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A METHOD FOR OPTIMIZING THE MERIDIONAL PASSAGE OF THE ROTOR IN CENTRIFUGAL COMPRESSORS

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Abstract

Based on the S_1,S_2 stream filament theory, a method is developed to generate an optimized meridional passage of the rotor in centrifugal compressors. Two optimization principles are adapted. First, the gradient of static pressure in stream line direction, which is mainly responsible for the boundary layer separation at the hub and shroud, should be minimized. Second, the gradient of static pressure from hub to shroud, which is generally considered the major reason of triggering the secondary flow in the passage, should be reduced as well. According to these two principles, a reasonable criterion is set up. In order to develop a computer code to generate the optimized meridional passage automatically, two practical schemes are used. The comparison of the static pressure distributions at hub and shroud between the initial and the optimized passages indicates that the methods achieved the anticipated result.

Introduction

The prevailing method of designing centrifugal compressors and blowers now is experiential based on a kind of one dimensional flow theory. Generally, this process proceeds a series of design-experiment-redesign-experiment circles until a reasonable result is reached. Thus ,it is wasteful both in time and manpower. With the highly advanced computer technology and the inner flow theory, it is possible to substitute those onerous works by using computer software packages. In order to establish a powerful centrifugal compressor CAD package, it is necessary to hand out an optimized meridional passage after the general geometries are determined.

Certain theories on flow lose discussed in reference (1) are inspiring, but are not enough to set up a practical formula for optimizing the meridional passages. Designer's experiences are still overwhelming. A general 3-dimensional flow theory on turbomachinery was set up in reference (2). Based on this theory, numerical computation methods and computer codes were developed in reference (3),(4),(5),(6), which can give a much more reasonable, more detailed analysis of the flow field than 1-dimensional theory and, therefore, can be used as a more powerful tool to handle the problem of meridional passage optimization. several main purposes should be accessed here: 1) To set up a quantitative optimization criterion, which can be used in computer programming directly, to qualify the meridional passage according to today's turbomachinery theory about inner flow and its loses; 2) To apply this criterion to develop a computer code which can generates an optimized meridional passage automatically; 3) To compare the static pressure distributions in the flow field before the optimization process with the distributions after the passage has been optimized to demonstrate whether the object of optimization is achieved actually.

Optimization Criterion

A generally accepted theory in the optimization of the meridional passage in centrifugal compressors is that to minimize the jet-wake structure can improve the efficiency of the compressors(1). Further researches indicate that the secondary flow and the boundary layer separation are mainly responsible for generating the jet-wake structure. In order to diminish the secondary flow , to alleviate the static pressure gradient in shroud-to-hub direction is necessary. A formula is presented to create the shroud generatrix on the basis of this consideration(1). But this formula is not enough to generate a reasonable and completed meridional passage. Experience is still necessary. Indeed, there must be an abrupt turn in the middle way of the passage by using this formula alone. That swerve will course a large pressure gradient in stream line direction, which will trigger the boundary layer separation and, hence, enlarge the flow lose. Therefore,it is not enough to optimize the meridional passage only through appeasing the static pressure gradient in shroud-to-hub direction.

According to the reasons above, it is necessary to alleviate the static pressure gradient in two directions (normal and stream line direction) simultaneously. Thus, we express the general optimization principle as follows: After the main sizes of the rotor have been determined, to minimize the static pressure gradient in two directions (shroud-to-hub and stream line directions) in the flow field by means of adjusting the geometry shape of the passage to reach the purpose of weakening the secondary flow and boundary layer separation.

Then, the relative quantitative criterion, which will be used in the computer code, is given as:

$$\Delta P_m = |\Delta P_{lm}| + |\Delta P_{nm}| \quad (1)$$

where, $\Delta P_{lm}, \Delta P_{nm}$ represent the largest pressure gradient in normal and stream line directions respectively. Thus, the optimization means that ΔP_m should be adjusted to its lowest point. Of course, (1) is not the unique form of the criterion.

Two Practical Schemes

Optimization is actually a sifting process in which the meridional passage will be modified again and again. Two practical schemes are introduced to generate a series of passage shapes and sift the optimized passage automatically.

1. Updating According to An Anticipated Static Pressure Distribution.

An anticipated static pressure distribution should be given by the user first. For example,

$\frac{\partial P}{\partial n}$ (gradient in normal direction) = 0, and

$\frac{\partial P}{\partial l}$ (gradient in stream line direction) = constant,

or other values based on the user's experiences. After the initial passage shape is given, change the shape repeatedly according to the given pressure distribution until the actual value is similar to the anticipated. Of course, generally, since there are certain limitations in geometric aspects, the two distributions, actual and anticipated, can not reach the same values, but there should be a lowest point in a series of ΔP_m values, and at this time the relative passage will be considered as the optimized. Quantitative analysis is as follows:

If P_s^n, P_h^n represent the local static pressure at shroud and hub respectively at nth step, P_{av} the local average pressure from shroud to hub, and P_l the anticipated local pressure value. Then the next time,

$$\begin{aligned} &\text{Shroud} \quad P_s^{n+1} = P_s^n + 0.5\left(\frac{P_{av}+P_l}{2} - P_s^n\right) \\ &\text{Hub} \quad P_h^{n+1} = P_h^n + 0.5\left(\frac{P_{av}+P_l}{2} - P_h^n\right) \end{aligned} \quad (2)$$

According to the thermodynamic relations, we have

$$\begin{aligned} W_s^{n+1} &= \left\{ (W_s^n)^2 + \frac{2k}{A(k-1)}\left[(P_s^n)^{(k-1)/k} - (P_s^{n+1})^{(k-1)/k} \right] \right\}^{0.5} \\ W_h^{n+1} &= \left\{ (W_h^n)^2 + \frac{2k}{A(k-1)}\left[(P_h^n)^{(k-1)/k} - (P_h^{n+1})^{(k-1)/k} \right] \right\}^{0.5} \end{aligned} \quad (3)$$

where $A = T_{i0}^{1/k} e^{-\Delta s/k}$, Δs is the entropy increment, T_{i0} the total temperature at the inlet, and $k=C_p/C_v$. From the continuity equation(2),

$$\frac{\partial}{\partial x^1}(\tau\rho\sqrt{a22}\ \sin\theta_{12} W^1) + \frac{\partial}{\partial x^2}(\tau\rho\sqrt{a11}\ \sin\theta_{12} W^2) = 0 \quad (4)$$

following relations can be derived

$$\begin{aligned} R_s^{n+1} &= R_s^n + f \cos\theta_2\ (W_s^n/W_s^{n+1} - 1) \\ R_h^{n+1} &= R_h^n - f \cos\theta_2\ (W_h^n/W_h^{n+1} - 1) \\ Z_s^{n+1} &= Z_s^n - f \sin\theta_2\ (W_s^n/W_s^{n+1} - 1) \\ Z_h^{n+1} &= Z_h^n + f \sin\theta_2\ (W_h^n/W_h^{n+1} - 1) \end{aligned} \quad (5)$$

where R,Z refer to the radial and axial coordinate respectively, f is a factor similar to a relaxation number. Other parameters are showed in figure 1.

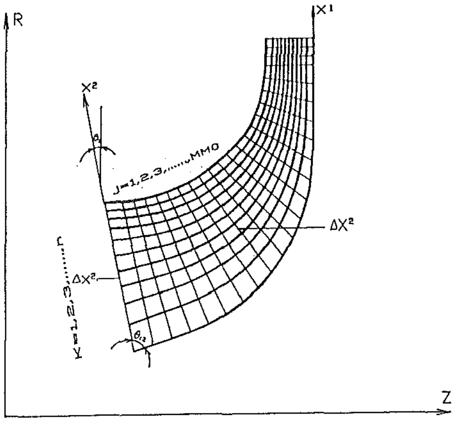

Figure 1. Coordinate and Numerical Grid

2. Updating by Using Quadratic Curves

The generatrix of the meridional passage is requested to be smooth curves, especially quadratic curves, in many cases of the technological process. The quadratic equation formed as follows is recommended here

$$f(X,Y) = X^2 + BY^2 + CXY + DX + EY + F = 0 \quad (6)$$

where the constants, B,D,E,F, can be determined according to the four conditions at the inlet and outlet of the passage

$$\begin{aligned} &\text{continuity} \quad f(X_1,Y_1) = 0 \\ &\qquad\qquad\quad\ f(X_2,Y_2) = 0 \\ &\text{smoothness} \quad f^1(X_1,Y_1) = \left(\frac{\partial f}{\partial X}dX + \frac{\partial f}{\partial Y}dY\right)\Big|_{X_1,Y_1} = C_1 \\ &\qquad\qquad\quad\ f^1(X_2,Y_2) = \left(\frac{\partial f}{\partial X}dX + \frac{\partial f}{\partial Y}dY\right)\Big|_{X_2,Y_2} = C_2 \end{aligned} \quad (7)$$

where subscript "1","2" refer to the values at the inlet and outlet respectively, C_1 ,C_2, which is related to the slope at the inlet and outlet, can be given first by the user. Then, the constant, C, should be determined according to the optimization criterion. The coordinates, R,Z, can be derived through relations

$$R = (R_1-R_2)Y+R_1$$
$$Z =-(Z_1+Z_2)X+Z_2 \qquad (8)$$

In the course of this sifting, after each passage shape has been given, the S2 inverse problem computer code(6) should be used to predict the flow field, present the static pressure distribution and the ΔP_m values. In order to solve the S2 inverse problem more exactly, as we know, a reasonable V_uR distribution , which refers to the load distribution in the rotor of the compressor, should be given in advance. Since the designers usually have a rough ideal about the load distribution at the beginning, to estimate the V_uR values is not difficult.

Results

Two examples are used to verify the two schemes respectively.

Figure 2a presents a comparison of the meridional passages between the initial and the optimized, while figure 2b shows the two static pressure distributions at shroud and hub corresponding to the two passages. This optimization process is proceeded by using the first scheme. Lineal static pressure distribution is assumed.

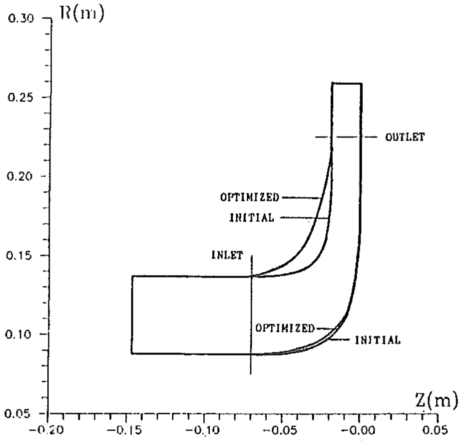

Figure 2a. Meridional Passages

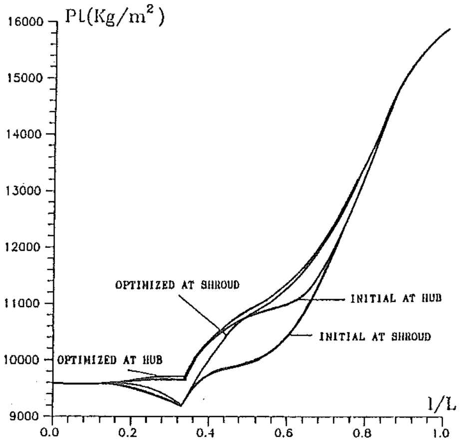

Figure 2b. Static Pressure Distributions

Figure 3a, 3b present the same comparisons. This process is proceeded by using the second scheme.

All two examples shows that the main purpose is reached. The pressure gradients are obviously appeased both in shroud-to-hub direction and in stream line direction.

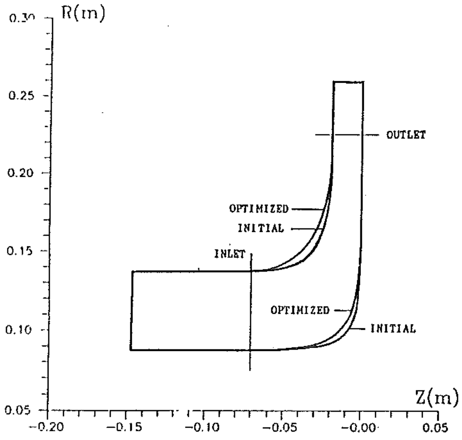

Figure 3a. Meridional Passages

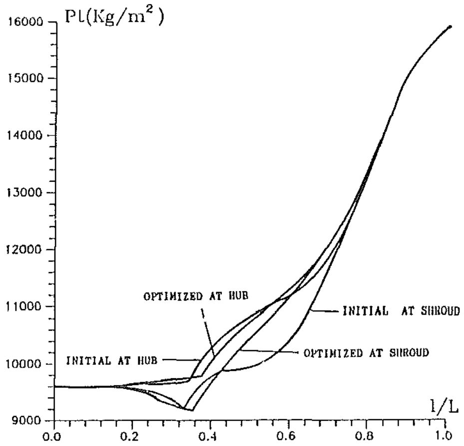

Figure 3b. Static Pressure Distributions

Conclusions

To compare the two optimization schemes, the first reaches a seemingly better result while the second is more practical and convenient.

Theoretically, the results acquired from the schemes above are not of necessarily the best passage, and after the impellers in the rotors are determined, the V_uR distribution should be revised according to solving the S1 direct problems. But the computation practices indicate that there is an "obtuse" region including the "optimum point". If the load distribution is estimated in comparative precision, the method recommended here is a useful way to attain an optimized geometric shape.

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