2ND ORDER NND SCHEME AND BOUNDARY LAYER CALCULATION

Liu Sen Lin Chaoqiang
Dept. Aircraft Engineering, Northwestern Polytech. Univ.
Xi' an, 710072, *P. R. China*

Abstract

As a new step to investigate the behaviour of TVD schemes in the calculation of viscous boundary layer, 2—D N—S equation has been solved with the 2—D flat plate laminar boundary layer and the interaction of oblique shock wave/2—D flat plate laminar boundary layer as test cases. It has been found that the original 2nd order NND scheme (Non-oscillatory, Non-free-parameter, Dissipative scheme, a kind of TVD scheme) is not suitable for the solution of boundary layer since it over-estimates the skin friction, even though it is very efficient to capture shock wave. The authors attribute this phenomenon to the improper numerical dissipation of the original NND scheme. By modifying the minmod terms of original scheme, a 3rd order improved NND scheme is obtained, which yields much better results than that of 2nd order NND scheme.

Introduction

From early 1980s, the solution of Euler equation developed in two directions, namely the central schemes (eg. Jameson's central schemes) and TVD schemes. The emergence of TVD schemes proposed by Harten, Yee(1), Osher and Chakravathy(2) are basically a milestone of numerical solution of Euler equation. In China, the representative work is the NND scheme(3) developed by Prof. Zhang H. X. of CARDC (China Aerodynamics Research and Development Center). Large amount of numerical works have shown that the NND scheme (virtually a TVD scheme) is both accurate and efficient for the solution of inviscid supersonic and hypersonic flows, which may include complex shock waves and expansion waves(4,5,6,7).

In resent years, the application of NND scheme has been extanded from inviscid flows to viscous flows around complicated configurations. However, at the very beginning, the objective to develop NND (or TVD) scheme is just to capture shock waves in a flow field, not to calculate viscous boundary layer; Numerical experiences have also told us that not all the methods suitable for inviscid flows are still suitable for viscous flows. As mentioned in Reference 8 that the improper numerical dissipation of upwind discretization may deteriorate the solution of viscous shear layer. Thus, as a basic research, the problems of 2—D flat plate laminar boundary layer and interaction of oblique shock wave/2—D flat plate laminar boundary layer are studied here by solving the N—S equation in LU-SGS (Lower-Upper-Symmetric-

Gauss-Seidel) approach. The results show that the original 2nd order NND scheme is not suitable for bounday layer calculation, yet the results could be greatly improved if the NND scheme is modified to 3rd order accurate.

Numerical Approach

Governing Equation

The governing equation here is the nondimensionalized 2—D Navier-stokes equation in conservative form:

$$\frac{\partial Q}{\partial \tau}+\frac{\partial E}{\partial \xi}+\frac{\partial F}{\partial \eta}=\frac{1}{Re}\left(\frac{\partial E_v}{\partial \xi}+\frac{\partial F_v}{\partial \eta}\right) \quad (1)$$

where Q is conservative variable, E and F are inviscid flux in ξ and η directions respectively, while E_v and F_v are viscous flux in ξ and η directions respectively.

Solution Procedure

LU-SGS Approach Eq. (1) is solved in the LU — SGS approach[9,10], whose final discretized equation is written as:

$$LD^{-1}U\delta Q_{ij}^{n+1}=-R_{ij}^{n} \quad (2)$$

where

$$L=\rho I-A_{i-1,j}^{+}-B_{i,j-1}^{+}$$

$$D=\rho I$$

$$U=\rho I+A_{i+1,j}^{-}+B_{i,j+1}^{-}$$

$$\rho=k\cdot[\rho(A)+\rho(B)]$$

$$R=\frac{\partial E}{\partial \xi}+\frac{\partial F}{\partial \eta}-\frac{1}{Re}\left(\frac{\partial E_v}{\partial \xi}+\frac{\partial F_v}{\partial \eta}\right)$$

Here I is unity matrix. $A=\frac{\partial E}{\partial Q}$, $B=\frac{\partial F}{\partial Q}$, $\rho(A)$ and $\rho(B)$ are their spectral radius respectively. k is a constant greater than one. A and B are approximately split as:

$$A^{\pm}=\frac{A\pm k\cdot\rho(A)I}{2}$$

$$B^{\pm}=\frac{B\pm k\cdot\rho(B)I}{2}$$

The solution is performed by sweeping from the bottom-left corner of computational domain to the upper-right corner and back to the bottom-left corner. It is noted that the discretization of $\frac{\partial E}{\partial \xi}+\frac{\partial F}{\partial \eta}$ in R is the main point of this paper.

NND Scheme The discretization of convective terms is written as:

$$\frac{\partial E}{\partial \xi_i}=\frac{1}{\Delta\xi}(\tilde{E}_{i+\frac{1}{2}}-\tilde{E}_{i-\frac{1}{2}}) \quad (3)$$

where

$$\tilde{E}_{i\pm\frac{1}{2}}=E^{+}_{i\pm\frac{1}{2},L}+E^{-}_{i\pm\frac{1}{2},R}$$

$$E^{+}_{i+\frac{1}{2},L}=E_i^{+}+\frac{1}{2}\text{minmod}(\Delta E^{+}_{i-\frac{1}{2}},\Delta E^{+}_{i+\frac{1}{2}}) \quad (4)$$

$$E^{-}_{i+\frac{1}{2},R}=E^{-}_{i+1}-\frac{1}{2}\text{minmod}(\Delta E^{-}_{i+\frac{1}{2}},\Delta E^{-}_{i+\frac{3}{2}}) \quad (5)$$

$$\Delta E^{\pm}_{i+\frac{1}{2}}=E^{\pm}_{i+1}-E^{\pm}_i$$

the limiter is defined as

$$\text{minmod}(x,y)=\begin{cases}0 & x,y<0\\ x & |x|\leqslant|y|\\ y & |x|>|y|\end{cases}$$

It could be verified that this is a 2nd order TVD scheme and is very efficient for flows including shock and expansion waves. It is evident that the scheme degenerates into 1st order upwind scheme when the limiter is taken to be zero.

Improved NND Scheme In order to reduce the numerical dissipation of NND scheme, a high order scheme is obtained by modifying those limiter terms of Eq. (4) and (5):

$$E^{+}_{i+\frac{1}{2},L}=E_i^{+}+\frac{1}{4}[(1-k)\nabla+(1+k)\Delta]\cdot E_i^{+} \quad (7)$$

$$E^-_{i+\frac{1}{2},R}=E^-_{i+1}-\frac{1}{4}[(1-k)\nabla+(1+k)\Delta]E^-_{i+1} \quad (8)$$

Here

$$\nabla E^+_i=\text{minmod}(\Delta E^+_{i-\frac{1}{2}},\beta\cdot\Delta E^+_{i+\frac{1}{2}})$$

$$\Delta E^+_i=\text{minmod}(\beta\cdot\Delta E^+_{i-\frac{1}{2}},\Delta E^+_{i+\frac{1}{2}})$$

$$\nabla E^-_{i+1}=\text{minmod}(\Delta E^-_{i+\frac{1}{2}},\beta\cdot\Delta E^-_{i+\frac{3}{2}})$$

$$\Delta E^-_{i+1}=\text{minmod}(\beta\cdot\Delta E^-_{i+\frac{1}{2}},\Delta E^-_{i+\frac{3}{2}})$$

$$\beta=\frac{3-k}{1-k}$$

if $k=1/3$, the scheme is 3rd order accurate. The new scheme is called improved NND scheme and it is similar to those MUSCL-type high order upwind schemes.

Numerical Results and Discussion

The main purpose of this paper is to check the behaviour of NND scheme in boundary layer calculation. Both the original 2nd order NND scheme and the improved 3rd order NND scheme are applied to the following test cases.

The first case is 2-D flat plate laminar boundary layer. Computational domain is shown in Fig. 1, in which 31 grid points are evenly distributed in x direction and 41 in y direction, clustering near solid wall with minmum normal spacing of 0.001. Free stream conditions are $M_\infty=0.5$ and $Re=10000$. The inflow boundary conditions are iso-entropy, constant total pressure and zero vertical velocity; At upper and outflow boundaries, the static pressure is specified as the free stream value while other variables are extrapolated. Non-slip condition is applied on solid wall.

Fig. 2 shows the skin friction with original NND scheme, which is greater than the Blasius solution. Accordingly, velocity profiles in boundary layer are compared in Fig. 3, showing that the profile of numerical solution is much more flat near the wall than that of Blasius solution. Fig. 4 and 5 are results of 3rd order NND scheme, which show the great improvement of the solution.

The second test case is the interaction of oblique shock wave/2-D flat plate laminar boudary layer, as shown in Fig. 6. Free stream conditions are $M_\infty=2.0$, $Re=0.296\times10^6$, shock impingement angle is 32.585 degree. 61 grid points are evenly spaced in x direction while 61 clustering in y direction with minmum normal spacing of 0.0003 at solid wall. All of the conservative variables are extrapolated at outflow boundary and the solid wall is adiabatic and non-slip.

The skin friction with 2nd and 3rd order NND schemes are shown in Fig. 7 and 8 respectively, where the most important phenomenon is the absence of flow separation in the result of 2nd order NND scheme in Fig. 7 (no negtive skin friction), which agrees with the first test case that this scheme over-estimates the skin friction. Fig. 9 and 10 are the iso-pressure countors corresponding to Fig. 7 and 8 respectively. Numerical experiments have shown that satisfactory results could be obtained if the discretization in y direction is 3rd order accurate, regardless of 2nd or 3rd order in x direction.

Conclusions

Based upon these numerical studies. it has been found that the original 2nd order NND scheme may over-estimate skin friction at a solid wall and result in unreliable study of viscous flows, even though it is very efficient to capture shock waves. The authors would attribute this deterioration of numerical results to the improper numerical dissipation and introduced a 3rd order NND scheme by modifying limiter terms of the original scheme. Numerical experiments have shown better results using this improved scheme in normal direction, regardless of 2nd or 3rd order in streamwise direction. However, theoretical explanation to these conclusions has not been systemetically completed , thus it remains to be an open problem.

References

1 Yee, H. C. , et al, "Implicit Total Variation Diminishing (TVD) Schemes for Steady State Calculations", AIAA paper 83—1902

2 Osher, S. and Chakravarthy, S. R. , "Very High Order Accurate TVD Schemes", UCLA Math Rept. , 1984

3 Zhang, H. X. , "Non-oscillatory, Non-free-parameter, Dissipative (NND) Scheme", Journal of Aerodynamics (in Chinese), Vol. 6, No. 2, 1989

4 Ye, Y. D. et al, "Space Marching in Hypersonic Inviscid Flow Field", Proc. of 5th National Conference on CFD (in Chinese), 1990, pp509—520

5 Guo, Z. Q. , "Numerical Investigation of Inviscid Flows around Complex Reentry Body", Proc. of 5th National Conference on CFD (in Chinese), 1990, PP530—540

6 Yang, Y. J. and Zhang, L. M. , "Inviscid Calculation of Flows around Straop-on Rockets at AOA", Proc. of 3rd Annual Conference on CFD at CARDC (in Chinese), 1991

7 Liu Sen, "Numerical Investigation of Flows Over Shrapnel/shell Discarding Configuration ", Proc. of 18th ICAS, pp1967—1972

8 Van Leer, et al, "A Comparision of Numerical Flux Formation for the Euler and Navier-Stokes Equations", AIAA paper 87—1104

9 Yoon, S. and Jameson A. , "Lower-Upper Symmetric-Gauss-Seidel Method for Euler and Navier-Stokes Equations", AIAA paper 87—0600

10 Liu Sen, "Numerical Investigation of Unsteady Transonic Viscous Flows Around Pitching Airfoil ", Ph. D Dissertation of Northwestern Polytechnical University (NPU), 1994

11 Hakkinen, R. J., et al, "The Interaction of An Oblique Shock Wave With a Laminar Boundary Layer", NASA Memo 2—18—59w, March 1959

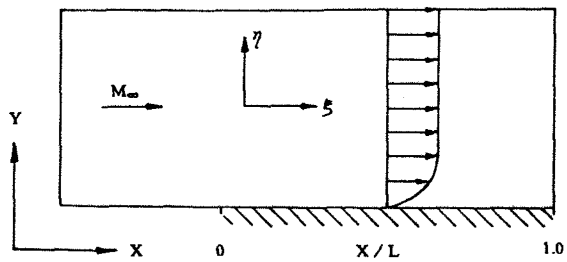

Fig. 1 Computational Domain of Test Case 1

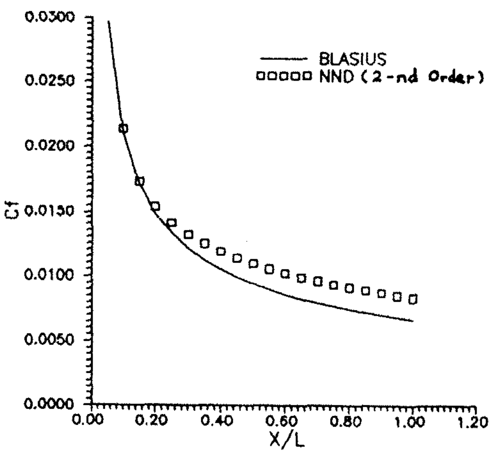

Fig. 2 Skin Friction with 2nd Order NND Scheme

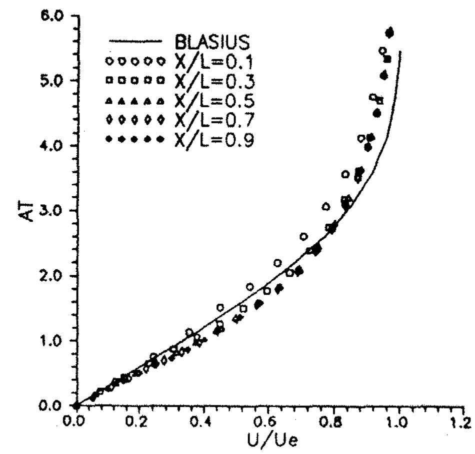

Fig. 3 Velocity Profile with 2nd Order NND Scheme

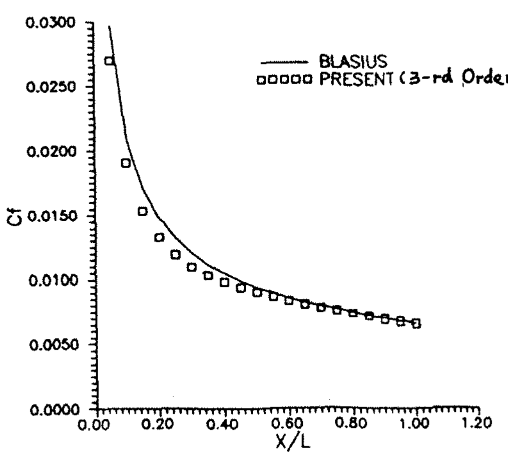

Fig. 4 Skin Friction with 3rd order NND scheme

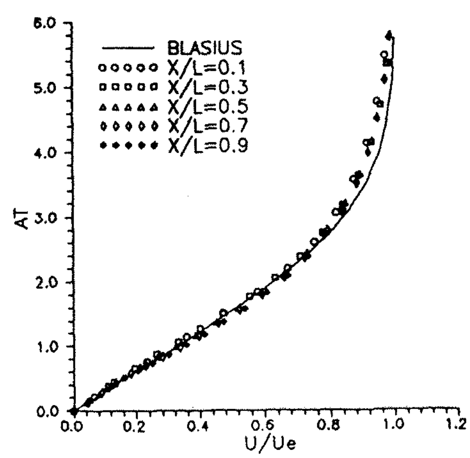

Fig. 5 Velocity Profile with 3rd Order NND Scheme

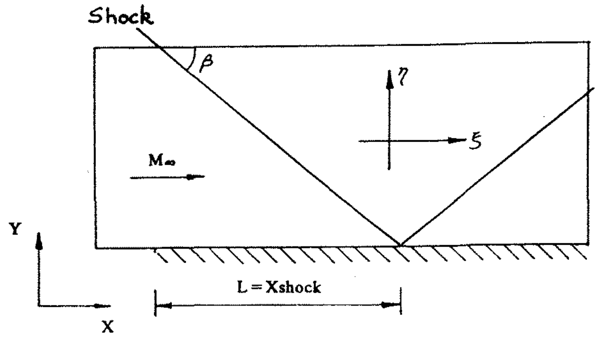

Fig. 6 Computational Domain of Test Case 2

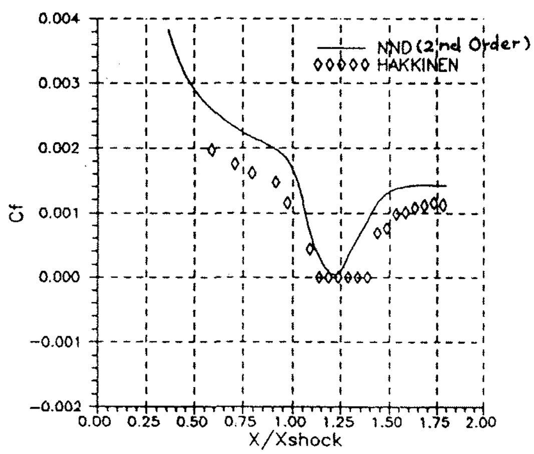

Fig. 7 Skin Friction with 2nd Order NND Scheme

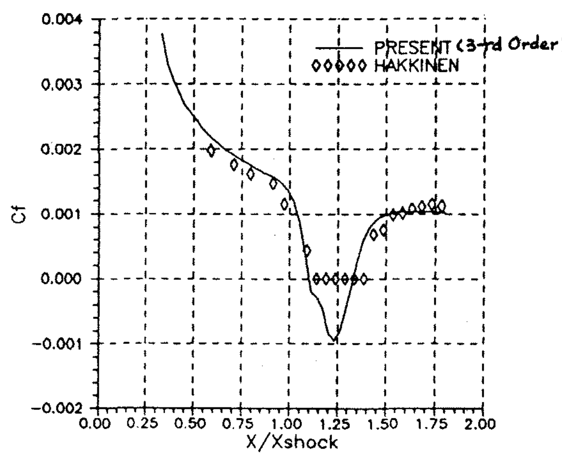

Fig. 8 Skin Friction with 3rd Order NND Scheme

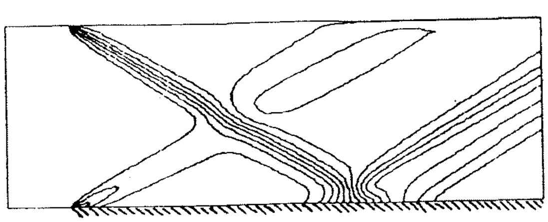

Fig. 9 Iso—pressure Countor with 2nd order NND Scheme

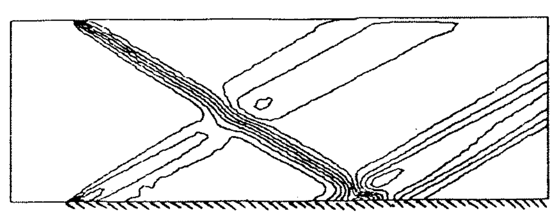

Fig. 10 Iso-pressure Countor with 3rd order NND Scheme