EFFECT OF NON-LINEARITIES ON THRUST VECTORING

ICAS-94-10.7.1

Péter GRÁNÁSY
PhD student
Technical University of Budapest
Hungary

Abstract: **Poststall manoeuvrability has become one of the important aspects project of military aircraft development. It can be controlled by thrust vectoring system. The motion of aircraft, in a simplified case, can be described by 6DOF system. This model uses a completely non-linear approach. None of this has not been published yet. It concentrates on effect on non-linearities on the aircraft motion using thrust vector control system. The investigation made about longitudinal motion. The results showed it is possible to reach the poststall domain and the aircraft is able to hold this position for a longer period. There is a unstable region at small pitch vector angle and mid thrust. The system has a little damped oscillation beyond the stall. The lift and moment hysteresis make unpredictable the lift and the pitching moment during a poststall. The lift changes about 10% while the moment change reached 400% at the maximum. Static hysteresis was not considered in this study.**

Nomenclature

b = reference span, m
c_D = drag coefficient, dimensionless
$\bar{c}$ = reference mean aerodynamic chord, m
c_L = lift coefficient, dimensionless
c_{L0} = static lift coefficient, dimensionless
c_{Lq} = lift derivative with respect to pitch rate
$c_{L\alpha}$ = lift derivative with respect to α
c_l = rolling moment coefficient, dimensionless
$c_{l\beta}$ = rolling moment derivative with respect to sideslip angle, 1/rad
c_{lp} = roll damping derivative, 1/rad
c_{lr} = rolling moment derivative with respect to yaw rate, 1/rad
c_m = pitching moment coefficient, dimensionless
c_{m0} = static pitching moment coefficient, dimensionless
c_{mq} = pitching moment derivative with respect to pitch rate
$c_{m\alpha}$ = pitching moment derivative with respect to α
c_n = yawing moment coefficient, dimensionless
$c_{n\beta}$ = yawing moment derivative with respect to sideslip angle, 1/rad
c_{np} = yawing moment derivative with respect to roll rate, 1/rad
c_{nr} = yaw damping derivative, 1/rad
c_y = side force coefficient, dimensionless
$c_{n\beta}$ = asymmetric side force derivative with respect to sideslip angle, 1/rad
c_{np} = side force derivative with respect to roll rate, 1/rad
c_{nr} = side force derivative with respect to yaw rate, 1/rad
g = gravitational constant, m/s^2
l_x = distance between centre of gravity and aerodynamic centre along x-axis, m
l_z = distance between centre of gravity and aerodynamic centre along z-axis, m
l_{xe} = distance between centre of gravity and engine thrust centre along x-axis, m
I_x = moment of inertia about the roll axis, kgm^2
I_{xz} = cross product of inertia between roll and yaw axis, kgm^2
I_y = moment of inertia about the pitch axis kgm^2
I_z = moment of inertia about the yaw axis kgm^2
m = aircraft mass, kg
p = roll rate, rad/s
q = pitch rate, rad/s
$\bar{q}$ = dynamic pressure, N/m^2
r = yaw rate, rad/s
S = reference area, m^2
T = thrust, N
u = x axis speed in body axis system, (also v_x), m/s
v = y axis speed in body axis system, m/s
w = z axis speed in body axis system, (also v_z), m/s
α = angle of attack, deg or rad (AoA)
β = sideslip angle, deg or rad
δ_{vp} = pitch vector thrust angle, deg or rad
δ_{vy} = yaw thrust vector angle, deg or rad
θ = pitch angle, deg or rad
ϕ = bank angle, deg or rad

Introduction

It's a need to enhance the manoeuvrability of the fighter aircraft's. The conventional aircraft is reaching its final stage. So a new approach has to be developed.
The new concept for controlling the aircraft is vectored thrust. The aircraft is controlled by turning the thrust vector and in this way generates moments about different axis instead of generating them by conventional aerodynamic surfaces (elevator, aileron, rudder). It can be used alone or with the conventional ones as well.
The advantage's of this control are at low speed it doesn't become ineffective, because of the small effect of speed on thrust and beyond the stall the conventional surfaces can't be used. Its agility was studied by different authors [1][2][3][4]. It mades possible poststall manoeuvres, which enhance the characteristics of the aircraft and make the aircraft able to do unexpected motions.
This paper doesn't want to debate agility. The purpose is to study the thrust vector characteristics of a given aircraft in the poststall domain and effects of non-linearities.

Vectored thrust aircraft

The idea originated from the VTOL aircraft (Harrier, Yak-36, Yak-38), that turned the thrust downward so lifting the aircraft from the ground. For the correct movement there should not be any moments around any axis. If there is a moment then the aircraft will turn around that axis. This can be used for controlling the aircraft.
For using this concepts there are two different possibilities: one is putting paddles on the nozzle (X-31), the other one is integrating this feature into the nozzle (axisymmetric, 2D-CD, SERN) [5].
To prove the advantages of this aircraft several tests and investigations have been made. The following results were obtained by computer simulation: at H=10900m and v=0.9M the killing ratio was 3.55:1 for the vectored thrust aircraft against a conventional one and at H=1500m and v=0.5M this ratio was up to 8.1:1 [6].
Flight testing has started. There were several projects flying using F-15, F/A-18, X-31, YF-22 (which crashed during landing [7]). Some results for example are: shortening the take off ground roll from 1100ft. to 900ft. with F-15 [8], stabilising X-31 at AoA 75° and reaching up to AoA 84° [9].
This February almost a hundred close-in combat engagements were made with an X-31 against an F/A-18. When the X-31 used poststall capabilities the killing ratio was 9.6:1. This ratio was 1:2.4 when it did not use these capabilities. In this program started to model the removal the vertical tail[10].
The Russian plan to install a vectoring nozzle on the Su-27.

Flight Domain of Vectored Thrust Aircraft

The characteristics of this kind of aircraft make possible use the flight domains that haven't been allowed previously because of the structural limit, human factor and flight control constraints.
It has made a redefinition of flight domain[11]. The thrust vector control has a special role in close-in combat engagements and in air-to-ground operations. To get this features the aircraft has to have thrust:weight ratio at least 1:1, a fly-by-wire system, full engine power up to AoA 70°, low wing loading and high wing sweep angle[12].
In a former study [13] it was found there are steady state flights from 0-0.9M with different thrust , angle of attack and pitch thrust angle. Steady turns were investigated with a simple model also. It was found the pitch vector angle was in a range -20°-20°.

Mathematical model

Some models describe the motion of the aircraft [14] [15] [16] based on the same two vector equations. The only differ is from the co-ordinate system representation. A study[17] made about minimum time turns with vectored thrust. It used an optimization method to get the trajectories, but the constraints are quite different. It applied a stall limit and did not use any limit for thrust vector or bank angle.
During modeling of the aircraft motion I have used Euler parameters to avoid singularities at high pitch angles.
The used model is the following[16].

$$\dot{u} = rv - qw - g\sin\theta + X + \frac{Tx}{m} \quad (1)$$

$$\dot{v} = pw - ru + g\cos\theta\sin\phi + Y + \frac{Ty}{m} \quad (2)$$

$$\dot{w} = qu - p\dot{v} + g\cos\theta\cos\phi + Z + \frac{Tz}{m} \quad (3)$$

$$\dot{p} = C41pq + C42qr + C43FR + C^{*}FP \quad (4)$$

$$\dot{q} = C51pr + C52(r^2 - p^2) + FQ - \frac{Tz}{Iy}lxe \quad (5)$$

$$\dot{r} = C61pq + C62qr + C63FP + C^{*}FR + \frac{C^{*}}{Iz}lxeTy \quad (6)$$

$$L = \bar{q}Sc_D \quad (7)$$

$$D = \bar{q}Sc_L \quad (8)$$

$$X = [-D\cos a + L\sin a]/m \quad (9)$$

$$Y = \bar{q}Sc_y \quad (10)$$

$$Z = [-D\sin a - L\cos a]/m \quad (11)$$

$$FP = (\bar{q}SbC_l - ml_zY)/I_x \quad (12)$$

$$FQ = (\bar{q}S\bar{c}C_m + m(l_zX - l_xZ))/I_y \quad (13)$$

$$FR = (\bar{q}SbC_n + ml_xY)/I_z \quad (14)$$

$$c_L = c_{L0} + \frac{\bar{c}}{2V}(c_{Lq}q + c_{L\dot{\alpha}}\dot{\alpha}) \quad (15)$$

$$c_m = c_{m0} + \frac{\bar{c}}{2V}(c_{mq}q + cm_{m\dot{\alpha}}\dot{\alpha}) \quad (16)$$

$$c_y = c_{y0} + c_{y\beta}\beta + \frac{\bar{b}}{2V}(c_{yp}p + c_{yr}r) \quad (17)$$

$$c_l = c_{l0} + c_{l\beta}\beta + \frac{b}{2V}(c_{lp}p + c_{lr}r) \quad (18)$$

$$c_n = c_{n0} + c_{n\beta}\beta + \frac{b}{2V}(c_{np}p + c_{nr}r) \quad (19)$$

$$\dot{a} = \frac{u\dot{w} - \dot{u}w}{u^2 + w^2} \quad (20)$$

$$T_x = T\cos\delta_{vp}\cos\delta_{vy} \quad (21)$$

$$T_y = T\cos\delta_{vp}\sin\delta_{vy} \quad (22)$$

$$T_z = T\sin\delta_{vp} \quad (23)$$

where

$$C^{*} = I_xI_z/\left(I_xI_z - I_{xz}^2\right) \quad (24)$$

$$C_{41} = I_{xz}\left(I_z + I_x - I_y\right)/\left(I_xI_z - I_{xz}^2\right) \quad (25)$$

$$C_{42} = \left(I_z\left(I_y - I_z\right) - I_{xz}^2\right)/\left(I_xI_z - I_{xz}^2\right) \quad (26)$$

$$C_{43} = I_xI_{xz}/\left(I_xI_z - I_{xz}^2\right) \quad (27)$$

$$C_{51} = \left(I_z - I_x\right)I_y \quad (28)$$

$$C_{52} = I_{xz}/I_y \quad (29)$$

$$C_{61} = \left(I_x\left(I_x - I_y\right) + I_{xz}^2\right)/\left(I_xI_z - I_{xz}^2\right) \quad (30)$$

$$C_{62} = I_{xz}\left(I_y - I_z - I_x\right)/\left(I_xI_z - I_{xz}^2\right) \quad (31)$$

$$C_{63} = I_xI_{xz}/\left(I_xI_z - I_{xz}^2\right) \quad (32)$$

Instead of using (1) and (3) as they stand we obtain sorted equations if we substitute X, D, L, c_D and c_L into (1) and Z, D, L, c_D and c_L into (3). Defining

$$C_b = \frac{\bar{q}S\frac{\bar{c}}{2V}c_{L\alpha}}{m\left(u^2 + w^2\right)} \quad (33)$$

and

$$B_q = 1 + \frac{uC_b\cos\alpha}{1 + wC_b\sin\alpha} \quad (34)$$

we got

$$\dot{w} = \frac{F_w + \dfrac{F_u wC_b\cos\alpha}{1 + wC_b\sin\alpha}}{B_q} \quad (35)$$

$$\dot{u} = \frac{F_u + u\dot{w}C_b\sin\alpha}{1 + wC_b\sin\alpha} \quad (36)$$

where

$$F_u = rv - qw - g\sin\theta - D\cos\alpha + \frac{T_x}{m} + \frac{\bar{q}S}{m}\sin\alpha\left(c_{L0} + \frac{\bar{c}}{2V}c_{Lq}q\right) \quad (37)$$

$$F_w = qu - pv + g\cos\theta\cos\phi - D\sin\alpha + \frac{T_z}{m} - \frac{\bar{q}S}{m}\cos\alpha\left(c_{L0} + \frac{\bar{c}}{2V}c_{Lq}q\right) \quad (38)$$

If we use unsorted equations and a small time step the effect of using the last value instead of the current value has only a small effect.

Effect of non-linearities on the motion of the aircraft

First I tried to compare a linerized and a non-linear system to understand what really happens. The linearized system with a constant pitch vector angle was not able to hold a poststall position and the non-linear system has a slightly damped oscillation around AoA 46°. The damping was so small that to stabilise takes at least 6minutes. There was big changes in the coefficients that the such system with linearization was not able to follow. It can give good results at small a angle of attack, but around and beyond the stall it is unusable.

This means that the linearized (using constant and first order function to interpolate c_D, c_L, c_{Lq}, $c_{L\alpha}$, c_m, c_{mq}, $c_{m\alpha}$) comparison with the non-linear is not worthwile.
Then the model was basically linearized, but after interpolation of the coefficient ($c_{L\alpha}(\alpha)$, $c_{Lq}(\alpha)$, $c_{mq}(\alpha)$ etc.) we got a highly non-linear system. The interpolation was made by atan(α) functions. The advantage of this interpolation is there is no place where the derivative of this function does not exist.
This equatiom system is very sensitive. Some numerical methods fail during getting the trim conditions.
The resultant vector angle was used everywhere, but the resultant and the geometric vector angle are not the same (17) (18) and depends on the nozzle pressure ratio, which in this case was constant. There is no effect on thrust vectoring by AoA according to (19).
Figure 1 and 2 describe a motion after pulling the stick 10° pitch vector angle with 35kN thrust from a trim condition at 0.3M.

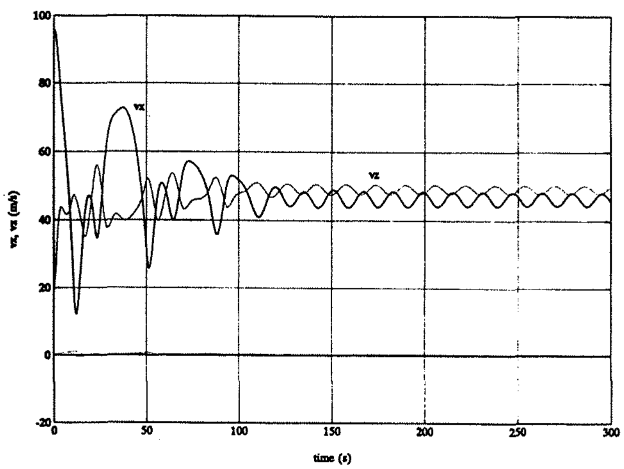

Fig.1.

In Figure 1. there are the changes of u and w. About 1 minute is needed to get through the transient zone and after that it starts an oscillation with small damping and about 30m/s vertical speed down.

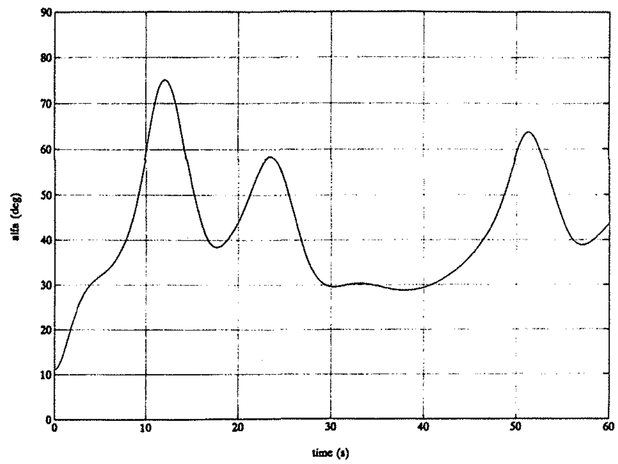

Fig.2.

I got very interesting results when I recorded the lift and pitching moment coefficient versus angle of attack. As can be seen above I did not use static hysteresis, which is the effect of detaching and reattaching of the flow around the wing.
Figure 3. shows the lift coefficient as the function of the angle of attack. We found a hysteresis loop beyond the stall where the biggest difference is around 10%.

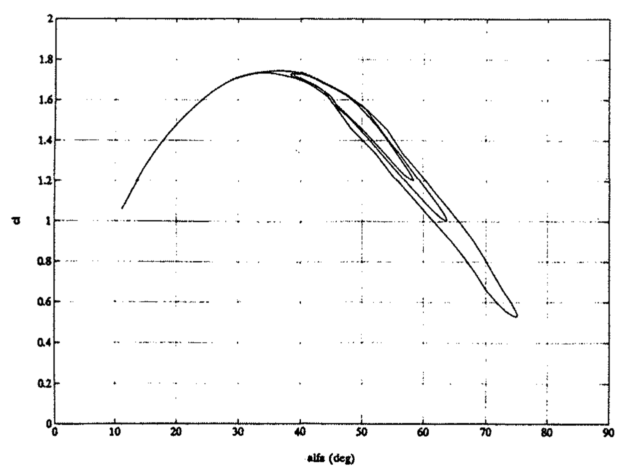

Fig.3.

In the Figure 4. there is the pitching moment coefficient versus angle of attack. Around the stall there is point where the maximum pitching moment value is four times bigger than the minimum value.

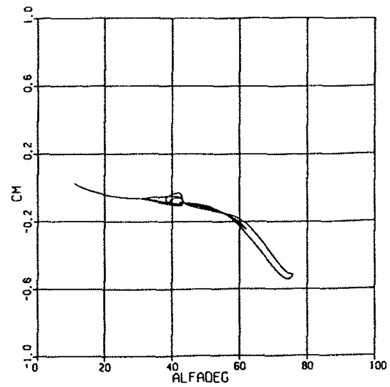

Fig.4.

On Figure 5. there is a stall made by conventional control surfaces. There is a bigger hysteresis loop around the stall. The constant pitch moment causes that while the conventional surfaces has decreasing of moment during the stall.

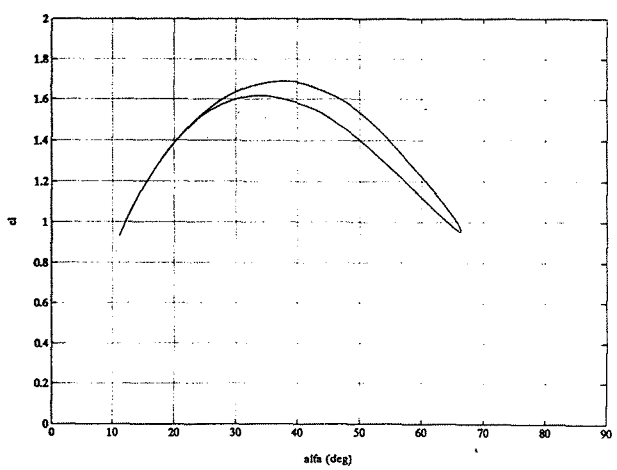

Fig.5.

This effect can cause some difficulties to identify where the aircraft really is.

As we can see on Figure 6. there are five different domains for a vectored aircraft. There is a thrust limit, under this there is no level flight. In all the region the thrust deflection produces an oscillation. This oscillation gives limit cycles, slightly damped and unstable regions.

At small vector angles and thrust the aircraft is moving before stall. There is a region where the aircraft oscillate between the poststall and before stall region. Between this two region there is another one where is the motiom of the aircraft is unstable.

Over the "before and poststall region" there is limit cycle poststall motion. In some point there is slightly damping, in eigenvalues is about 10E-5.

If we use too big thrust deflection we "overpull" the aircraft and we get the over 90° region.

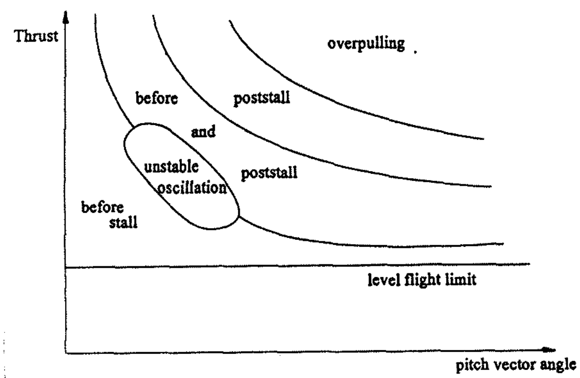

Fig.6.

Conclusions

1. There is an unstable flight domain at mid thrust and small vane deflection where the aircraft starts an unstable oscillation.
2. The pilot is able to stabilise the aircraft in poststall region. It needs enough big thrust and pitch thrust deflection.
3. There is a danger to "overpull" the aircraft and get to the flight domain over 90°.

4.The hysteresis has bigger effect on conventional surfaces, but it has significant effect in some vectored thrust cases.

Special thanks to my supervisor Prof.Rohacs at Technical University of Budapest and Mr.Thomasson at Cranfield Institute of Technology.

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