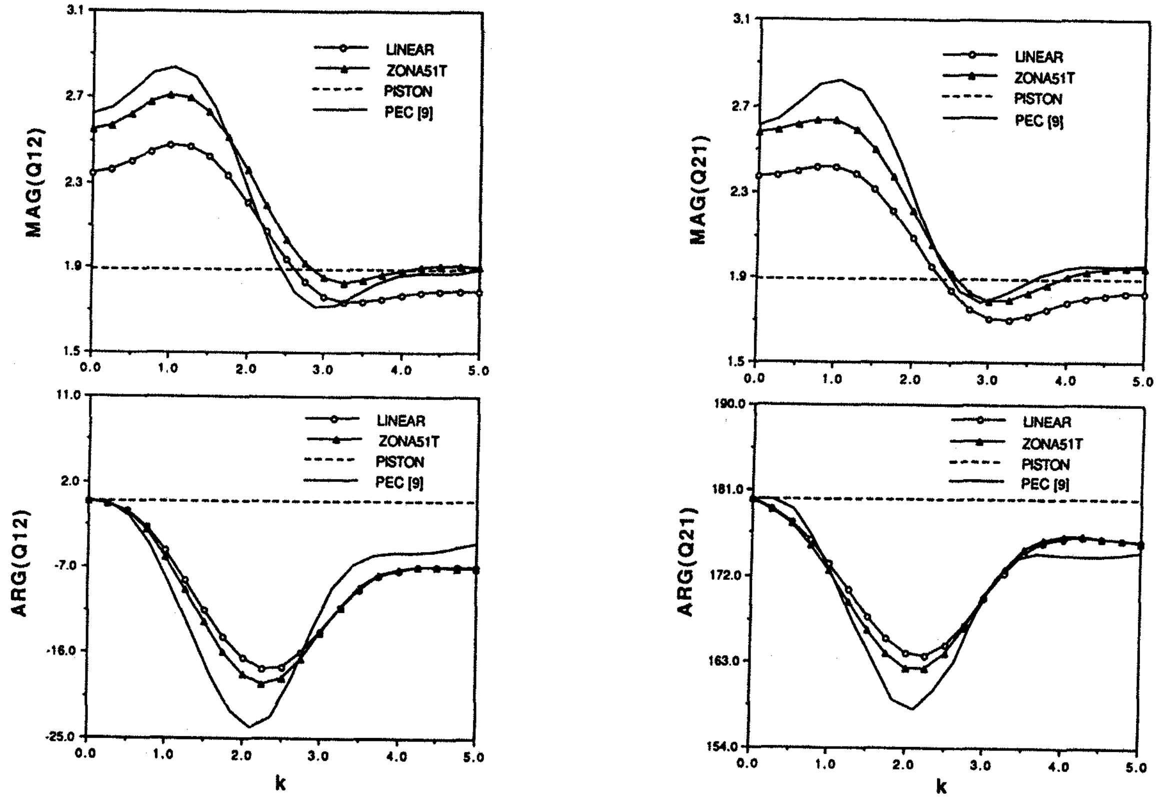

Fig. 15. Effect of Reduced Frequency on Generalized Aerodynamic Forces for an Oscillating Panel: M = 1.5, $\sigma = 2^{\circ}$, N = 2.

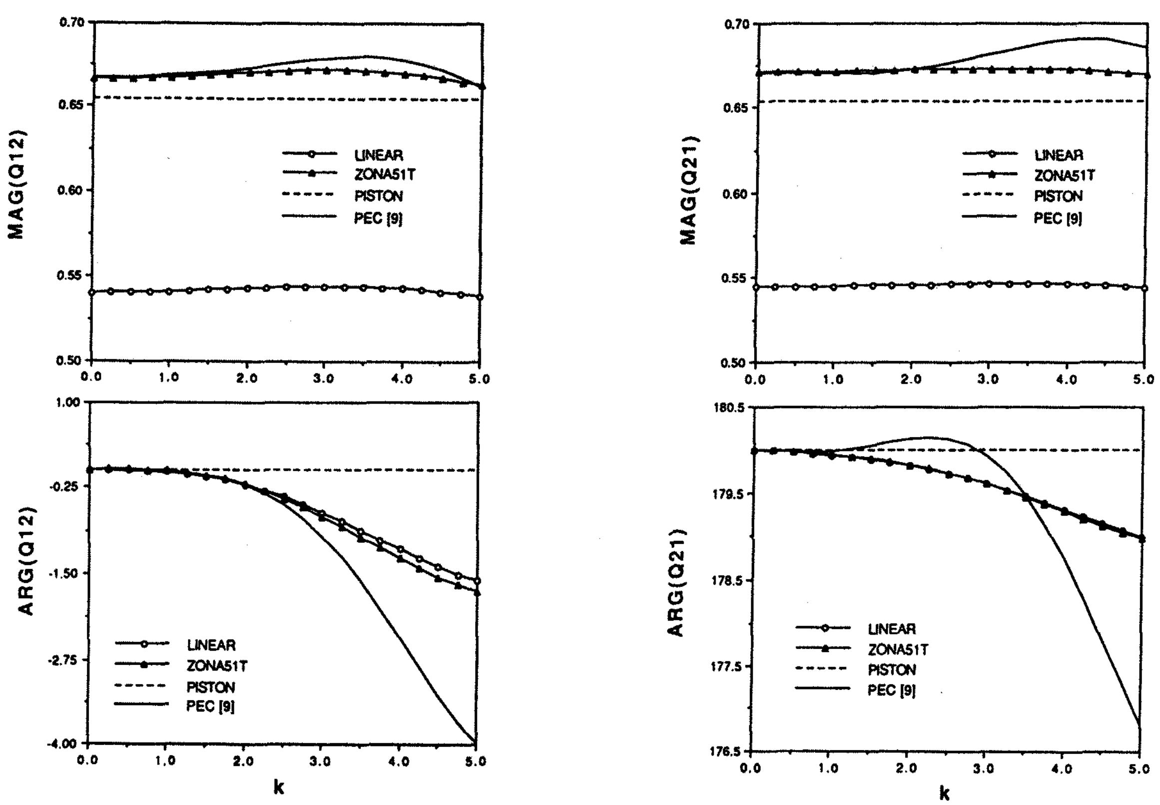

Fig. 16. Effect of Reduced Frequency on Generalized Aerodynamic Forces for an Oscillating Panel: M = 5.0, $\sigma = 2^{\circ}$, N = 2.

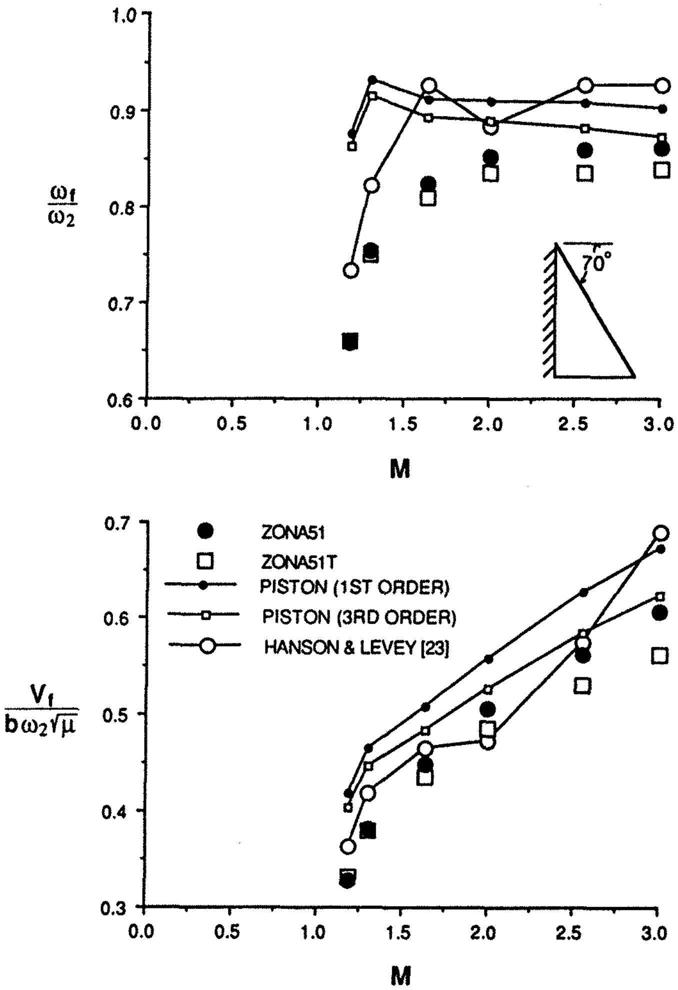

Fig. 17. Flutter Boundary for a 70-Degree Delta Wing with 6% Thick Biconvex Airfoil Section.

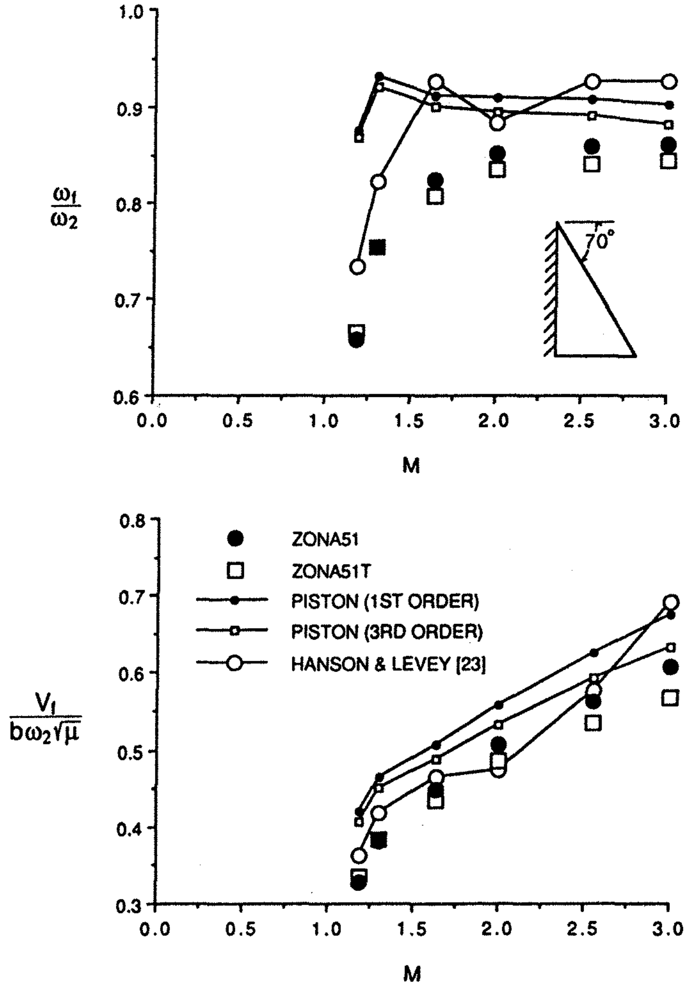

Fig. 18. Flutter Boundary for a 70-Degree Delta Wing with 6% Thick Diamond Airfoil Section.

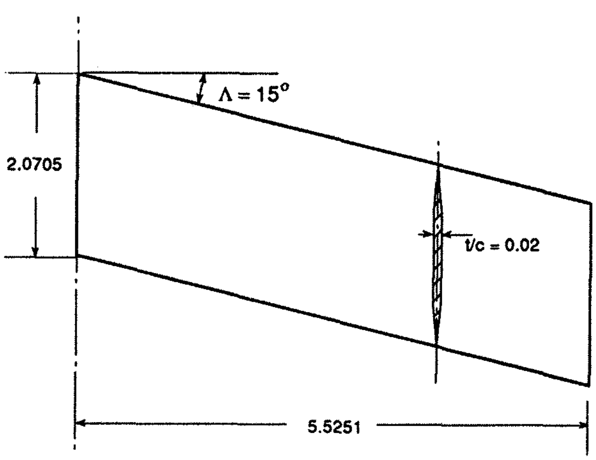

Fig. 19. 15-Degree Swept Untapered Wing Showing Dimensions

Table 1. Flutter Speed and Flutter Frequency for a 15-Degree Swept Untapered Wing: M = 1.3 & 3.0

	M = 1.3		M = 3.0	
	V_f (ft/s)	f_f (Hz)	V_f (ft/s)	f_f (Hz)
Test [24]	1280	102	2030	146
Rodden [25]	1397	124	1913	149
ZONA51	1591	125	2415	151
ZONA51T	1483	124	1824	143