MULTIOBJECTIVE OPTIMIZATION DESIGN OF TRANSONIC AIRFOILS*

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Abstract

An efficient multiobjective optimization design method for the multiple design-point problem of transonic airfoil is presented. The procedure is developed by coupling the hierarchical optimization method with an aerodynamic analysis program which is based on combination of full potential and boundary layer flow analysis. By using present method the supercritical airfoils or the conventional low-speed airfoils can be modified to be trasonic airfoils which have lower drag coefficients at two or more design mach numbers. The presented method is very attractive for practical use because of its quick convergence.

Introduction

One of the major challenges that the high speed aircraft designers are facing is to minimize the drag at cruising condition to improve the fuel efficiency. It is very promising to develop new transonic airfoils which are of low drag coefficient. Modern computational fluid dynamics methods in conjunction with single objective optimization techniques have been applied to the design of transonic airfoils in recent years. Examples and reviews can be found in references[1-4]. Although the existing single objective optimization design methods can generate automatically transonic airfoils having low drag coefficient at a single design-point, they are difficult to deal with the multiple design-point problems.

For the reason of practical use in wing design, it is very important that airfoil has low drag coefficient at two or more flight Mach number. This paper attempts to present a multiobjectiv optimization design method for the multiple design-point problems of transonic airfoils. The results of some representative design cases are presented.

Design Method

Described in this paper is an efficient multiobjective optimization design method for the multiple design-point problems of transonic airfoils. The drag coefficients of multiple design-point are considered as objective functions and the hierarchical optimization method is adopted. The objective functions are listed in order of importance in the optimization process. Every objective function is minimized by the direct search EXTREM method and subjected to the extra constraints to increments of high level objective function. The aerodynamic analysis program is based on combination of full potential and viscous boundary layer flow analysis.

The optimization design procedure starts with an assumed initial airfoil. The initial airfoil shape is represented in the program by the following equeations:

$$Y_{us} = Y_{usb} + \Sigma a_i f_i \tag{1}$$

$$Y_{ls} = Y_{lsb} + \Sigma b_i g_i \tag{2}$$

where Y_{usb} and Y_{lsb} are the ordinates of the upper and lower surface of the initial airfoil, f_i and g_i are the shape functions,

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a_i and b_i are the design variables. The new airfoil coutour is determined by the value of the design variables and the shape functions. The communication between the flow solver and the optimization program is established through the objective function and the design variables. The design variables is changed to generate a new airfoil. The aerodynamic characteristics of this new airfoil is calculated by the flow solver, from which the drag coefficients i.e. the new value of the objective function is obtained. Then the design variables is changed again in a specific manner according to the decision made by the optimization algorithm. This process is repeated until a final airfoil with minimum drag coefficient is obtained.

In the following sections we will discuss the shape functions, transonic flow solver and numerical optimization algorithm because they play the key role in the design process.

Airfoil shape functions

Seven shape functions which have been used successfully by Hicks and Vanderplaats[1][3] are chosen having the following form:

$$f_i = X^{ni}(1-X) / e^{mix} \qquad i=1 \qquad (3)$$
$$f_i = \sin^3(\pi X^{ni}) \qquad i=2,3,4 \qquad (4)$$
$$f_i = \sin^3[\pi(1-X)^{ni}] \qquad i=5 \qquad (5)$$
$$f_i = \sin[\pi(1-X)^{ni}] \qquad i=6,7 \qquad (6)$$

In the above functions, $m1=10$, $n1=0.4$ is selected for the first function, it's peak occur at 5 percent chord position. For $i=2,\ 3,\ 4,\ 6,\ 7$, $ni=\log 0.5 / \log Xi$ is used to place the peak at Xi, Xi are 0.2, 0.4, 0.6, 0.06 and 0.13 respectvely. $n5=n2$ is chosen for the fifth function, and it's peak occur at 8 percent chord.

The flow solver for viscid transonic airfoil

The transonic airfoil analysis code used in this paper is the NPUFOIL program developed by Qiao et al. [5]. The program is based on combination of full potential and boundary layer flow analysis which includes the calculations of laminar boundary layer, turbulent boundary layer, natural or fixed transition prediction and separation prediction. The inviscid flow is obtained by solving the unconservational full potential equation. The laminar boundary layer is solved by Thwaites' method under stewartson transformation, and the turbulent boundary layer is solved by Nash's method. the natural or fixed transition is predicted by Granville method and the separation of the turbulent boundary layer is predicted by Nash–Macdonald criterion.

From which the displacement thickness and other parameters of the boundary layer are obtained. The aerodynamic characteristics such as lift and drag coefficient are calculated through viscous / inviscid iteration.

The main reason for using NPUFOIL program as the objective function of the optimization design is that the predicted drag coefficient is very good agreement with experimental data.

Numerical optimization algorithm

The individual objective function is minimized by the direct search EXTREM method which is developed by Jacob[6]. The hierarchical optimization method is adopted for the multiobjective optimization design of transonic airfoil. In the following the optimization methods are introduced briefly.

The EXTREM method is a direct search method and can be used efficiently for solving the multivariable constrained optimization problem. The search direction is determined just the same as Rosenbrock's method. If it is a constrained optimization problem with N variables, the first main search direction is determined by the initial value $\vec{C}$ and the initial search step $\vec{DC}$ in following manner:

$$\vec{S}^{(1)} = \vec{C} + \vec{DC} \qquad (7)$$

By means of a Gram–Schmidt orthogonalization, N–1 secondary search directions are determined. The next main search direction always results from connecting the optimum of the last and the next to the last secondary search direction:

$$\vec{S}^{(k+1)} = \vec{X}^{(k+1)} - \vec{X}^{(k)} \qquad (8)$$

The optimization process in one direction is implemented by a parabolic extrapolation, just like Powell's method:

$$\vec{C}_4 = \vec{C}_1 + \frac{\vec{DC}}{|F_3 - 2F_1 - F_2|} \cdot \frac{F_2 - F_1}{2 \cdot N} \qquad (9)$$

where the maximum is obtained when $N=+1$ and the minimum is obtained when $N=-1$.

All kinds constraints can be take into account, but the optimization variables in each search or extrapolation process must be checked whether they have violated the given constraits. If the variables have violated, they will be managed in following way:

(a) In search process, if $\vec{C}_2 = \vec{C}_1 + \overline{DC}$ violats the given constraints, $\vec{C}_2$ will be changed to $\vec{C}_{2new} = \vec{C}_1 - \overline{DC}/2$.

(b) In extrapolation process, if $\vec{C}_4$ violates the given constraints, $\vec{C}_4$ will be changed to $\vec{C}_{4new} = \vec{C}_1 + (\vec{C}_4 - \vec{C}_1)/4$.

The method discribed as above have been shown having quick convengence[6].

In the hierarchical optimization method process, the objective functions are listed in order of importance. Every objective function is minimized and subjected to the extra constraints to the increments of high level objective function. i.e.

$$\text{Minimize} \quad f_i(\vec{X}) \tag{10}$$

$i = 1,2,3,\cdots\cdots k$

from $i = 2$ to $i = k$ subject to

$$f_{j-1}(\vec{X}) \leqslant (1 + \varepsilon_{j-1}/100) f_{j-1}(\vec{X}^{(k-1)}) \tag{11}$$

$j = 2,3,\cdots\cdots,i$

where ε_{j-1} is the given increment of j-1th objective function.

So the optimum point $X^{(k)}$ is finally obtained.

This optimization procedure can maintain the quick convergence of EXTREM method and eliminate some restraints to the initial airfoil data.

Design examples and discussion

In various design problems, different constraints may be applied to the airfoil geometry depending on the specific requiements. In present paper, we allow both upper and lower surfaces to vary in the same manner under the constraint of constant cross sectional area.

design case 1

The existing supercritical airfoil RAE2822 is chosen as the initial airfoil in this case. We want to get a optimal airfoil which has lower drag coefficients at two design point ($M = 0.75$ and $M = 0.77$) when it's lift coefficient $Cl = 0.7$ and Reynolds number $Re = 2.0 \times 10^6$.

The described shape functions when $i = 2,3,4,5,6,7$ are chosen in this design case. The airfoil's drag coefficient at $M = 0.75$ is considered as the first level objective function and is minimized by the direct search EXTREM method. The six variable become:

$a_1 = 0.001655 \qquad a_2 = -0.001485$
$a_3 = -0.001320 \qquad a_4 = -0.002125$
$a_5 = 0.000099 \qquad a_6 = -0.003225$

$\varepsilon_1 = 10$ is chosen as the increment of first level objective function, the drag coefficient at $M = 0.77$, which is considered as the second level objective function, is minimized and subjected to

$$CD \leqslant (1 + \varepsilon_1/100) CD_{min}$$

We obtain:

$a_1 = 0.000202 \qquad a_2 = -0.000427$
$a_3 = 0.000127 \qquad a_4 = -0.000435$
$a_5 = 0.000266 \qquad a_6 = -0.000155$

The comparision of drag coefficients, pressure distributions and geometries are shown in figure 1, figure 2 and figure 3, respectively. In order to compare the geometris of the airfoils clearly, the proportion of Y axis in figure 3 is enlarged.It shows that the drag coefficient of the final airfoil at $M = 0.77$, which compares with the single objective optimal airfoil, has got 10% reduction although it's increased 5.5% at $M = 0.75$.

design case 2

The symmetric airfoil NACA0012 is chosen as the initial airfoil, which is usually used for low speed regime and wound exhibit low performance if flying at transonic speed. It is desired to modify it's shape to minimized the drag coefficients at two design Mach number ($M = 0.77$ and $M = 0.79$) while It's lift coefficient $Cl = 0.5$ and the Reynolds number $Re = 6.5 \times 10^6$keeps unchanged.

The shape functions when $i = 1,2,3,4,5$ are chosen in this case. According to the hierarchical optimization method, the drag coefficient of the airfoil at $M = 0.77$ and $M = 0.79$ are considered as two objective functions. The drag coefficient at $M = 0.77$, which is the main design–point, is selected as the first level objective function and the drag coefficient at $M = 0.79$ is selected as the second. $\varepsilon_1 = 15$ is chosen as the increment. After the first level objective function is minimized, we obtain the five variables as follows:

$a_1 = -0.022195 \qquad a_2 = -0.005685$

$a_3 = 0.002408 \qquad a_4 = 0.002886$

$a_5 = 0.014311$

After the second level objective function is minimized while subjected to the constraint of the first level objective function increment, we obtain the final optimal airfoil. The five design variables become:

$a_1 = 0.000647 \qquad a_2 = 0.000251$

$a_3 = 0.003578 \qquad a_4 = -0.001241$

$a_5 = 0.000295$

The comparision of drag coefficient, pressure distributions and geometries of the initial and final airfoil are shown in figure 4, figure 5 and figure 6, respectively. In order to compare the geometris of the airfoils clearly, the proportion of Y axis in figure 6 is enlarged.

The time required to complete this design case is about five hour on a Intel AT486 personal computer.

Conclusions

An efficient multiobjective optimization design method for multiple design–point problem of transonic airfoil is presented by coupling hierachical optimization method with a reliable aerodynamic analysis program. The results of design cases show that:

(a) The supercritical airfoil or the conventional low speed airfoil can be modified to be transonic airfoil which have lower drag coefficients at two or more design Mach number.

(b) The total drag coefficient of the multiobjective optimal airfoil can be reduced by 20% to 70% and the wave drag coefficient can be reduced by 60% to 95%. The aerodynamic characteristics get a significant improvement.

(c) The procedure is successful for transonic airfoil designs in which the objective functions are highly nonlinear or discontinuous and is very attractive for practical engineering application because of the quick convergence. It can be extended to the multiobjective optimization design for the airfoils of propeller and rotor blade.

Acknowledgments

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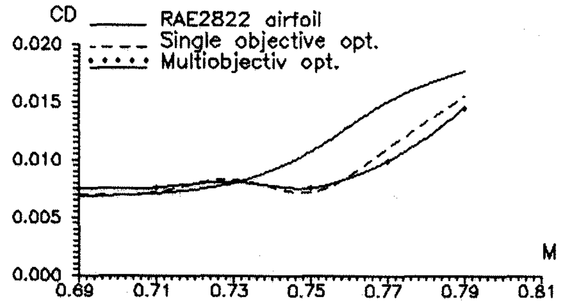

Fig.1 The comparision of drag coefficients for initial airfoil and optimal airfoil

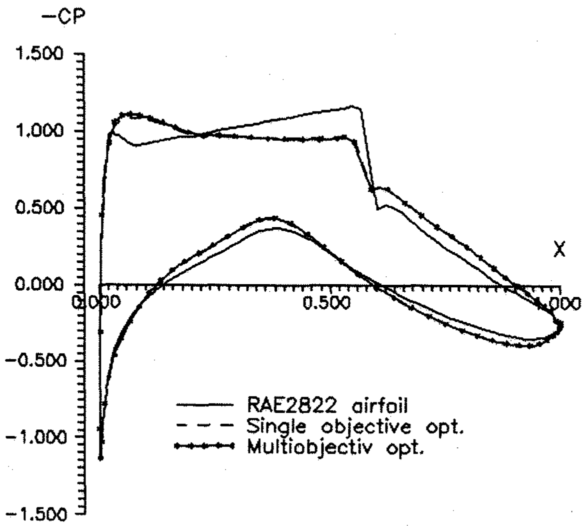

Fig.2 The comparision of pressure distributions for initial airfoil and optimal airfoil

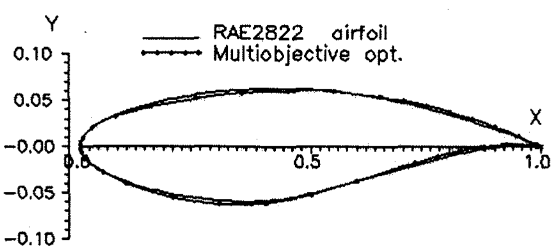

Fig.3 The comparision of goemetries for initial airfoil and optimal airfoil

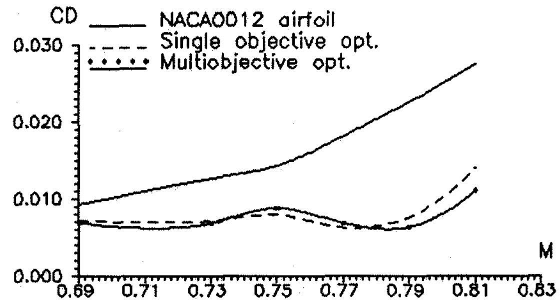

Fig.4 The comparision of drag coefficients for initial airfoil and optimal airfoil

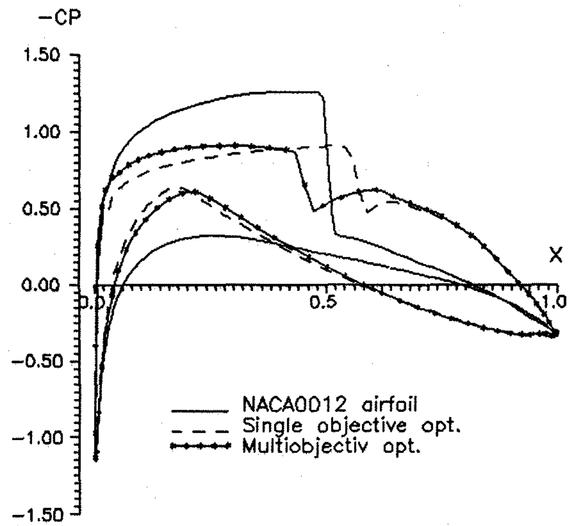

Fig.5 The comparision of pressure distributions for initial airfoil and optimal airfoil

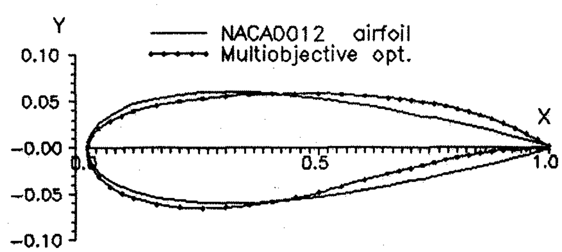

Fig.6 The comparision of goemetries for initial airfoil and optimal airfoil