Calculation of Transonic Flow Over Bodies of Varying Complexity Using Singular Perturbation Method

Fu Qiang
Northwestern Polytechnical University
Department of Aircraft Engineering
Shaan Xi Province, Xi'an , 710072, P.R. China

Abstract

This paper is devoted to examples of the use of Singular Perturbation Method (SPM),which was proposed by Whitecomb and Oswatitch et.al,in transonic flow, the starting point is the full potential equation. At the assumption of small angle of attack and slender bodies, the asymptotic expansion can be used. The rule shows that the originalthree dimensional problem is devided into two simpler component problems, the near field and the far field [3] . The near field is described by a crossflow Laplace equation's boundary value problem and can be solved by Panel Method; the far field is described by a nonlinear transonic small disturbance equation over a body of revolution having the same longitudinal area distribution as the asymmetric body, which can be solved by AF2 scheme(Approximate Fractorization). The two component solutions are combinated to obtain the complete solution.

The calculations predicted the pressure results with good accuracy and the computing speed withAF2 scheme is faster than SLOR method.

I. Introduction

The transonic flow problem is important, since most military airplane and most civil aircraft maneuer in this field; it is also mixed and nonlinear flow, so it is complex; the body's drag ,in this field, is senstive to the change of bodies' shape [2] ,so it is necessary to design the shape of aircraft with low drag. With the advent of computational fluid dynamic procedures, the transonic problem can be handled by Full Potential Equation and Euler Equation, and higher accurate solution have been obtained, but the computing time is longer , so it is also expensive, especially in the optimization procedure. So it is essential to develop quicker, simpler and reliable method to treat the transonic flow problem and to set a base for the optimization design.

At the assumption of small angle of attack and slender bodies, the limit process asymptotic expansion method (also called Slender Body Theory) can be used in transonic flow [3] J.D.Cole etal. use this method to have handled some complex flows [1] . In his method, as a first step to achive the simplification, the velocity potential is expanded asymptotic in both the near and far field from the body. Substituting these expansions into potential equation gives two boundary value problems, the near field is an imcompressible cross flow, is Laplace's equation and is subject to a flow tangency boundary condition on body, the far field is the flow over a body of revolution with the same axial area distribution as the origi–nal body. The near field problem can be handled by Panel Method, the far field problem can be handled by AF2 scheme.

II. Basic Equation

Referring to the configuration defined in Fig.1, with the indicated coordinate system and the X axis aligned with the freestream direction, the full potential equation in these coordinate is [3] ,

$$\begin{aligned}(a^2-\phi_x^2)\phi_{xx}+(a^2-\phi_R^2)\phi_{RR}+a^2\phi_R/R\\+(a^2-\phi_\theta^2/R^2)\phi_{\theta\theta}/R^2-2\phi_x\phi_R\phi_{Rx}\\-2\phi_x\phi_\theta\phi_{\theta x}/R^2-2\phi_R\phi_\theta\phi_{R\theta}/R^2=0\end{aligned} \quad (1)$$

θwhere the subscripts denote partial differentiation.

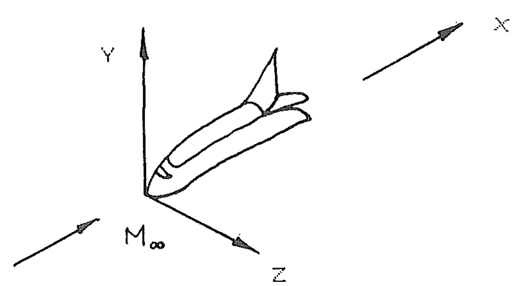


FIG. 1

Inner expansion

The appropriate asymptotic expansion for ϕ in the inner region for the case of zero sideslipe [1] :

$$\begin{aligned}\phi_{inner}=U_\infty\,[x+(\delta^2 ln\delta)2S(x)\\+\delta^2\phi^*(x,r^*,\theta,k,A,B)+O(\delta^4)]\end{aligned} \quad (2)$$

which hold in an "inner limit"

$$x = X / C, r^* = R / (\delta C), k = (1 - M_\infty^2) / \delta^2$$

$$A = \alpha / \delta, B = b / \delta$$

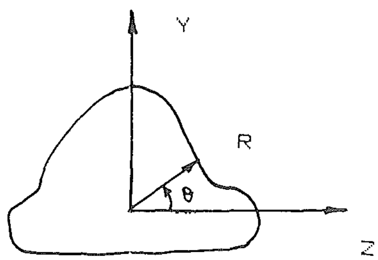
fixed as $\delta \to 0$

where M_∞ is the freestream Mach number, δ is a maximum thickness ratio, c is a dimensional chord such as the wing root chord, and s is a source strength determined from asymptotic matching.

Referring to Fig.2 the equation of the cross sectional shape in a plane X = constant is

$$R = \delta C F(x, \theta) \tag{3}$$

FIG. 2



then, the cross sectional area A(x) is

$$A(x) = 0.5 \oint F^2 \tag{4}$$

Upon substitution of Eq.(1) into the full potential equation and retaining only terms of order 1, a boundary value problem for the Mach number indepen–dent part of ϕ(denoted as ϕ_2^*) is obtained. The theory shows that near the body, the cross flow described by the perturbation velocity potential, ϕ_2^*, is incompressible in the sence that it satisfies Laplace's equation [1]

$$\phi^*_{2r^*r^*} + \phi^*_{2r^*} / r^* + \phi^*_{2\theta\theta} = 0 \tag{5}$$

The condition of tangency of the flow to the body surface gives the normal (Neumamn) derivative boundary condition[1]

$$\phi^*_{2n} = F F_x / \sqrt{F^2 + F_\theta^2} \qquad 0 < X < 1 \tag{6}$$

then,

$$\oint \phi^*_{2n} ds = A'(x) \tag{7}$$

where s is the arc length along the cross sectional boundary. The far field of ϕ^*_{2n} is asymptotically a source flow in the sence that

$$\phi_2^* = A'(x) / (2\pi) Ln r^* \qquad as \qquad r^* \to \infty \tag{8}$$

therefore, the source strength,

$$S(x) = A'(x) / (2\pi) \tag{9}$$

Equation (8) is the crucial link that determines the nonlinear compressible part of the near field which is defined as the function g(x,k). Therefore, ϕ^*, can be considered to consist of two parts, ie.

$$\phi^* = \phi_2^* + g(x,k) \tag{10}$$

Outer Expansion

To obtain g(x,k), the outer (far field) flow has to be treated. In this zone, the approximate representation for the perturbation potential is an asymptotic expansion of different form than Eq.(2). This is [1]

$$\phi_{outer} = U_\infty [x + \delta^2 \phi(x, \tilde{r}, k) + O(\delta^4)] \tag{11}$$

where, $\tilde{r} = \delta R / C$ is a strained coordinated.

Substitution of Eq.(11) into Eq.(1) and retention of the dominant terms gives

$$[k - (\nu + 1)\phi_x] \phi_{xx} + (\tilde{r}\phi_{\tilde{r}})_{\tilde{r}} / \tilde{r} = 0 \tag{12}$$

By matching with the inner solution, the behavior of the solution to Eq.(12) as $\tilde{r} \to 0$ is needed, it can be written as

$$\lim_{\tilde{r} \to 0} (\tilde{r}\phi_{\tilde{r}}) = S(x) \tag{13}$$

$$g(x, K) = \lim_{\tilde{r} \to 0} (\phi - S(x) \ln \tilde{r}) \tag{14}$$

and the pressure coefficient can be given as Ref. [1].

Once ϕ is determined, g(x,k) can be evaluated from Eq.(14) and the evalution of Cp over the body can be completed.

III. Numerical Analysis

Near–Field Problem

The Panel Method can be used to solve the problem described by Eqs.(5) and (6) [1]. Accordingly, the body is represented as the cumulative effect of constant strength source panel. Since the potential of each of these panels is a solution of Eq.(5). It is only necessary to adjust the strength of all of them so that their cumulative effect at any point along their boundary is such that the boundary condition (6) is satisfied.

Assumping the total panel number is N, the potential at a point due to one of the line source can be evaluated by integration of the effect of point sources along the line

This effect can be written as

$$\phi_i = \sum_{j=1}^{N} \lambda_j / (2\pi) \int_j \ln r_{ij} ds_j \quad (j = 1, 2, \cdots, N) \tag{15}$$

where $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ then

$$\phi_{ni} = \sum_{j=1}^{N} \lambda_j / (2\pi) \int_j (\ln r_{ij})_{ni} ds_j \tag{16}$$

Denoting ϕ_{ni}, specified Neumann condition in Eq.(6)

$$f_i = (F F_x)_i / \sqrt{F_i^2 + F_{\theta i}^2} \tag{17}$$

the densities λ_j can be obtained from the system of equations

$$\sum_{j=1}^{N} A_{ij} \lambda_j = f_i \tag{18}$$

where

$$A_{ij} = 1 / (2\pi) \int_j (\ln r_{ij})_{ni} ds_j \tag{19}$$

once the λ_i are known, the potential are also known.

Far–Field Problem

In this paper, the nonlinear transonic small disturbance Eq.(12) was solved using AF2 scheme (Approximate Fractorization) [4], the far field boundary conditions are assumed as

$$\phi = 0, \qquad \bar r \to 0 \tag{20}$$

$$\phi_x = 0 \qquad x \to \infty \tag{21}$$

the condition at $\bar r \to 0$ is Eq.(13)

The AF2 scheme can be given as:

$$(\sigma\overleftarrow{\delta}_x - \delta_{\bar r\bar r} - \delta_{\bar r} / \bar r) f_{ij}^{(n)} = \sigma\Omega L\phi_{ij}^{(n-1)} \tag{22}$$

$$[\sigma - A_{ij}(1-\mu_{ij})\overleftarrow{\delta}_x - \mu_{i-1}A_{i-1,j}\overrightarrow{\delta}_x]\ C_{i,j}^{(n)} = f_{ij}^{(n)} \tag{23}$$

where the operators $\overleftarrow{\delta}_x, \overrightarrow{\delta}_x$ are, respectively, first-order-accurate, backward-difference and foreward-difference operators, $L\phi_{ij}^{(n-1)}$ is the nth interation residual operator, Ω is a relaxition factor, σ is an acceleration parameter. μ_{ij} is same as [4].

IV. Results

The method was used to predict the flow over an elliptic cone, two parabolic arc of revolution body. Fig.3 illustrates the pressure results for the flow over an elliptic cone for various angles of attack. Fig.4 shows the pressure results for a parabolic arc of revolution body with $\tau=0.14$ at different Mach. Fig.5 shows the Cp distribution for another parabolic arc of revolution body with $\tau=0.1667$. The calculation predicate the pressure results with good accuracy.

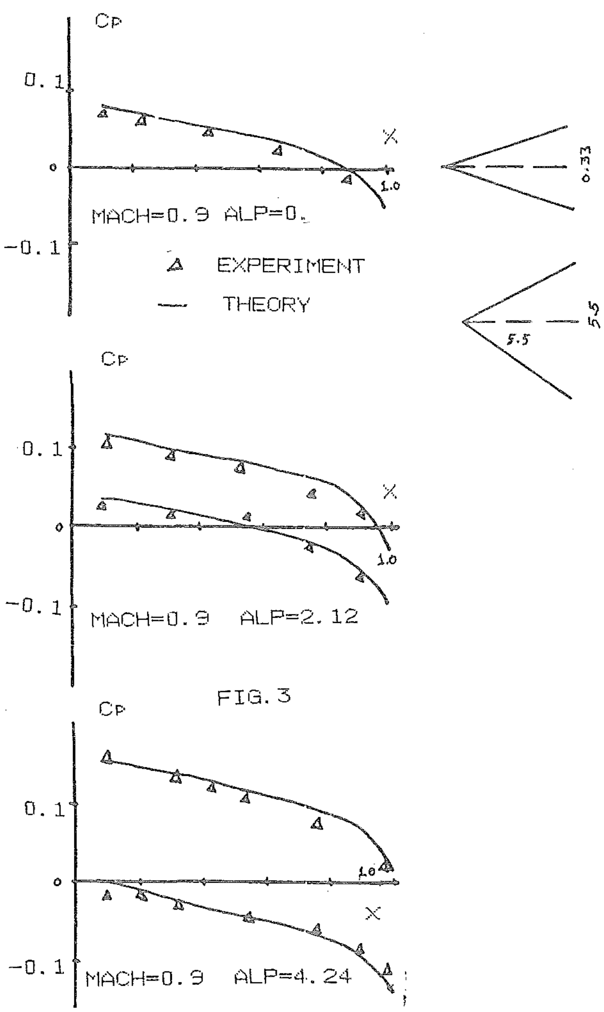
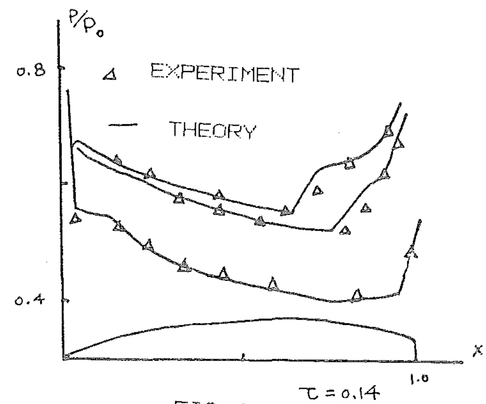


FIG. 3



τ = 0.14

FIG. 4

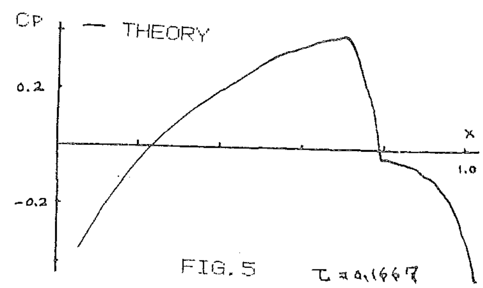


FIG. 5 τ = 0.1667

V. Conclusion

The results indicate that the method can be used transonically to provide useful preliminary design estimates. An important finding is that, by using AF2 scheme, the computating speed increased obviously.

Reference

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