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THE RECEARCH OF THE INFLUENCE OF TEMPERATURE LOADS ON THE DEPLOYMENT OF SPACE LONG PANEL STRUCTURES

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Abstract

The creation of constractions which might be unfold in orbit with various technical purposes and parameters tukns to be one of the main development path for space oriented technologies.

The main task of construction deploying in space is accentuated on eliminating temperature strains in panels caused solar radionce and thus different heat levels of the frow and below panel surfaces.

The paper contains the results of temperature effects on constraction elements hing joints and strain compensation methods taking plase in panels during constructure deploying. The construction math model is provided and the temperature difference of panel surfoces are colculated. The strain function in joints and the panels scrolling axis deformation as a function of coordinates and the rotation angle of construction elements are calculated. On the basis of calculation results the strain compensation method is provided by resilient joint construction. At the same time this joint provides the strictness of the whole construction according technikal requirments at the end of the deploying process.

1. The review of space constructions and the aspects of their deployment

One of the most promising directions of space technology is the development of orbital universal collapsable structures. Because of the strict limitotions on dimensions of spacecraft cargo compartment these constructions are being delivered to the Earth orbit in furled state. In this connection they may be classified, in terms of the way of their installation, into three groups: prefobricated, deployable and made-on-orbit structures.

Prefabricated space constructions are transported to orbit as a set of large number of unified pieces and units. Then a robot-manipulator or the cosmonauts assemble a predetermined construction. Such a method requires sophisticated systems and equipment that, in its turn, provide an excess weight and reduce the space for pay load and reliability. Involvement of men for assembly is labourous and quite ineffective.

Among the made-on-orbit construstions are the structures made of composite materials. In this case the thermositting of matrix and binder lasts during the shaping. This method is rather complex and isn't yet completely elaborated. In addition, it isn't suitable for structures "filled" with electronic equipment.

The most developed is the third type of constructions - the space deployable structures. Among these are framework and rigid-panel constructions, that are designed to be deployed in an orbit by means of an electrical or thermomechanical drive.

The most popular are panel deployable constructions such as solar power supply arroys, phosed antenna arrays etc.

Set us consider the deploying of a large space rigid-panel construction. The fragments of this structure are to be unfolded in two perpendicular directions: along the longitudinal axis and along the transversal one of ready construction. The process of deploying has several successive phases. And at second or at third stage the structure must be unfolded along the longest joint of a construction.

The problems of a long space structure deployment derive from the phenomena of panel deformations caused by many facts. Among tgese are: errors of technology and manufacturing, technique of assembly and justification, mechanical loods (dynamic loads when launching and tragectory alterations, vibrations from a jet engine etc.), temperature gradients due to solar radiation. At present it is indicated, that the most considerable deformations result from the temperature loads. So we have to study this problem. By way of example let us consider a structure that consists of two long rigid panels.

2. The features of space structures

The features of space structures design is determined by their service conditions. The structures must have adequate stiffness and strength to withstand dynamic loods (overload and vibrations) when testing and launching, thermal loods during deploying and life-time. The essential feature required is to provide minimum weight and dimensions of designed cinstruction. In this connection the composite materials, having higher relative strength and lesser temperature extention factor then metals, are widely used. What is more the composites have good stiffness and strength. Also weight goin is attained by using box frames and shell structures, consisting of rigid framework and thin outshell. The similar structures have low thermal conductivity and are, therefore, exposed to high temperature load due to formation of great temperature gradients in the thickness of panels. This lood is one of the most dangerous ones imposed on a construction during the deploying

So, let us consider a construction of two conpled panels. Each panel has commonly used structure: a composite (Fig.3) framework as a central part of the panel, faced with two thin composite plates on both sides. A clearance between panels is denoted "H". The hinges, joining the panels are made of netal or alloy, used in space. The places where the hinges are fastened to the thin-plate faced panel are most critical from the standpoint of structure strength.

3. The determination of forces and displacements produced un oanels' hinges during a deployment of structure

3.1. The desoription of nathematical model

Let us consider two panels of a construction that are linked by hinges along their longest edges, Since the ratio of panel side lengthes is enough small ($a/\ell \ll 1$; $(a/\ell)^2 < 0,1$)), it is possible to regard the structure as a model of two joined girders. In this case the girders can resist twisting load (the width of panels is taken in account). After removal of the constrains and substituting them for forces, as shown in fig. 1,2, we have the following model for calculation of the strain-stress of the structure.

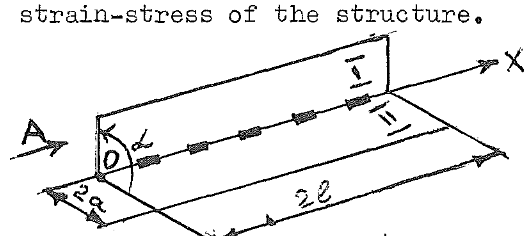

Fig.1

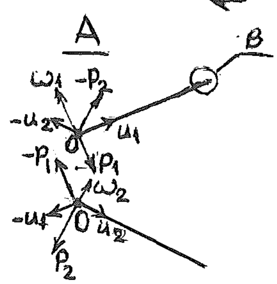

Fig.2

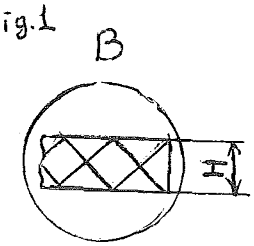

Fig.3

The denotions are:

ω_1 and ω_2 - strains of the panels caused by 1^{st} and 2^{nd} temperature load respectively.

They are directed at right angle to the plane of panel.

P_1, P_2 -forces, induced by bucbling of the panels; are exerted bt one panel on another.

P_1 -force, exerted by the bucbled panel N1 on panel 2.

u_1, u_2-strains in the plane of panels.

Now we can set up a system of differential equations for the state of bending and torsion of two girders.

3.2. The determination of forces and disolecements within the mathematical model

$$
\mathrm{I}\quad
\begin{cases}
EJ_z \dfrac{d^4\omega_1}{dx^4} = -P_1(x) + P_2(x)\cos\alpha \\
EJ_z \dfrac{d^4\omega_2}{dx^4} = -P_2(x) + P_1(x)\cos\alpha \\
GJ_p \dfrac{d^2\varphi_1}{dx^2} = a\left(-P_1(x) + P_2\cos\alpha\right) \\
GJ_p \dfrac{d^2\varphi_2}{dx^2} = a\left(-P_2(x) + P_1(x)\cos\alpha\right) \\
EJ_y \dfrac{d^4 u_1}{dx^4} = P_2(x)\sin\alpha \\
EJ_y \dfrac{d^4 u_2}{dx^4} = -P_1(x)\sin\alpha
\end{cases}
$$

In the system (I):

1 and 2 - are the differential equations of bending in normal plane;

3,4 - the equations of the panel torsion about panel symmetry axes;

5,6 - the equations of bending in the plane of the panels;

$\varphi_{1(2)}$- angle of twisting (an angle between the planes of warped and flot panel 1 (2)).

J_z, J_y- axial moment of inertia.

E, G - modulus of elasticity and torsion respectivily;

a - a holf of panel's width;

α - opening angle;

$P_1(x)$ and $P_2(x)$ - the songht - for functions.

Thus, there are 8 unknown parameters in 6 equations: $P_1, P_2, \omega_1, \omega_2, \varphi_1, \varphi_2, u_1, u_2$. Therefore we add two equations of continuity in the system (I).

$$
\mathrm{II}\quad
\begin{cases}
(\omega_1 + a\varphi_1 + \omega_{1T}) - (\omega_2 + a\varphi_2 + \omega_{2T})\cos\alpha + u_2\sin\alpha = \dfrac{P_1}{K}\sin^2\alpha \\
(\omega_2 + a\varphi_2 + \omega_{2T}) - (\omega_1 + a\varphi_1 + \omega_{1T})\cos\alpha - u_1\sin\alpha = \dfrac{P_2}{K}\sin^2\alpha
\end{cases}
$$

The equation of continuity indicates, that the combined deformation in normal plane of a panel is equal to the sag of a panel in the same plane caused by the respective component of a force (note that the sag of a panel is limited by its stiffness until destruction). The system (II) contains:

$$K = \frac{K_1 \cdot K_2}{1/2\,(2-\cos\alpha)\,K_1 + K_2\sin\alpha}$$

where $K_1 = \dfrac{\pi \cdot 2d \cdot h \cdot E}{-2 \cdot A \cdot 2a \cdot \ln(2d/2a)}$; $K_2 = \dfrac{4 \cdot G \cdot h \cdot H^2 \cdot 2a}{A(H+2a)}$

K - stiffness of Structure (may be calculated as $K = K(\alpha)$); here $2d$ - width of a rigid inset of a hinge in the panel structure.

h - thikness of facing (see the item "The feotures of space structures);

A - distance between hinges,

$2a$ - width of a panel.

Herein we assume the hinge to be absolutely rigid ($K_{hinge} = \infty$) relative to rigidness of a panel ( ), i.e. $K_{hinge} \gg K$.

ω_{1T}, ω_{2T} - displaument functions of temperature lood for loose panel. Set them be the quadratic functions:

$$\omega_{1T} = \frac{\alpha_0 x^2}{2H}\Delta T_1$$

$$\omega_{2T} = \frac{\alpha_0 x^2}{2H}\Delta T_2$$

where

α_0 - temperature extention factor of panel material;

H - overall height (see the item "The features ...);

$\Delta T_1, \Delta T_2$- temperature gradient between the facing plates of a panel.

We are coming now to the solution of given equations.

3.3 The solution of the mathematical model

Let us differentiate fowr times the 1^{st} equation of continuity from system (II) and substitute the result functions $\frac{d^4\omega_{1,2}}{dx^4}$, $\frac{d^4 u_{1,2}}{dx^4}$, $\frac{d^2 P_{1,2}}{dX^2}$ for their expressions from the system (I). Then the functions ω_{1T} and ω_{2T} will equal zero. So we have:

$$\frac{1}{EJ_z}\left(-P_1+P_2\cos\alpha\right)-\frac{1}{EJ_z}\left(-P_2+P_1\cos\alpha\right)\cos\alpha+\frac{a^2}{GJ_p}\left(-\frac{d^2P_1}{dx^2}+\frac{d^2P_2}{dx^2}\cos\alpha\right)-\frac{a^2}{GJ_p}\left(-\frac{dP_2}{dx^2}+\cos\alpha\frac{d^2P_1}{dx^2}\right)\cos\alpha-\frac{1}{EJ_y}P_1\sin^2\alpha=\frac{\sin^2\alpha}{K}\frac{d^4P_1}{dx^4}$$

The 2^{nd} equation of continuity can be rearranged in a similar manner:

$$\frac{1}{EJ_z}\left(-P_2+P_1\cos\alpha\right)-\frac{1}{EJ_z}\left(-P_1+P_2\cos\alpha\right)\cos\alpha+\frac{a^2}{GJ_p}\left(-\frac{d^2P_2}{dx^2}+\cos\alpha\frac{d^2P_1}{dx^2}\right)-\frac{a^2}{GJ_p}\left(-\frac{d^2P_1}{dx^2}+\frac{d^2P_2}{dx^2}\cos\alpha\right)\cos\alpha-\frac{1}{EJ_y}P_2\sin^2\alpha=\frac{\sin^2\alpha}{K}\frac{d^4P_2}{dx^4}$$

After grouping the terms we get the system:

$$\text{III}\begin{cases}\frac{1}{EJ_z}\left(-(1+\cos^2\alpha)P_1+2P_2\cos\alpha\right)+\frac{a^2}{GJ_p}\left(-(1+\cos^2\alpha)\frac{d^2P_1}{dx^2}+2\cos\alpha\frac{d^2P_2}{dx^2}\right)-\frac{1}{EJ_y}P_1\sin^2\alpha=\frac{\sin^2\alpha}{K}\frac{d^4P_1}{dx^4}\\ \frac{1}{EJ_z}\left(-(1+\cos^2\alpha)P_2+2P_1\cos\alpha\right)+\frac{a^2}{GJ_p}\left(-(1+\cos^2\alpha)\cdot\frac{d^2P_1}{dx^2}+2\cos\alpha\frac{d^2P_2}{dx^2}\right)-\frac{1}{EJ_y}P_2\sin^2\alpha=\frac{\sin^2\alpha}{K}\frac{d^4P_2}{dx^4}\end{cases}$$

This system is symmetrical with respect to the functions $P_1(x)$ and $P_2(x)$.

Let us designote the combination of all operations on function $P_1(x)$ in 1^{st} equation of system (III) by operator L_1. Similarly L_2 will be the combination of all operations on function $P_2(x)$ in the same equation.

Then the system (III) may be transformed in following manner:

$$\text{IV}\begin{cases}L_1(P_1)+L_2(P_2)=0\\ L_1(P_2)+L_2(P_1)=0\end{cases}$$

where L_1 and L_2 are the operators

$$L_1=\frac{\sin^2\alpha}{K}\frac{d^4}{dx^4}+\frac{a^2}{GJ_p}(1+\cos^2\alpha)\frac{d^2}{dx^2}+\frac{1}{EJ_z}(1+\cos^2\alpha)+\frac{1}{EJ_y}\sin^2\alpha$$

$$L_2=-\frac{a^2}{GJ_p}2\cos\alpha\frac{d^2}{dx^2}-\frac{1}{EJ_z}2\cos\alpha.$$

We can rearrange the system (IV) by addition and subtraction 2^{nd} equation from 1^{st}. This gives:

$$\text{V}\begin{cases}(L_1+L_2)\cdot(P_1+P_2)=0\\ (L_1-L_2)\cdot(P_1-P_2)=0\end{cases}$$

Let us designate:

$$P_1+P_2=y_1$$
$$P_1-P_2=y_2$$

Then the system shows:

$$\text{VI}\begin{cases}\frac{d^4y_1}{dx^4}+2\gamma_1\frac{d^2y_1}{dx^2}+\beta_1y_1=0\\ \frac{d^4y_2}{dx^4}+2\gamma_2\frac{d^2y_2}{dx^2}+\beta_2y_2=0\end{cases}$$

The coefficients $\gamma_1, \gamma_2, \beta_1, \beta_2$ from the system (V) are:

$$2\gamma_1=\frac{a}{GJ_p}(1+\cos^2\alpha-2\cos\alpha)/\frac{\sin^2\alpha}{K}=$$
$$=\frac{a(1-\cos\alpha)^2}{GJ_p}/\frac{\sin^2\alpha}{K}=\frac{Ka(1-\cos\alpha)}{GJ_p(1+\cos\alpha)}$$

$$2\gamma_2=\frac{a}{GJ_p}(1+\cos^2\alpha+2\cos\alpha)/\frac{\sin^2\alpha}{K}=\frac{a(1+\cos\alpha)^2}{GJ_p}/$$
$$/\frac{\sin^2\alpha}{K}=\frac{Ka(1+\cos\alpha)}{GJ_p(1-\cos\alpha)}$$

$$\beta_1=\left[\frac{1}{EJ_z}(1+\cos^2\alpha-2\cos\alpha)+\frac{1}{EJ_y}\sin^2\alpha\right]/\frac{\sin^2\alpha}{K}=\frac{K(1-\cos\alpha)}{EJ_z(1+\cos\alpha)}+\frac{K}{EJ_y}$$

$$\beta_2=\left[\frac{1}{EJ_z}(1+\cos^2\alpha+2\cos\alpha)+\frac{1}{EJ_y}\sin^2\alpha\right]/\frac{\sin^2\alpha}{K}=\frac{K(1+\cos\alpha)}{EJ_z(1-\cos\alpha)}+\frac{K}{EJ_y}$$

The system VI has the solution. Characteristic equations

$$\alpha_i^4+2\gamma_i\alpha_i^2+\beta_i=0\ ;\quad i=1,2$$

the roots

$$\alpha_i^{1...4} = \pm\sqrt{-\gamma_i \pm \sqrt{\gamma_i^2 - \beta_i}}$$

or

$$\alpha_i^{1...4} = \pm i\sqrt{\gamma_i \pm \sqrt{\gamma_i^2 - \beta_i}} \quad ; \; i = \sqrt{-1}$$

Since the functions P_1 and P_2 are symmetrical the solutions will be also symmetrical (cos)

$$\overline{VII} \begin{cases} y_1 = 2A_1 \cos\alpha_1 x + 2A_2 \cos\alpha_2 x \\ y_2 = 2A_3 \cos\alpha_3 x + 2A_4 \cos\alpha_4 x \end{cases}$$

where

$$\alpha_{1,2} = \sqrt{\gamma_1 \pm \sqrt{\gamma_1^2 - \beta_1}}$$

$$\alpha_{3,4} = \sqrt{\gamma_2 \pm \sqrt{\gamma_2^2 - \beta_2}}$$

Addition and subtraction y_1 and y_2 gives:

$$\overline{VIII} \begin{cases} P_1 = A_1\cos\alpha_1 x + A_2\cos\alpha_2 x + A_3\cos\alpha_3 x + A_4\cos\alpha_4 x \\ P_2 = A_1\cos\alpha_1 x + A_2\cos\alpha_2 x - A_3\cos\alpha_3 x - A_4\cos\alpha_4 x \end{cases}$$

Let us determine the factors A_1, A_2, A_3 and A_4. For this purpose we have to set the integral conditions. Our system is in equilibrium (it is neither constrained nor moving). Consequently the following integrals of the functions P_1 and P_2 over full length must be equated to zero

$$\int_0^{\ell} P_1(x)dx = 0 \; ; \; \int_0^{\ell} P_2(x)dx = 0$$

Hence it is right for y_1 and y_2 too:

$$\int_0^{\ell} y_1(x) = 0 \; ; \; \int_0^{\ell} y_2(x) = 0$$

After integration we get

$$\frac{\overline{A_1}}{\alpha_1}\sin\alpha_1\ell + \frac{\overline{A_2}}{\alpha_2}\sin\alpha_2\ell = 0$$

$$\overline{A_1} = -\overline{A_2}\frac{\alpha_1\sin\alpha_2\ell}{\alpha_2\sin\alpha_1\ell}$$

$$\overline{A_2} = -\overline{A_1}\frac{\alpha_2\sin\alpha_1\ell}{\alpha_1\sin\alpha_2\ell} = -A_1\alpha_2\sin\alpha_1\ell$$

$$\overline{A_4} = -\overline{A_3}\frac{\alpha_4\sin\alpha_3\ell}{\alpha_3\sin\alpha_4\ell} = -A_3\alpha_4\sin\alpha_3\ell$$

Note the denotion

$$A_1 = \frac{\overline{A_1}}{\alpha_1\sin_2\ell}$$

$$A_3 = \frac{\overline{A_3}}{\alpha_3\sin\alpha_4\ell}$$

Then the solution will be

$$\overline{IX} \begin{cases} P_1 = A_1(\alpha_1\sin\alpha_2\ell\cos\alpha_1 x - \alpha_2\sin\alpha_1\ell\cos\alpha_2 x) + \\ + A_3(\alpha_3\sin\alpha_4\ell\cos\alpha_3 x - \alpha_4\sin\alpha_3\ell\cos\alpha_4 x) \\ P_2 = A_1(\alpha_1\sin\alpha_2\ell\cos\alpha_1 x - \alpha_2\sin\alpha_1\ell\cos\alpha_2 x) \\ -(\alpha_3\sin\alpha_4\ell\cos\alpha_3 x - \alpha_4\sin\alpha_3\ell\cos\alpha_4 x) \end{cases}$$

The last two unknown factors A_1 and A_3 can be found from boundary conditions.

3.4. The determination of the factors A_1 and A_3

Let us set boundary conditions. Inosmuch as the torques and cutting forces on free ends of the girder in both directions (orthogonal planes) are equal to zero, then we can write

$$\frac{\partial^2\omega_{1,2}}{\partial x^2} = \frac{\partial^3\omega_{1,2}}{\partial x^3} = 0$$

$$\frac{\partial^2 u_{1,2}}{\partial x^2} = \frac{\partial^3 u_{1,2}}{\partial x^3} = 0$$

if $x = \ell$ and $x = -\ell$

For a fixed point of the panel 1:

$$x = 0 \; : \; \frac{\partial\omega_{1,2}}{\partial x} = \frac{\partial^2\omega_{1,2}}{\partial x^2} = 0$$

(Dieplacement and slew angle, with referance to frive fixing place, are equal to zero. In fact, this point is the zero of coordinates).

Also if $x = \ell$

$$\frac{d\varphi}{dx} = 0$$

Let us next integrate the firs equation of system (I) four times.

$$\omega_1 = \frac{1}{EJ_z}\int_0^x\int_0^x\int_0^x\int_0^x((-P_1(\xi) + P_2(\xi)\cos\alpha)d\xi)dx\,dx\,dx +$$

$+P_1x^3+a_1x^2+d_1x+b_1.$

In view of that system is symmetrical, the factors at add terms will be equol to zero. Suppose also that $b_1=P_1=d_1=0$. Differentiating of the equation two times and inserting $x=\ell$ into it gives:

$$\frac{1}{EJ_z}\int_0^{\ell}\left(\int_0^x(-P_1(\xi)+P_2(\xi)\cos\alpha)\,d\xi\right)dx+2a_1=\left.\frac{d^2\omega_1}{dx^2}\right|_{x=\ell}$$

One to the boundary conditions the right part of this equation equals zero. Hence:

$$a_1=-\frac{1}{2EJ_z}\int_0^{\ell}\left(\int_0^x(-P_1(\xi)+P_2(\xi)\cos\alpha)\,d\xi\right)dx$$

In a similar manner from the 2nd equation of system (I) we derive

$$a_2=-\frac{1}{2EJ_z}\int_0^{\ell}\left(\int_0^x(-P_2(\xi)+P_1(\xi)\cos\alpha)\,d\xi\right)dx$$

In what follows we define the expression for φ_1,φ_2 and u_1,u_2 from the system (I). Double integration of 3d and 4th equations of the system (I) gives:

$$a\varphi_1=\frac{a^2}{GJ_p}\int_0^x\left(\int_0^x(-P_1(\xi)+P_2(\xi)\cos\alpha)\,d\xi\right)dx+C_{11}x+C_{10}$$

$$a\varphi_2=\frac{a^2}{GJ_p}\int_0^x\left(\int_0^x(-P_2(\xi)+P_1(\xi)\cos\alpha)\,d\xi\right)dx+C_{21}x+C_{20}$$

Owing to symmetry of the system the coefficients $C_{11}=C_{21}=0$. The functions u_1 and u_2 can be expressed by integration of the equations 5 and 6 from system (I). The factors at x and x^3 are equal to zero. The factors at x^2 we can define after double differentiation in much the same way as the factors a_1 and a_2 in the equations with ω_1,ω_2. Substitution of the coefficients gives

$$u_1=\frac{1}{EJ_y}\int_0^x\int_0^x\int_0^x\left(\int_0^x P_2(\xi)\sin\alpha\,d\xi\right)dx\,dx\,dx-\frac{x^2}{2EJ_y}\int_0^{\ell}\int_0^x P_2(\xi)\sin\alpha\,d\xi\,dx$$

$$u_2=\frac{1}{EJ_y}\int_0^x\int_0^x\int_0^x\left(\int_0^x(-P_1(\xi)\sin\alpha)\,d\xi\right)dx\,dx\,dx-\frac{x^2}{2EJ_y}\int_0^{\ell}\int_0^x(-P_1(\xi)\sin\alpha\,d\xi)\,dx$$

We next insert the obtained expressions of $\omega_1,\omega_2,\varphi_1,\varphi_2,u_1$ and u_2 into the continuity equation (II). Equating of the factors at x^2 in these equations gives two lacking equations to define A_1 and A_3.

$$\begin{cases}-\dfrac{1}{2EJ_z}\displaystyle\int_0^{\ell}\left[\int_0^x(-P_1+P_2\cos\alpha-P_2\cos\alpha+P_1\cos^2\alpha)\,d\xi\right]dx-\dfrac{\sin\alpha}{2EJ_y}\displaystyle\int_0^{\ell}\int_0^x-P_1\sin\alpha\,d\xi\,dx=\dfrac{\alpha_0}{H}(\Delta T_2\cos\alpha-\Delta T_1)\\ -\dfrac{1}{2EJ_z}\displaystyle\int_0^{\ell}\left[\int_0^x(-P_2+P_1\cos\alpha-P_1\cos\alpha+P_2\cos^2\alpha)\,d\xi\right]dx+\dfrac{\sin\alpha}{2EJ_y}\displaystyle\int_0^{\ell}\int_0^x P_2\sin\alpha\,d\xi\,dx=\dfrac{\alpha_0}{H}(\Delta T_1\cos\alpha-\Delta T_2)\end{cases}$$

After simplificative manipulations we get

$$\mathrm{XII}\begin{cases}\left(\dfrac{1-\cos^2\alpha}{2EJ_z}+\dfrac{\sin^2\alpha}{2EJ_y}\right)\displaystyle\int_0^{\ell}\int_0^x P_1\,d\xi\,dx=\dfrac{\alpha_0}{H}(\Delta T_2\cos\alpha-\Delta T_1)\\ \left(\dfrac{1-\cos^2\alpha}{2EJ_z}+\dfrac{\sin^2\alpha}{2EJ_y}\right)\displaystyle\int_0^{\ell}\int_0^x P_2\,d\xi\,dx=\dfrac{\alpha_0}{H}(\Delta T_1\cos\alpha-\Delta T_2)\end{cases}$$

Addition and subtraction of 1st and 2nd equations gives:

$$\mathrm{XIII}\begin{cases}\left(\dfrac{1-\cos^2\alpha}{2EJ_z}+\dfrac{\sin^2\alpha}{2EJ_y}\right)\displaystyle\int_0^{\ell}\int_0^x(P_1+P_2)\,d\xi\,dx=-\dfrac{\alpha_0}{H}(\Delta T_1+\Delta T_2)(1-\cos\alpha)\\ \left(\dfrac{1-\cos^2\alpha}{2EJ_z}+\dfrac{\sin^2\alpha}{2EJ_y}\right)\displaystyle\int_0^{\ell}\int_0^x(P_1-P_2)\,d\xi\,dx=-\dfrac{\alpha_0}{H}(\Delta T_1-\Delta T_2)(1+\cos\alpha)\end{cases}$$

From (VII) we get

$$\mathrm{XIV}\begin{cases}P_1+P_2=2A_1(\alpha_1\sin\alpha_2\ell\cos\alpha_1x-\alpha_2\sin\alpha_1\ell\cos\alpha_2x)\\ P_1-P_2=2A_3(\alpha_3\sin\alpha_4\ell\cos\alpha_3x-\alpha_4\sin\alpha_3\ell\cos\alpha_4x)\end{cases}$$

Let us next integrate two times each of equations

$$\text{XV}\quad\begin{cases}\displaystyle\int_0^{\ell}\int_0^{x}(P_1+P_2)\,d\xi\,dx = 2A_1\left(\frac{\sin\alpha_2\ell}{\alpha_1}(1-\cos\alpha_1\ell)-\frac{\sin\alpha_1\ell}{\alpha_2}(1-\cos\alpha_2\ell)\right)\\[2ex] \displaystyle\int_0^{\ell}\int_0^{x}(P_1-P_2)\,d\xi\,dx = 2A_3\left(\frac{\sin\alpha_4\ell}{\alpha_3}(1-\cos\alpha_3\ell)-\frac{\sin\alpha_3\ell}{\alpha_4}(1-\cos\alpha_4\ell)\right)\end{cases}$$

Simultaneous solution of (XIII) and (XV) jields A_1 and A_3

$$A_1=\frac{-\frac{\alpha_0}{H}(\Delta T_1+\Delta T_2)(1-\cos\alpha)}{2\left(\frac{1-\cos^2\alpha}{2EJ_z}+\frac{\sin^2\alpha}{2EJ_y}\right)}\cdot\frac{1}{\left(\frac{\sin\alpha_2\ell}{\alpha_1}(1-\cos\alpha_1\ell)-\frac{\sin\alpha_1\ell}{\alpha_2}(1-\cos\alpha_2\ell)\right)}$$

$$A_2=\frac{-\frac{\alpha_0}{H}(\Delta T_1-\Delta T_2)(1+\cos\alpha)}{2\left(\frac{1-\cos^2\alpha}{2EJ_z}+\frac{\sin^2\alpha}{2EJ_y}\right)\left(\frac{\sin\alpha_4\ell}{\alpha_3}(1-\cos\alpha_3\ell)-\frac{\sin\alpha_3\ell}{\alpha_4}(1-\cos\alpha_4\ell)\right)}$$

or

$$\text{XVI}\quad\begin{cases}A_1=\dfrac{-E\alpha_0(\Delta T_1+\Delta T_2)}{H(1+\cos\alpha)\left(\frac{1}{J_z}+\frac{1}{J_y}\right)\left(\frac{\sin\alpha_2\ell}{\alpha_1}(1-\cos\alpha_1\ell)-\frac{\sin\alpha_1\ell}{\alpha_2}(1-\cos\alpha_2\ell)\right)}\\[3ex] A_2=\dfrac{-E\cdot\alpha_0(\Delta T_1-\Delta T_2)}{H(1-\cos\alpha)\left(\frac{1}{J_z}+\frac{1}{J_y}\right)\left(\frac{\sin\alpha_4\ell}{\alpha_3}(1-\cos\alpha_3\ell)-\frac{\sin\alpha_3\ell}{\alpha_4}(1-\cos\alpha_4\ell)\right)}\end{cases}$$

The sought for functions are:

$$\text{XVII}\quad\begin{cases}P_1=A_1(\alpha_1\sin\alpha_2\ell\cos\alpha_1x-\alpha_2\sin\alpha_1\ell\cos\alpha_2x)+\\ \quad+A_3(\alpha_3\sin\alpha_4\ell\cos\alpha_3x-\alpha_4\sin\alpha_3\ell\cos\alpha_4x)\\ P_2=A_1(\alpha_1\sin\alpha_2\ell\cos\alpha_1x-\alpha_2\sin\alpha_1\ell\cos\alpha_2x)-\\ \quad-A_3(\alpha_3\sin\alpha_4\ell\cos\alpha_3x-\alpha_4\sin\alpha_3\ell\cos\alpha_4x)\end{cases}$$

The pressure exerted on a hinge is:

$$P=\sqrt{(P_1-P_2\cos\alpha)^2+P_2^2\sin^2\alpha}=$$
$$=\sqrt{P_1^2-2P_1P_2\cos\alpha+P_2^2}$$

if the hinge is continuous.

In case of discrete hinge the maximum force can be estimated by the following integrals:

$$P_1=\int_{\ell-L}^{\ell}P_1(x)\,dx\,;\quad P_2=\int_{\ell-L}^{\ell}P_2(x)\,dx$$

where L - distance from the last hinge till the end of a panel plus a half of space to the next hinge. Then the formula for pressure can be transformed by means of integration of P_1 and P_2 with further insertion of them into (XVII)

$$P=\sqrt{2\left[(A_1f_1)^2(1-\cos\alpha)+(A_3f_3)^2(1+\cos\alpha)\right]}$$

where f_1 and f_3 are the integrals of P_1 and P_2 :

$$f_1=\sin(\alpha_1\ell)\sin[\alpha_2(\ell-L)]-$$
$$-\sin(\alpha_2\ell)\sin[\alpha_1(\ell-L)]$$
$$f_2=\sin(\alpha_3\ell)\cdot\sin[\alpha_4(\ell-L)]-$$
$$-\sin(\alpha_4\ell)\sin[\alpha_3(\ell-L)].$$

The function of displacement one can obtain by substituting P_1 and P_2 (XVII) into (XI). Herein all the coefficients (a, b, c, d) are already found. Insertion and integration of P_1 and P_2 gives the function of displacements

$$\omega_1\cdot EJ_z=-\left[A_1\left(\frac{\sin\alpha_2\ell}{\alpha_1^3}\cos\alpha_1x-\frac{\sin\alpha_1\ell}{\alpha_2^3}\cdot\cos\alpha_2x\right)+A_3\left(\frac{\sin\alpha_4\ell}{\alpha_3^3}\cos\alpha_3x-\frac{\sin\alpha_3\ell}{\alpha_4^3}\cdot\cos\alpha_4x\right)\right]+\cos\alpha\left[A_1\left(\frac{\sin\alpha_2\ell}{\alpha_1^3}\cos\alpha_1x-\frac{\sin\alpha_1\ell}{\alpha_2^3}\cos\alpha_2x\right)-A_3\left(\frac{\sin\alpha_4\ell\cos\alpha_3x}{\alpha_3^3}-\frac{\sin\alpha_3\ell}{\alpha_4^3}\cos\alpha_4x\right)\right]+\frac{x^2}{2}\left\{-\left[A_1\left(\frac{\sin\alpha_2\ell}{\alpha_1}\cos\alpha_1\ell-\frac{\sin\alpha_1\ell\cos\alpha_2\ell}{\alpha_2}\right)+A_3\left(\frac{\sin\alpha_4\ell\cos\alpha_3\ell}{\alpha_3}-\frac{\sin\alpha_3\ell\cos\alpha_4\ell}{\alpha_4}\right)\right]+\cos\alpha\left[A_1\left(\frac{\sin\alpha_2\ell\cos\alpha_1\ell}{\alpha_1}-\frac{\sin\alpha_1\ell\cos\alpha_2\ell}{\alpha_2}\right)-A_3\left(\frac{\sin\alpha_4\ell\cos\alpha_3\ell}{\alpha_3}-\frac{\sin\alpha_3\ell\cos\alpha_4\ell}{\alpha_4}\right)\right]\right\}$$

The function ω_2 is determined analogously.

4. Method compensation of the forces

Analysis of calculations of real constructions whows thet during the process of unfolding of deformed panels, the forces generated in the most vulnerable areas (places where hinges are fastened to panels) can exceed allowable loads. To lessen stresses in these critical places, the authors suggest the techniques of temporal alteration of structure regidity during the process of deployment. It is associated with the fact that decreasing of parameter K from system (I) leads to reduction of the efforts P_1 and P_2 that, consequently, lessens stresses in the fastenning areas.

One of the design solution proposed by the authors,[1] consists in substitution of conventional rigid hinges in the present construction for special "resilient" joints, that reduce rigidity of structure in the process of deploying. At the end of deployment process these "resilient" joints take their initial position and are locked. Hence on completion of the deployment such joints are equal in properties to usual rigid hinges.

The performance of the proposed joints is shematicolly shoun in fig.4-6 and technical realisation - in fig.7 The "resilient" joint design includes two levers (holders). Each of the levers is fastened on its panel by means of supporting axle (pos. 1)with allowace of limited turning about the supporting axle. The first arm of the lever is joined by swivel axle (pos.2) with the first arm of other lever,and the second arm of each lever, constrained by restoring spring, rests on the retaining plate surface (pos.3) of a panel. Then at the outset of structure unfolding (fig.5) the swivel axle is shifred in the phone of one of the panels, and the corresponding lever, therewith, is being slewed about its supporting axle and depresses the restoring with its second arm.

As this takes place, a load on critical places of the construction is decreased owing to reduction of structure rigidity because of elasticity of the "resilient" joints.

At the opening angle of 90° the rigidity of the structure is maximum, because at this moment one panel repre-

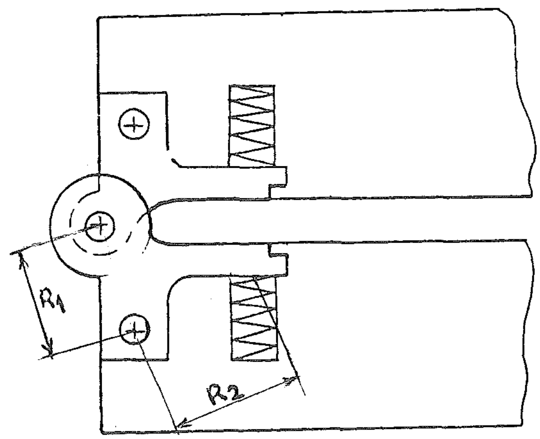

Fig. 4

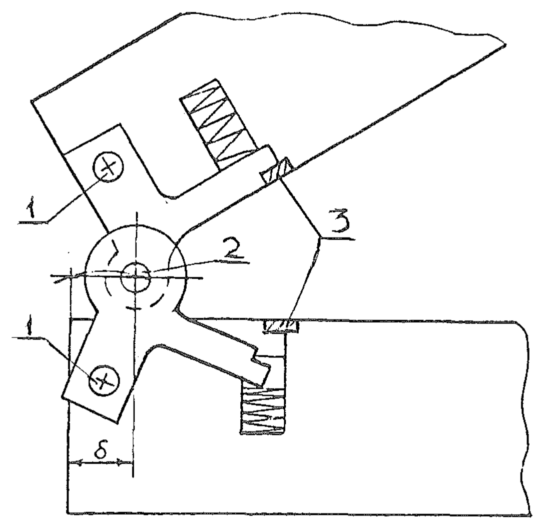

Fig 5

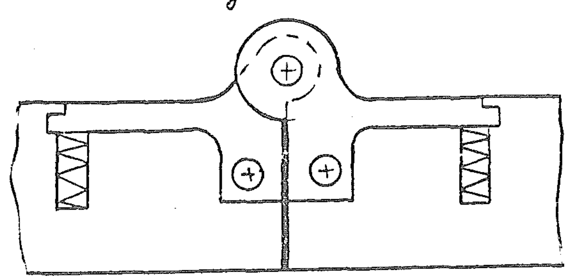

Fig 6

sents a stiffener of another. In this case a lood on the joints is maximum and displacements of swivel axles, that pivot the panels, are extreme.

On the completion of temperature equalisation on both sides of the construction (quasi-stationary process is considered - speed of unfolding is low. The restoring spring of each hinge on expasion returns the second arm of a lever to initial position.

When the construction is deployed and the panels are turned through (angle of) 180° (fig.6) the joint flanges, positioned on the (pos.4) first arm of each lever, come into contact. The second arm of each lever therewith rests on the retaining plate surface of its panel. Hence in the servicing state all parts of the structure are firmly locked. After locking of the construction in operating state (α = 180°), it will be a stiff structure.

The described joints can be efficiently applied to the construction in all places except for one or more central hinges.

Compressive elosticity of a restoring apring should be determined separately for each particular joint. Stiffness of the spring should be reciprocal to value of effort exerted to a joint. In this way the most elastic springs are installed in morginal joints whereas the stiffist springs are in central joints.

This is necessory for even loading of the construction.

The solution for structure with rigid hinges given in this paper, is applicable for calculations of forces in the construction with "resilient" joints. One should only properly change the regidity parameter $K: K_{general} = \frac{K \cdot K_{hinge}}{K + K_{hinge}}$,

where $K_{hinge} = \frac{K_{spring} \cdot R_1}{R_2}$ (Fig.4)

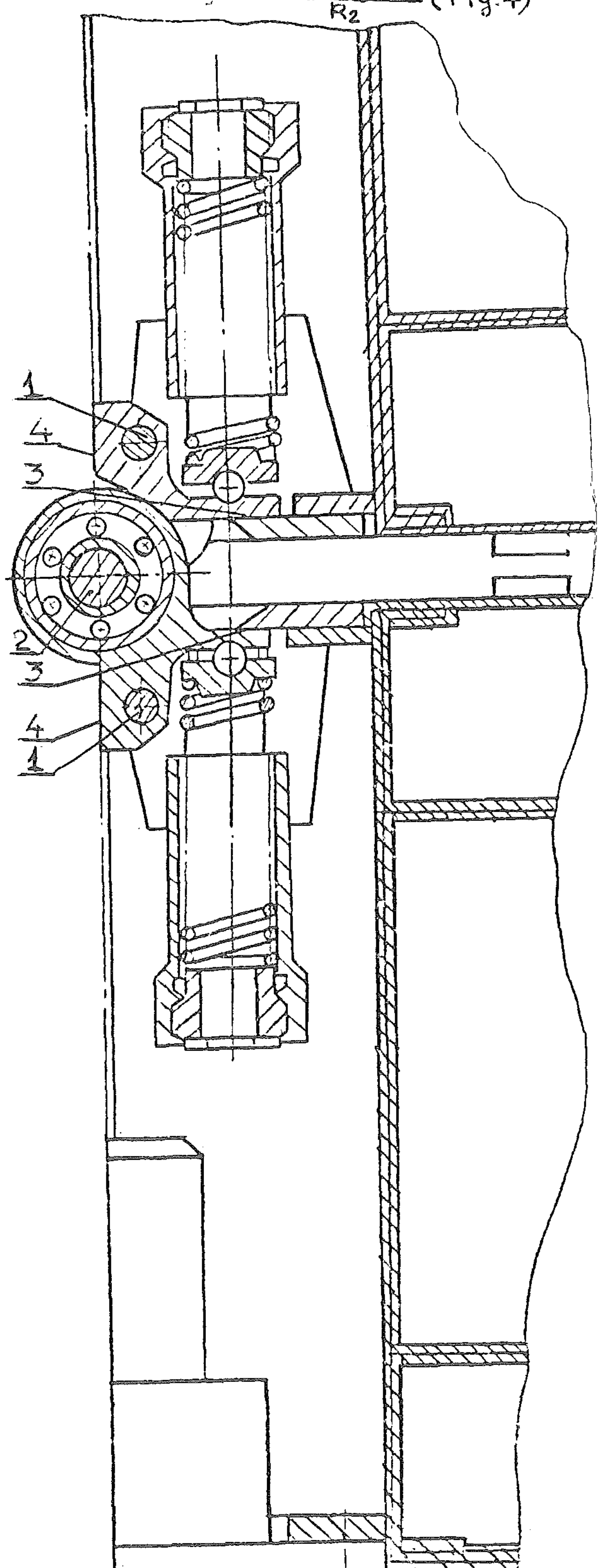

Fig.7

1. USSR invention:
SPACE DEPLOYABLE ANTENNA. Inventors: P.Tushnov, A.Samostvetov, I.Sternin.

Invention Number: 4885290/23 (cl. B64 G 1/44)
Date of invention: Nov.26, 1990.