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AEROSERVOELASTIC ANALYSIS OF AN AIRCRAFT MODEL INCORPORATING THE MINIMUM STATE METHOD FOR APPROXIMATING UNSTEADY AERODYNAMICS

R.P. Roberts, Stirling Dynamics Ltd., 42 Priory Avenue, Bristol BS9 4BZ, U.K.

ABSTRACT

The representation of frequency-dependent unsteady aerodynamics in aeroelastic analysis has been the subject of considerable research. The objective is to provide a convenient method to approximate the frequency dependence of the aerodynamics for time domain analysis, frequency response and eigenvalue evaluation (for root locus and flutter). This cannot be accomplished without a state space or equivalent formulation. The work reported here uses one of the existing rational function approximation techniques and illustrates its application to an aircraft configuration. The aerodynamics have been approximated using the Minimum State method developed by Karpel. This relies on fitting the variation of the real and imaginary parts of the aerodynamics as calculated from the **B** and **C** matrices (U.K. notation) for discrete values of frequency parameter. Additional states are required in the model to reproduce this variation but, because of the form of solution, the increase in dimension need only be of the order of one fifth of the number of states in the structural dynamic model. This represents a considerable saving when compared to alternative techniques. The method of solution for the fits is described and it is shown how the resulting matrices can be introduced into the state space form of simple aeroelastic systems.

Eigenvalues and frequency and time responses have been obtained for the aeroelastic system to validate these fits, and some assessment is made of the effect of varying the accuracy of the approximations. Where possible the results are compared with a "baseline" response from current procedures. Time responses are compared with values synthesized using an interpolated transfer function which includes the frequency dependent unsteady aerodynamics and the Fourier coefficients of an input signal. Once a suitable fit has been determined, the method allows normal linear analysis methods to be applied without the need for further interpolation to determine the unsteady aerodynamic forces. A simple manoeuvre controller for the aircraft has then been designed. This has been tested against pilot handling qualities criteria for up-and-away flight and found to be satisfactory. The results obtained for the aircraft with its control system using the minimum state method show good correlation with the results obtained from the Fourier coefficient model. These validate the method in this application. The only significant problem encountered was caused by the interaction of one of the introduced aerodynamic roots with an actuator root. This was solved by moving the aerodynamic root and producing a new fit. The final part of the study involved introducing a rate limit to the actuator and checking its effect on the performance of the aircraft. It would be difficult and time consuming to do this using existing methods. It can be foreseen that this ability to include unsteady aerodynamic effects in a nonlinear time domain model could be one of the main benefits of this type of method.

1. INTRODUCTION

The Minimum State method is one of a number of techniques for representing unsteady aerodynamics in the form of rational function approximations. These methods use transfer function fits to data sets generated by unsteady aerodynamics programs. The fits produce simple differential equations which can be combined with the existing representation of the aircraft to produce an approximation to the aeroservoelastic behaviour over a range of frequencies. Such a method was used in the design of the B-52 flutter suppression system described in Reference 1. Because the procedure is general the way the aerodynamic data have been calculated does not affect it. This means that any refinements introduced at this stage will automatically be approximated by the fits. Conversely, the weaknesses of the aerodynamics programs will also be carried over to the final model. The main advantage of this particular technique is that the number of states added by the method is small enough to make it a practical tool in aeroservoelastic analysis.

The reasons that such approximations are needed are several. It has been shown that the inadequate representation of unsteady aerodynamic effects in control law design can lead to the inclusion of instabilities in the final system. An example of this was the YF-16 "missiles off" instability in roll described in Reference 2. It was shown in a detailed analysis (Reference 3) that one of the requirements for predicting the exact frequency of the instability was a good representation of the unsteady aerodynamics. If initial estimates of the unsteady aerodynamics could be included at the design stage there would be a much better chance of predicting this type of behaviour and so designing to prevent it. Gust load alleviation and flutter suppression systems have to respond to higher frequencies than a basic flight control system (FCS) and thus need a better representation of frequency dependent aerodynamics if they are to be analysed accurately. This could be done using single frequency parameter aerodynamics but this would give a less satisfactory indication of the response to different frequencies than an overall approximation. With all control systems, checks on FCS/structural coupling have to be performed across quite a wide range of frequencies, so again some approximation to the behaviour of the aerodynamics across this spectrum would be useful. As well as these control law design applications, the method can be used for simple flutter analysis. Because of the fits, only one model need be used to evaluate the roots for all speeds up to flutter. This obviates the need for iteration to match the aerodynamics at the frequency and frequency parameter of flutter and so simplifies the computation considerably. It can also be applied to structural loading calculations using the loading equation. Finally, the method can be used in time domain simulations using numerical integration. This enables nonlinearities to be introduced into the control and actuation systems, permitting a more detailed analysis of complex aeroservoelastic phenomena.

With all these applications there is naturally an initial overhead in validating the approximations to ensure that they are sufficiently accurate. This will of course reduce the time saved, but the method still makes a lot of analysis possible which would otherwise be extremely laborious.

2. AERODYNAMIC EQUATIONS AND DATA

In order to follow how the approximations are put into the aircraft model, it is necessary to see the matrices they replace and how they are used in the aircraft flutter response equation. This is written (in U.K. notation) as

$$[A + \rho G]\ddot{q} + [D + \rho V B]\dot{q} + [E + \rho V^2 C]q = \text{input function.} \tag{2.1}$$

where q is the generalized coordinate vector,
A the structural mass matrix,
D the structural damping matrix,
E the structural stiffness matrix,
G the aerodynamic inertia matrix,
B the aerodynamic damping matrix,
and C the aerodynamic stiffness matrix.

The input function may be one of several - gusts, control demands, or zero for a flutter solution. As the aerodynamic inertia matrix has been neglected in all the data supplied it will not be included in the analysis. Both the **B** and **C** matrices depend on the frequency parameter ν. This variation is best shown by plotting the real and imaginary parts of the aerodynamics as vectors in the complex plane. This can be obtained by considering the aerodynamic matrices. Let

$$Q_A = [\rho V B s + \rho V^2 C]q$$

Replacing s by $j\omega$ and substituting the frequency parameter $\nu = \omega\ell/V$, we obtain

$$Q_A = \rho V^2[C + B(\nu/\ell)j]q \tag{2.2}$$

The real and imaginary parts can be approximated by equations in the nondimensional Laplace variable s. These are then included in the aircraft equations by multiplying through by ρV^2 and the generalized coordinate vector q.

The data set for this study is representative of a delta wing fighter aircraft, with aerodynamics for a Mach number of 0.8. The **B** and **C** matrices are given for 17 values of frequency parameter between 0.001 and 5.0. These were calculated at British Aerospace (BAe) Military Aircraft Division using methods based on those described in Reference 4, with the semi-span as the reference length. The matrices include rigid heave and pitch degrees of freedom, the first four wing modes, and two flexible and two rigid control modes. The fuselage is effectively rigid and only represented by mass and inertia terms. A modal displacement matrix was supplied for determining structural deflections.

If the input function to the equation includes gust terms which vary with frequency parameter then these will also have to be fitted. As this has not been done in this work, the formulation will not be detailed here or in subsequent sections. A further topic that will be omitted is the application of the method to the loading equation. This is used to obtain structural loads or other pure outputs of the system in terms of the generalized coordinate vector, its derivatives and the input function. The main problem here would be obtaining consistent loads as the fits will introduce discrepancies.

3. METHOD

The technique relies on fitting transfer functions to the unsteady aerodynamic data. This form of representation is not really the most appropriate for the subsonic data as disturbances shed into the flow will affect the later behaviour of the wing (Reference 5). In fact the behaviour of lift cannot be strictly described by a rational function in the Laplace plane since it is a multivalued function with a branch point at the origin and a branch cut along the real axis (Reference 6). However, as noted in Reference 7, rational functions do produce the most convenient form for analysis since simple differential equations are the result of the approximations. Also, whatever the strict mathematical limitations, this form of approximation has been successfully used in the design of flutter suppression systems in the past (Reference 1). The method is based on a variant of Roger's approximation (Reference 8) where the equations are cast slightly differently to reduce the number of additional states required. The original form would result in the addition of $(m \times n)$ states to the model. However Karpel (Reference 9) showed how to work back from the desired state space form of the solution to the approximation method appropriate to that formulation. This only adds a number of states equal to the number of denominators in the approximation. This is at the expense of reducing the accuracy of the fit for a given number of extra denominator terms and increasing the complexity of the solution, which now requires iteration.

3.1 State Space Form

The form of the approximation to the unsteady aerodynamics is

$$Q(s) \approx P_0 + P_1 s + P_2 s^2 + M[sI - R]^{-1}Ns \tag{3.1.1}$$

where M is an $(m \times n)$ matrix, R is a diagonal matrix of the denominator coefficients $(n \times n)$ and N is an $(n \times m)$ matrix.

Thus if we substitute for s ($= s\ell/V$) we obtain

$$Q(s) \approx P_0 + P_1(\ell/V)s + P_2(\ell/V)^2 s^2 + M[(\ell/V)sI - R]^{-1}N(\ell/V)s$$

As $[(\ell/V)sI - R]$ is diagonal, its inverse is the reciprocal of each of the diagonal elements. Hence we can multiply this term by the external (ℓ/V) factor and dimensionalize the equation by multiplying through by ρV^2 to obtain

$$\rho V^2 Q(s) \approx \rho V^2 P_0 + \rho\ell V P_1 s + \rho\ell^2 P_2 s^2 + \rho V^2 M[sI - (V/\ell)R]^{-1}Ns$$

If we now say $X_A = [sI - (V/\ell)R]^{-1}Nsq$ and substitute back into the original flutter equation (Equation 2.1) then

$$[A + \rho\ell^2 P_2]\ddot{q} + [D + \rho V\ell P_1]\dot{q} + [E + \rho V^2 P_0]q + \rho V^2 M X_A = Fu \tag{3.1.2}$$

For the aerodynamic states we have $sX_A + (V/\ell)RX_A = Nsq$

$$\text{so } \dot{X}_A = N\dot{q} - (V/\ell)RX_A \tag{3.1.3}$$

We can now write the state space equations for the full aircraft in matrix form as given by Equation 3.1.4 below.

As **R** has the dimension of the number of denominator terms in the approximation, only this number of states has been added to the original model. Thus the size of the additional "aerodynamic dimension" is independent of the size of the original model. Hence in general fewer states are added than with other methods of this type. However for a specified accuracy the number of states required will still be dependent on the size of the existing model due to the need to produce reasonable fits and the way the solution is formulated.

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \\ \dot{X}_A \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -[A + \rho\ell^2 P_2]^{-1}[E + \rho V^2 P_0] & -[A + \rho\ell^2 P_2]^{-1}[D + \rho V\ell P_1] & -\rho V^2[A + \rho\ell^2 P_2]^{-1}M \\ 0 & N & -(V/\ell)R \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ X_A \end{bmatrix} + \begin{bmatrix} 0 \\ [A + \rho\ell^2 P_2]^{-1}F \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \tag{3.1.4}$$

3.2 Method of Solution

Constraints are added to the fits to remove the need for some of the iteration (Reference 9) and to ensure that the final solution is closely related to the physical problem (References 10 and 11). This technique allows a direct solution for three of the matrices in terms of the others. Consider the form of the approximation given in Equation 3.1.1. At zero frequency parameter there is no imaginary part to the aerodynamics so $\mathbf{P}_0$ must be constrained to equal the real part of the data. There must also be a direct correspondence between the pitch column of the $\mathbf{C}$ matrix and the heave column of the $\mathbf{B}$ matrix. This means that the rate of change of the heave column of the imaginary aerodynamics with ν must be set equal to the aerodynamic stiffness in pitch at zero frequency. For this column, this demands that

$$[\mathbf{P}_1]_Z = [\mathbf{P}_0]_\theta/\ell + [\mathrm{MR}^{-1}\mathrm{N}]_Z \qquad 3.2.1$$

For the heave column this provides two constraints, the third being given by forcing the real part to be exact at some other point. For all other columns of the matrix, the approximation is required to be exact at some frequency parameter other than zero. If the other constrained point is at a frequency parameter of ν_c then evaluating the real and imaginary parts of the approximation we obtain

$$\mathbf{Q}_R(\nu_c) = \mathbf{P}_0 - \mathbf{P}_2\nu_c^{\,2} + \mathrm{M}[\nu_c^{\,2}\mathbf{I} + \mathbf{R}^2]^{-1}\mathrm{N}\nu_c^{\,2} \qquad 3.2.2$$

$$\mathbf{Q}_I(\nu_c) = \mathbf{P}_1\nu_c - \mathrm{M}[\nu_c^{\,2}\mathbf{I} + \mathbf{R}^2]^{-1}\mathbf{R}\mathrm{N}\nu_c \qquad 3.2.3$$

As $\mathbf{P}_0$ is equal to the real part of the data at zero frequency parameter $\mathbf{Q}_R(0)$ then

$$\mathbf{P}_2 = [\mathbf{Q}_R(0) - \mathbf{Q}_R(\nu_c)]/\nu_c^{\,2} + \mathrm{M}[\nu_c^{\,2}\mathbf{I} + \mathbf{R}^2]^{-1}\mathrm{N} \qquad 3.2.4$$

$$\mathbf{P}_1 = \mathbf{Q}_I(\nu_c)/\nu_c + \mathrm{M}[\nu_c^{\,2}\mathbf{I} + \mathbf{R}^2]^{-1}\mathbf{R}\mathrm{N} \qquad 3.2.5$$

Thus $\mathbf{P}_0$, $\mathbf{P}_1$ and $\mathbf{P}_2$ can all be determined for given matrices M and N. These solutions give the two matrices in terms of each other so their evaluation requires an iterative least squares procedure. This involves performing least squares solutions for the columns of the N matrix and then using the same technique to give the rows of the M matrix. A suitable procedure is as follows:

1. Set $\mathbf{P}_0 = \mathbf{Q}_R(0)$.
2. Set an initial M matrix.
3. Solve for N in terms of M.
4. Solve for M using this value of N.
5. Calculate $\mathbf{P}_1$ and $\mathbf{P}_2$ using 3.2.1, 3.2.4 and 3.2.5.
6. Calculate the total approximation error.
7. Repeat steps 3 to 6 to convergence.

This solution is not linearly independent, so that elements which are difficult to fit can cause a deterioration in the fit to other elements. Because of the iteration involved, the solution requires considerably more computer time than for other similar techniques. However, this one-off cost should be more than offset by the reduction in run times when using the final model. More detail on the solution can be found in References 11 and 12.

3.3 Errors and Weighting

Having seen the form of solution, it is important to appreciate that weightings may well be needed to produce a sensible fit. The form of weighting depends upon what is required in the end use of the approximation. As the fit quality can only be gauged with respect to the whole matrix, elements of small magnitude will effectively be ignored in the solution if there is no corresponding weighting, since errors in them will make an insignificant contribution to the overall error. Unless it is known that the elements are small for the physical reason that they do not greatly influence the behaviour of the system, it is dangerous to allow this to happen. Similarly the magnitude of the coefficients may vary quite significantly with frequency parameter. Hence, although the size of the error may remain approximately constant, the percentage error it represents in the magnitude of the coefficient may vary considerably with frequency parameter. These comments are intended to be general because the only real measure of the quality of the fit is whether the final model satisfactorily performs the tasks required of it. Thus, for the design of a flutter suppression system it would be important to match the behaviour of the unsteady aerodynamics accurately around the frequency parameter of the flutter, whereas for a more general FCS/structural coupling analysis it would be more important to get a fairly even fit. For this work the weightings have been set to normalize the error on each point of the aerodynamic matrix. This was done to remove the magnitude problems mentioned above and because it was thought to be a good basis for introducing more refined weightings at a later stage. Weighting is introduced by adding weighting matrices to the least squares solutions for the M and N matrices.

4. FITS

Space precludes a detailed discussion of the accuracy of the fits obtained, but a few remarks would be appropriate. The method will fit simple vector shapes quite well with relatively few denominators. Generally, the elements which cause most problems are those with loops in the variation of the vector. For a five mode data set, it was found that removing one such element and replacing it with a simpler one reduced the sum of the squared errors by 50%. This was mainly because of the error on that element, but was also due to the improvement gained in the fits to other elements once it had been removed. Thus such "difficult to fit" elements can have an influence disproportionate to their importance to the aeroservoelastic problem. Hence there might be justification for decreasing the weighting on such elements.

In order to solve for the numerator matrices in the Minimum State method it is necessary to know the values of the denominator coefficients in the $\mathbf{R}$ matrix. This can be done by specifying their values in advance of the calculation but the solution obtained will not be an optimum one. As a result it may be desirable to improve the fit without adding extra states to the final model by optimizing the fit with respect to these denominator terms. This is a nonlinear problem so there is no direct analytical solution. The method used for this work is the adaptive simplex procedure suggested by Nelder and Mead in Reference 13. It has been found to be reliable (Reference 14) and relatively efficient in previous work (Reference 15). It also requires no gradient calculations, which helps to reduce the amount of computation required. To prevent unstable transfer functions being produced, a barrier function and linear gradient have been placed at a real value of zero. The main problem is that there are numerous function evaluations involved in the procedure. This means that the computing time required to run the optimizer can be quite high. The Minimum State method also needs considerable time to iterate for each solution, so the process can be time consuming. Optimizing the fits produces somewhat unpredictable improvements to the model. In some cases the fits are noticeably better but in general the reduction in error is less than 10%. As might be expected, this depends largely upon whether substantial improvements can be made to the "difficult to fit" elements. The optimizer also requires quite significantly varying run times. In general it was found that the more difficult it was to improve a fit, the longer the run time needed for the program. As yet, no way has been found of testing whether any significant improvements can be made without doing the full run. The experience gained thus far supports the conclusions reached by Karpel that it is generally more profitable simply to add an extra state to the fit than to attempt to optimize the denominator coefficients. As these terms appear as poles of the aeroelastic system, there is also some benefit to be gained from placing them in a specific place, rather than allowing them to appear anywhere in the complex plane.

This study used three different fits to the aerodynamics. Two had three denominator terms and one six. Generally these denominators were spread evenly across the frequency parameter range of interest but one of the three denominator approximations had them biased towards the higher frequency end of the data. This was for reasons which will be explained later.

5. VALIDATION OF AEROELASTIC MODELS

Several checks were performed on the aerodynamic models to ensure that they matched the results predicted by the data. The first of these was a visual inspection of the fits, simply to ascertain that the approximation method was producing reasonable variations in each of the elements. More rigorous comparisons of the results were also made.

5.1 System Roots

The roots of each of the aeroelastic models were calculated for the conditions of Mach 0.8, sea level for which the aerodynamic data set is valid. This required an eigenvalue evaluation of the matrix given in Equation 3.1.4. To provide a check, circular interpolation and iteration for the frequency and frequency parameter of the roots were used to calculate the eigenvalues based on the actual data. The results are given in Tables 1 to 4, which show that all the fits produce a good estimate of the root positions. It should be noted that the roots based on the circular interpolation are not the "correct" ones but merely results representative of conventional techniques.

As another check on the behaviour of the roots, flutter plots were produced using the aerodynamic models. Again these were compared with results obtained by an iterative circular interpolation approach. Figures 1 to 4 illustrate the results. For the fits based on evenly spaced denominators, the results tend to agree quite well but the modified three state model shows noticeably different behaviour for high dynamic pressures. The calculated flutter speeds are given in Table 5, confirming the initial impression. The mode with a frequency of 30 Hz has been omitted from this data. However the circular interpolation indicates that it goes marginally unstable at 1555 kts while only the modified three state fit produces this instability, albeit at the rather low speed of 1349 kts. It might be that a slightly different interpolation would suggest that this mode should remain stable.

5.2 Frequency Response

The second validation test was to check the frequency response of the aircraft heave acceleration to an inboard control input. The different approximations were run for conditions on Mach 0.8, sea level. The three fits were once again compared with results based of circular interpolation of the aerodynamic data. The overall comparison is shown in Figure 5, which illustrates how good the match is for all the models. This is not surprising since the roots of the system are also well matched for these conditions.

5.3 Time Response

The final check was to run time response simulations of the Minimum State models and compare them with results obtained from the original data. The response of the wing tip leading edge to an inboard control input was investigated as it showed a noticeable response to many of the modes in the model. Producing comparable results from the aerodynamic data was more difficult than in previous cases. It involved calculating the transfer function using circular interpolation for the aerodynamics, then taking the Fast Fourier Transform (FFT) of the input signal and band limiting it to the range of frequencies contained in the aerodynamic data. These Fourier coefficients were combined with the transfer function to give the Fourier coefficients of the output signal. The inverse FFT was then taken to give the time response. This process is illustrated schematically in Figure 6.

The frequency limited input signal was injected into the Minimum State models and the response left to settle into a periodic form. This was necessary since periodicity is one of the underlying assumptions of the FFT method. The two results could then be compared. Space precludes the inclusion of all the results so only the acceleration has been assessed, as this is the most interesting. Again, the test conditions were set to Mach 0.8, sea level. The responses produced by the different models and methods are given in Figure 7. This shows that their shape is approximately what would be expected in all cases, with the six state fit (the most accurate) being closest both in form and magnitude to the results produced using circular interpolation. This figure illustrates that the small differences visible in the frequency responses do not have a great effect on the time domain responses.

On the basis of this evaluation, it can be seen that all three of the approximations match the expected behaviour quite well. The main reservation concerns the accuracy of the modified three state model at high values of dynamic pressure but for operation around a design point for which the aerodynamics were calculated, the other results show that the fit should be perfectly satisfactory. It would be possible to perform many more checks but this validation was considered sufficient for the purposes of this study.

6. DESIGN OF MANOEUVRE CONTROLLER

A block diagram of the control scheme is given in Figure 8. Most of the main elements required in a representative controller are present, although some are not detailed models. Notable by their absence are sensor dynamics and notch filters. For this work, it was assumed that the gyro was ideal for the range of frequencies under consideration, and notch filters were not required because structural modes did not significantly affect the pitch rate feedback. This was mainly because the fuselage was assumed rigid in the data supplied. The actuator was simplified to the form of a first order lag with a break frequency of 7 Hz. Only real stiffness terms were associated with it since for time domain work, a hydraulic simulation of the device would be needed to include the complex impedance properly. The sampling frequency was 44 Hz and the anti-aliasing filter was set for 20 Hz.

The guidelines for acceptable aircraft behaviour were obtained from Reference 16. This gives limits for frequency and time domain responses to a pilot input and is based to some extent on MIL-F-8785. The study only attempted to comply with the requirements demanded for up-and-away flight. Because one of the specifications involves the step response of the vehicle, the Minimum State models had to be used as a basis for designing the system. Interpolated data could be used to check some of the results and ensure that the numbers produced were reasonable. The controller was only designed to function at a Mach number of 0.8 at sea level and no provision was made for gain scheduling with flight condition. However it was intended only that the process should give some insight into any problems which might arise during a more detailed design exercise.

The procedure followed was to increase the frequency of the short period roots whilst maintaining a reasonable damping by altering the gains in the proportional plus integral controller. This was designed to make the aircraft respond more rapidly to the pilot input. Once a satisfactory response had been obtained, the lead/lag filter on the input was used to advance the pitch response and ensure that it still remained within the desired boundaries. The final step was to take the full system transfer function between the aircraft pitch attitude and the demand signal and to use the input gain to force the gain crossover frequency to occur at 0.3 Hz. Boundaries obtained from Reference 16 could then be checked.

This process was first performed using the six state fit, since this was the most accurate. When the three state fit with evenly spaced denominator terms was used, it was found that one of the aerodynamic roots interfered with the actuator root. Since these roots are not present in the real aircraft but are rather a mathematical convenience used for fitting the aerodynamics, it was decided that to avoid this problem a modified three state model should be used, with the denominator terms biased towards the higher frequencies. All subsequent references to a three state model refer to this modified case. The responses for the final designs for the three and six state aerodynamic models to a step pitch rate demand are shown in Figures 9 and 10. The gain between the demand and the feedback loop has been set to unity to show more clearly the amount by which the peak response overshoots the final steady state response. These illustrate that there is effectively zero drop-back in the pitch attitude response and that the peak values of pitch rate and acceleration occur within the time constraints suggested by Reference 16. The control anticipation parameter was calculated for both models and found to be equal to 0.27 rad/s^2/g, slightly outside the range suggested in the reference of between 0.28 and 3.6 rad/s^2/g. As the object of the study was to evaluate unsteady aerodynamic models in aeroservoelastic systems and not to produce a perfect controller, this performance was deemed satisfactory. Finally, the input gain was modified to give a gain crossover frequency of 0.3 Hz in the pitch attitude response to the pilot input. The gains required were slightly different, 1.74 for the six state model and 1.77 for the three state one. This represents a disparity of 1.7% and is a measure of how much the aerodynamic approximations differ at this frequency for these flight conditions. The transfer functions obtained from the two models are shown in Figures 11 and 12. These also illustrate the acceptable boundaries for up-and-away flight given in Reference 16 and show that in both cases the overall performance just satisfies these criteria. They can be compared with one generated using interpolated data given in Figure 13. This required a gain of 1.84 to produce the desired crossover frequency.

7. TIME DOMAIN RESULTS

As in the case of the pure aeroelastic model, results from the Minimum State approximations were compared with those from the original data to check their validity. The time domain responses have had a steady state pitch rate error removed to allow for the slightly different conditions between them and the Fourier transform method. The pitch rate responses to different band-limited square wave inputs are shown in Figure 14 and this confirms that the models are once again true to the predicted results. In order to demonstrate the potential of the method, it was decided to introduce an actuator rate limit to make the system nonlinear. This was not intended to be representative in terms of magnitude but was purely for illustrative purposes. For a nonlinearity to be included in the interpolation method would be difficult and would require describing functions to be evaluated for each input amplitude. In contrast, the Minimum State approach allows the rate limit to be programmed directly into the time domain simulation.

The effect of the rate limit on the six state model can be seen by comparing Figures 10 and 15. The latter shows that the actuator rate reaches a maximum and delays the growth of the pitch acceleration and pitch rate. It also introduces a small amount of drop-back to the pitch attitude response. A transfer function analysis showing the effect of the rate limit on the response of the actual pitch rate to the demanded pitch rate is shown in Figure 16. As would be expected, as the amplitude is increased, the gain and phase responses drop off at a lower frequency. While this is basically a low frequency system in which the frequency dependence of the aerodynamics is less important, similar effects would need to be modelled for high frequency control applications.

8. FUTURE WORK

There are several main areas where further work is required. The first is to apply the method to more data sets to check for any particular weaknesses. This would include studies of supersonic conditions, models including more modes and applications to commercial transport aircraft. As well as wing modes it would be beneficial to study the effect of including fuselage flexible modes in the model. For the type of model developed during this study, it would be useful to include sensor dynamics, notch filters and comprehensive actuator simulations to allow for impedance and physical limits within the actuator. It will also be necessary to study the interaction of the introduced aerodynamic roots and other eigenvalues within the control system. This should be aimed at finding ways of producing good aerodynamic approximations which will not interfere with the real dynamics of the control and actuation system. It seems almost certain that this will require some prior knowledge of the roots of the control system.

As yet, the gust response of the aircraft has not been evaluated. This will be important since the aerodynamics in all modes can be excited over a range of frequencies determined by the particular gust spectrum applied. In the civil field where gust responses can determine the maximum structural design loads, this approximation method would be particularly useful, especially with the increasing trend towards fly-by-wire systems. It would permit unsteady aerodynamics to be included in studies of the effects of control system nonlinearities on the aircraft gust response, with worst case gusts evaluated by methods such as those described in References 17 and 18.

Another field which needs consideration is the loading equation. It does not appear that this method could give consistent loads throughout an airframe, simply because by definition the approximations cannot be guaranteed to be consistent. However it should produce results which are sufficiently accurate to make these inconsistencies small in comparison to the magnitudes of the structural loads. Thus it should certainly be suitable for preliminary design work.

Finally, more experience is needed in applying the technique and using the results, in order to check for any problems that might arise. This can only be done by using different parameters and data sets to calculate the approximations and then running complete aeroservoelastic models. Ultimately this could be aimed at automating the whole procedure to reduce the operator involvement to a minimum and yet still produce valid approximations.

9. CONCLUSION

Overall, the Minimum State method is a powerful technique because it adds considerably fewer states to the final model than other methods whilst still accurately approximating the variation of the aerodynamics with frequency parameter. Its main disadvantage is that it needs significantly longer computer time than other methods to generate the fits since iteration is required for the solution. It is sensitive to elements which are difficult to fit in the matrix and these tend to degrade the overall approximation. Although this study has illustrated that the introduced states can cause problems by interacting with other roots in the aeroservoelastic system, this difficulty will be correspondingly more severe in other methods where more states have to be added. This, together with the high dynamic pressure flutter behaviour of the modified three state model, shows that some validation of the overall model will be required.

However, once this has been done, it is possible to perform a wide range of tasks with a single aeroelastic model, saving both on time and complexity. This model has the advantage that the states added to represent the unsteady aerodynamics are no longer the dominant component in the number of states in the overall model. The simulation used for this study had at most six aerodynamic states, compared with twenty structural ones and eleven control system states. This means that including unsteady aerodynamics in the model is actually a practicable proposition. The final benefit of the technique is that it allows control system nonlinearities to be included in the same model as the unsteady aerodynamics. This could be extremely useful as many control system problems have occurred when the response is influenced by such nonlinearities.

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NOTATION

The following list contains most of the symbols used in the text. Those not defined here are defined locally. Bold type is used to denote a matrix.

F Input matrix.
I Identity matrix.
j Complex variable, $\sqrt{-1}$.
ℓ Reference length.
M Premultiplying numerator matrix.
m Number of modes in data.
N Postmultiplying numerator matrix.
n Number of denominators used in fit.
P Aerodynamic numerator approximation matrices.
$\mathbf{Q}_A$ Aerodynamic terms in flutter equation.
$\mathbf{Q}_I$ Imaginary part of aerodynamics.
$\mathbf{Q}_R$ Real part of aerodynamics.
q Generalized coordinate vector.
R Diagonal denominator matrix.
s Laplace operator.
s Nondimensional Laplace operator (= $s\ell/V$).
u Input vector.
V Velocity (T.A.S., m/s).
$\mathbf{X}_A$ Aerodynamic state vector.
ν Frequency parameter (= $\omega\ell/V$).
ρ Density (kg/m^3).
ω Frequency (radians/s).

Real Part	Imaginary Part	Frequency (Hz)	Damping Ratio
-42.13	1 248	198.63	0.0337
-30.35	749.2	119.24	0.0405
-6.507	354.9	56.484	0.0183
-35.85	302.4	48.129	0.1177
ν = 5.0			
-11.02	234.1	37.250	0.0471
-7.979	152.6	24.286	0.0522
-1.954	124.4	19.799	0.0157
-1.783	46.46	7.3949	0.0383
-2.249	4.233	0.6737	0.4693
0.000	0.000	0.0000	0.0000
0.000	0.000	0.0000	0.0000

TABLE 1. ROOTS USING INTERPOLATED DATA

Real Part	Imaginary Part	Frequency (Hz)	Damping Ratio
-43.67	1 269	202.04	0.0344
-30.59	755.7	120.26	0.0405
-8.199	355.1	56.494	0.0231
-38.53	304.2	48.413	0.1257
ν = 5.0			
-10.99	234.2	37.273	0.0469
-8.066	152.9	24.341	0.0527
-2.602	125.0	19.891	0.0208
-2.078	46.54	7.4069	0.0446
-2.158	4.031	0.6416	0.4720
0.000	0.000	0.0000	0.0000
0.000	0.000	0.0000	0.0000

TABLE 2. ROOTS OF THREE STATE MODEL

Real Part	Imaginary Part	Frequency (Hz)	Damping Ratio
-44.58	1 273	202.64	0.0350
-25.80	752.6	119.79	0.0343
-6.526	355.5	56.587	0.0184
-35.60	304.9	48.533	0.1159
ν = 5.0			
-11.05	234.3	37.297	0.0471
-8.361	153.0	24.349	0.0546
-2.789	125.1	19.906	0.0223
-2.089	46.58	7.4127	0.0448
-2.207	4.107	0.6536	0.4734
0.000	0.000	0.0000	0.0000
0.000	0.000	0.0000	0.0000

TABLE 3. ROOTS OF MODIFIED THREE STATE MODEL

Real Part	Imaginary Part	Frequency (Hz)	Damping Ratio
-44.74	1 273	202.59	0.0351
-23.83	749.5	119.29	0.0318
-7.525	355.1	56.516	0.0212
-37.18	304.5	48.464	0.1212
ν = 5.0			
-11.13	234.3	37.293	0.0474
-7.830	152.5	24.274	0.0513
-2.066	124.2	19.762	0.0166
-1.915	46.45	7.3935	0.0412
-2.180	4.007	0.6377	0.4779
0.000	0.000	0.0000	0.0000
0.000	0.000	0.0000	0.0000

TABLE 4. ROOTS OF SIX STATE MODEL

	12 Hz Flutter		48 Hz Flutter	
	Velocity (kts)	Frequency (Hz)	Velocity (kts)	Frequency (Hz)
Three State	1448	12.3	1458	47.1
Three State (Mod)	1426	12.1	1455	48.6
Six State	1448	12.7	1436	47.6
Interpolation	1446	12.4	1419	47.2

TABLE 5. FLUTTER SPEEDS AND FREQUENCIES

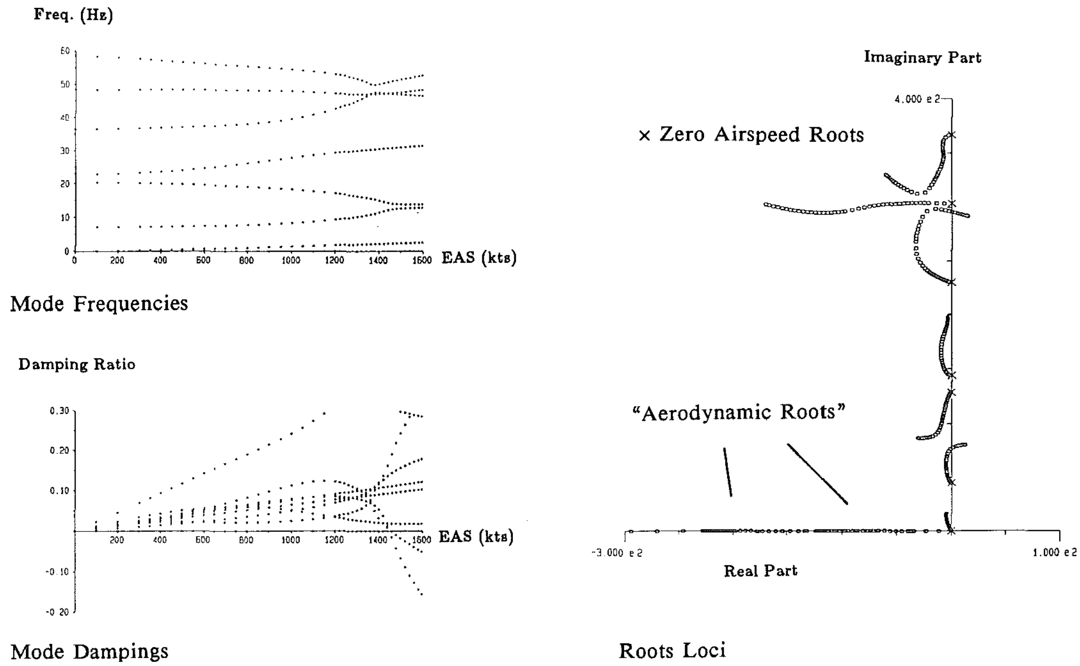

FIG. 1 SYSTEM ROOTS. THREE STATE FIT TO AERODYNAMICS

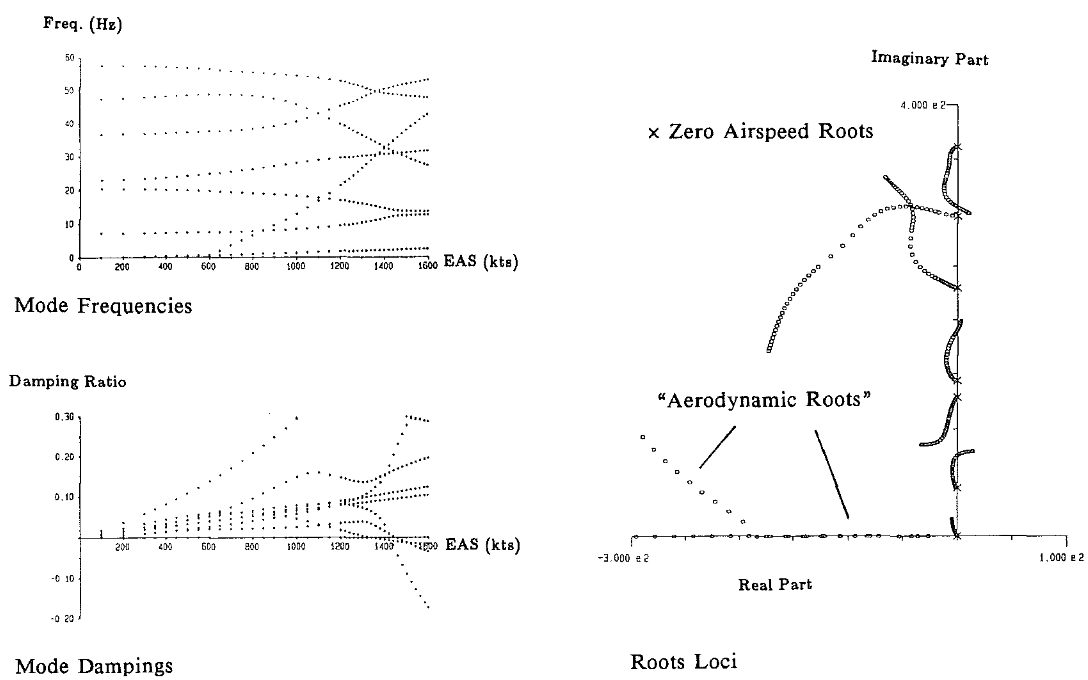

FIG. 2 SYSTEM ROOTS. MODIFIED THREE STATE FIT TO AERODYNAMICS

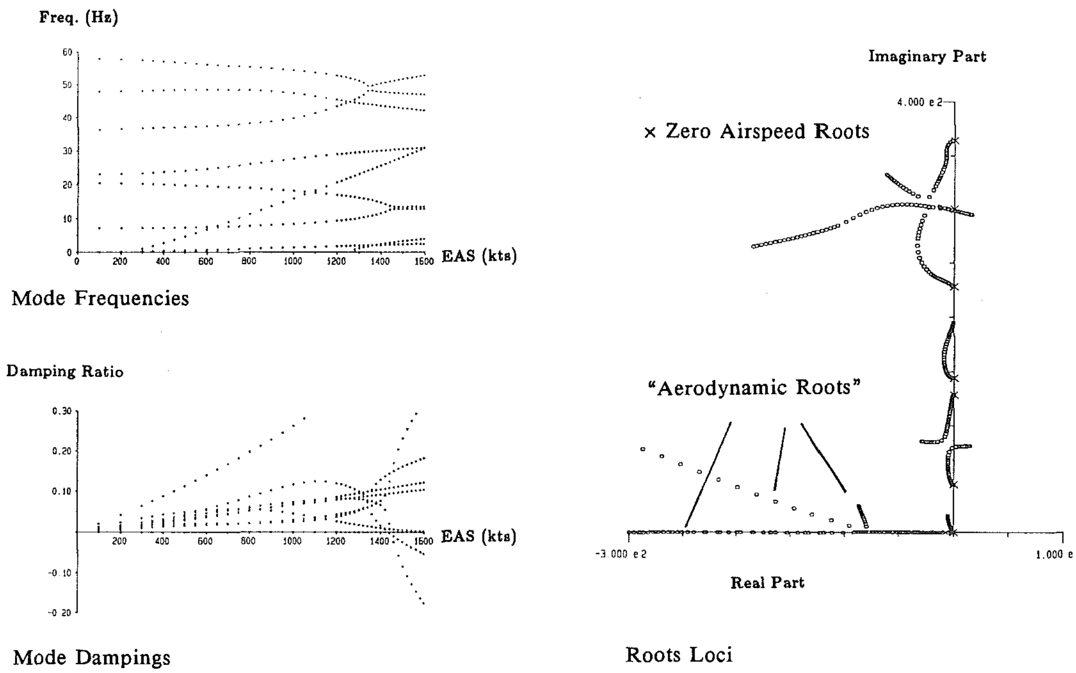

FIG. 3 SYSTEM ROOTS. SIX STATE FIT TO AERODYNAMICS

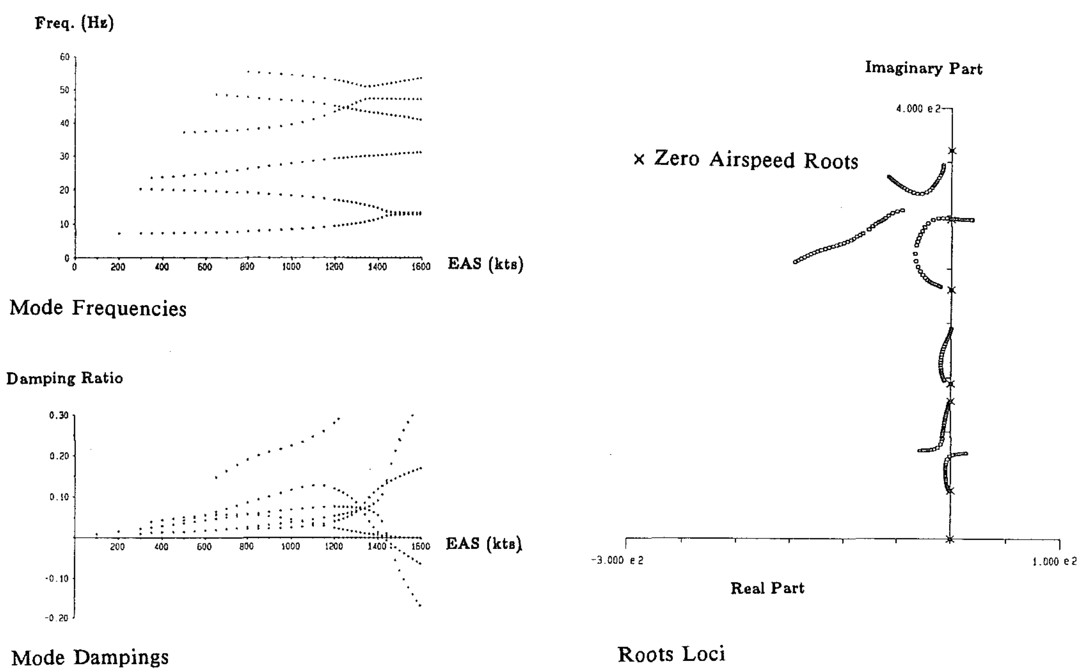

FIG. 4 SYSTEM ROOTS. INTERPOLATED AERODYNAMICS

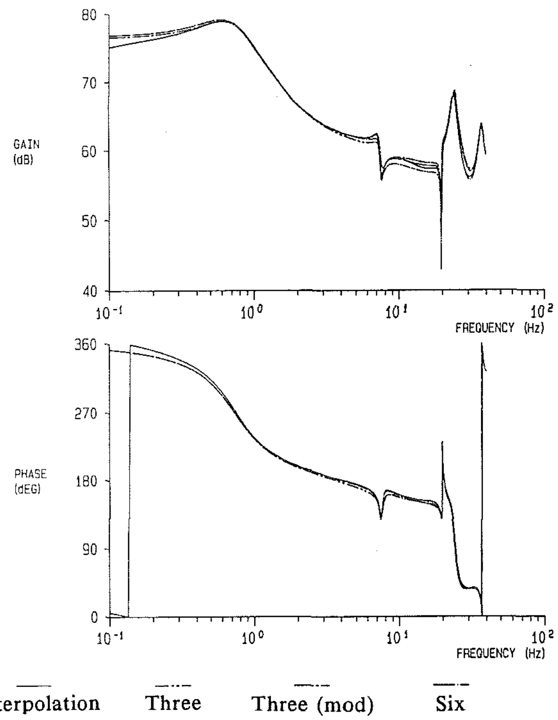

FIG. 5 FREQUENCY RESPONSE. HEAVE ACCELERATION / INBOARD FLAP DEFLECTION. COMPARISON OF FITS.

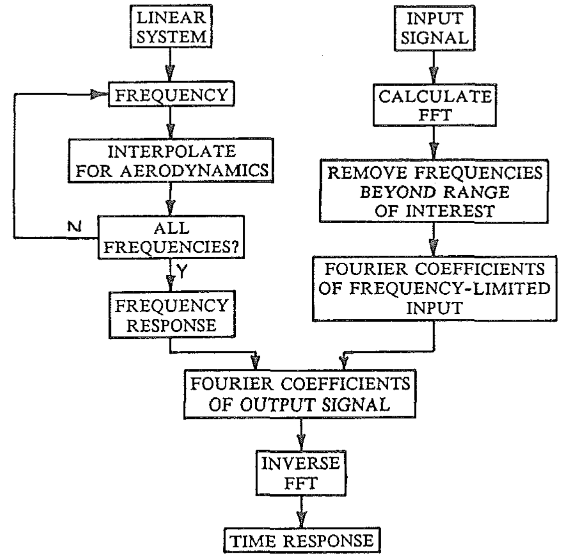

FIG. 6 OBTAINING TIME RESPONSES FROM ORIGINAL AERODYNAMIC DATA.

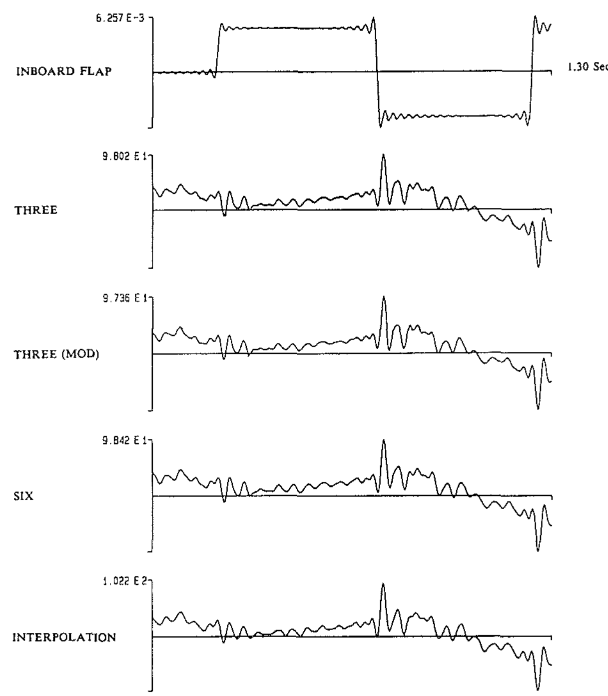

FIG. 7 RESPONSE OF WING-TIP LEADING EDGE ACCELERATION TO INBOARD FLAP.

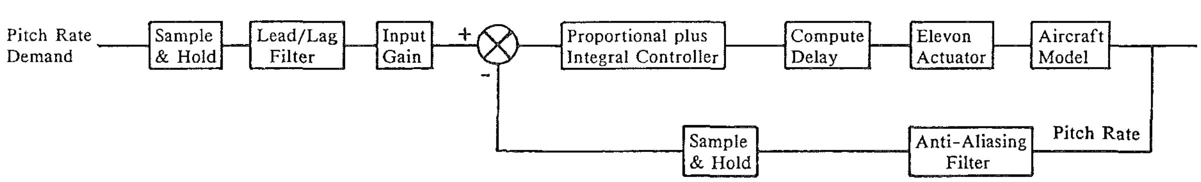

FIG. 8 CONTROL SYSTEM BLOCK DIAGRAM.

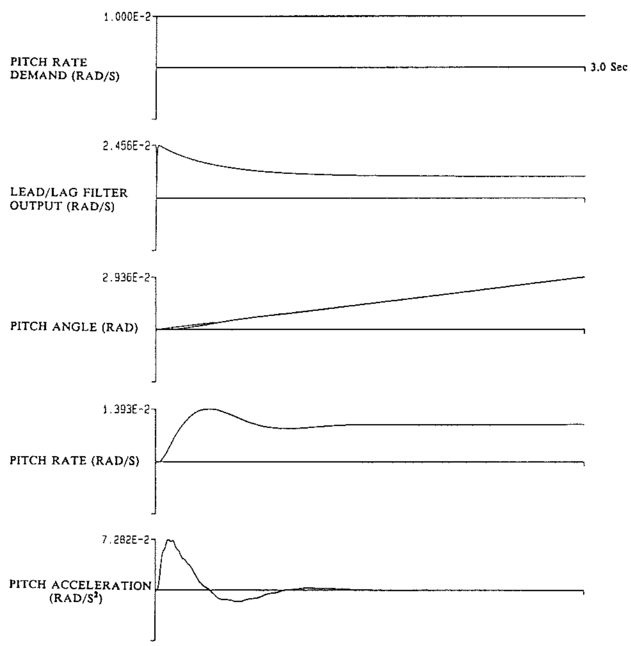

FIG. 9 PITCH RESPONSE OF AIRCRAFT TO A STEP DEMAND IN PITCH RATE. 3 STATE FIT.

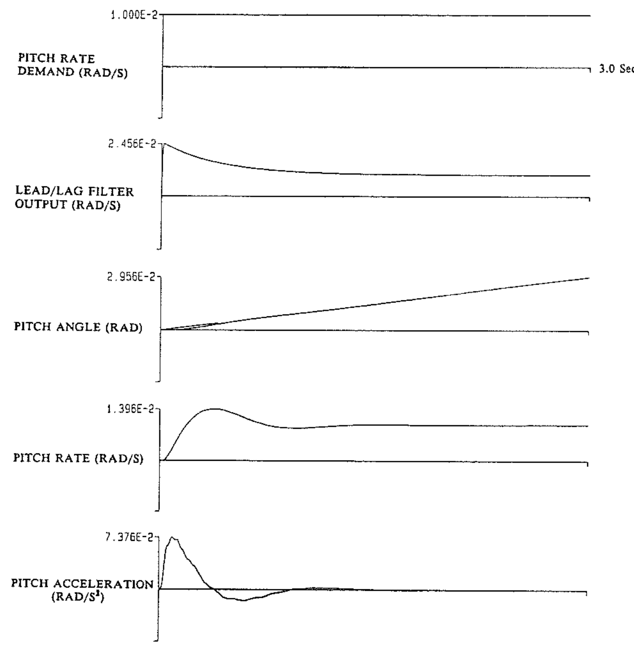

FIG. 10 PITCH RESPONSE OF AIRCRAFT TO A STEP DEMAND IN PITCH RATE. 6 STATE FIT.

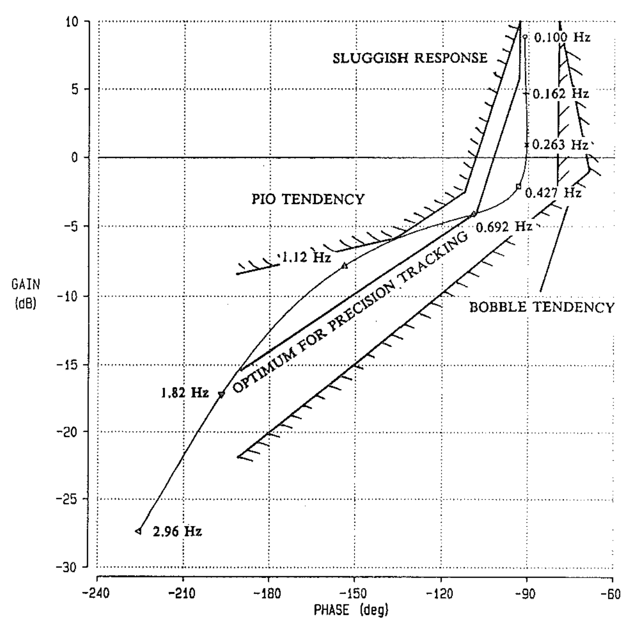

FIG. 11 RESPONSE OF AIRCRAFT PITCH ATTITUDE TO PITCH RATE DEMAND. 3 STATE FIT.

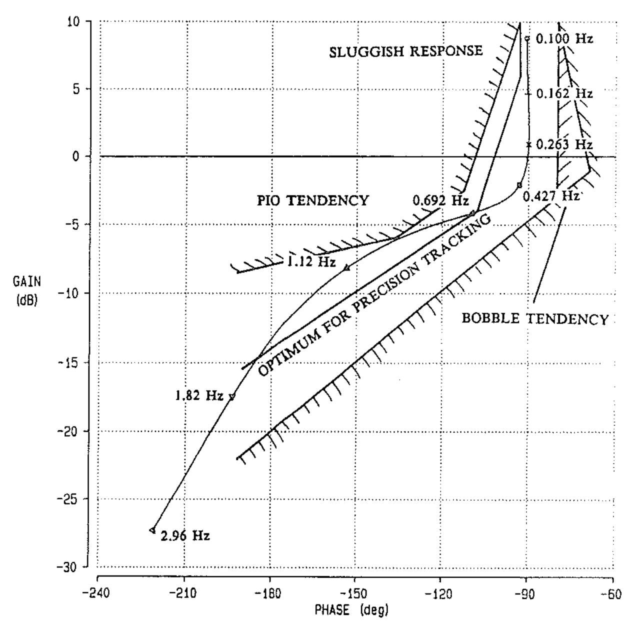

FIG. 12 RESPONSE OF AIRCRAFT PITCH ATTITUDE TO PITCH RATE DEMAND. 6 STATE FIT.

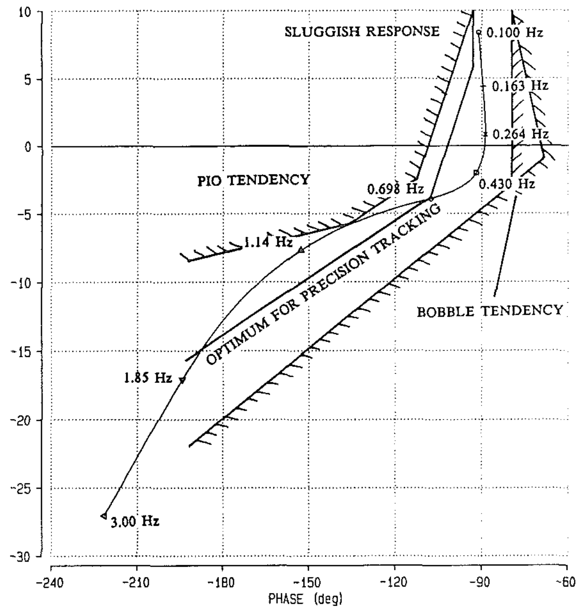

FIG. 13 RESPONSE OF AIRCRAFT PITCH ATTITUDE TO PITCH RATE DEMAND. INTERPOLATED AERODYNAMICS.

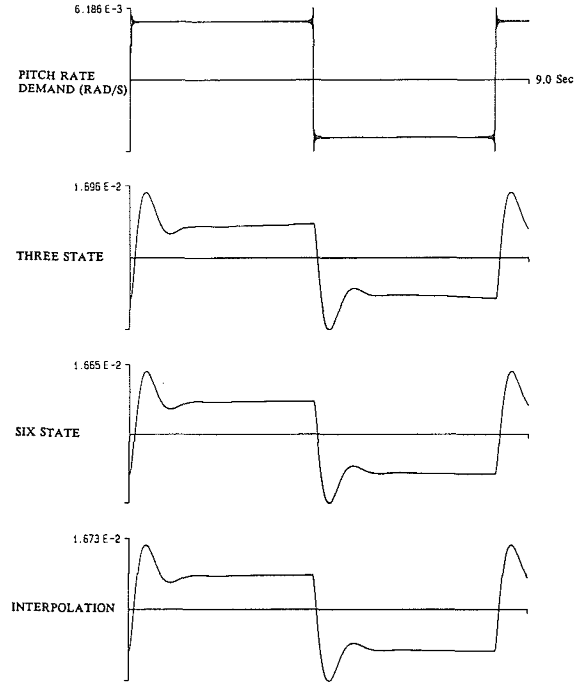

FIG. 14 RESPONSE OF AIRCRAFT PITCH RATE TO INPUT DEMAND. COMPARISON OF FITS AND INTERPOLATION.

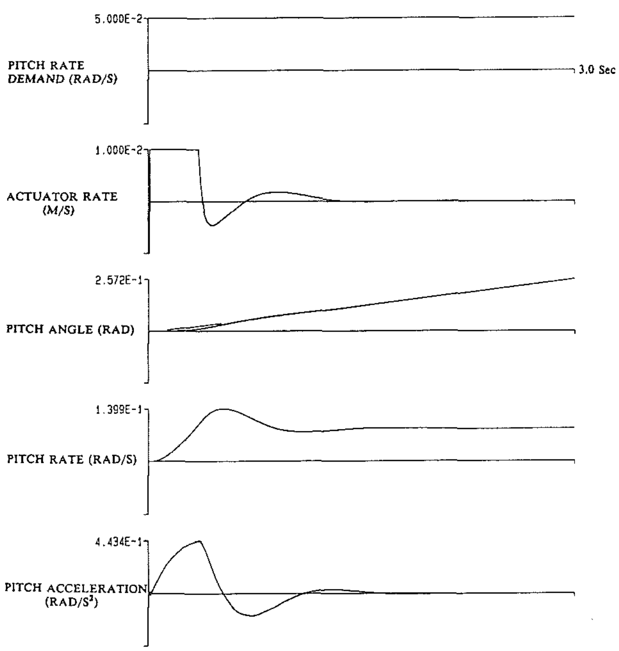

FIG. 15 PITCH RESPONSE OF AIRCRAFT TO A STEP DEMAND IN PITCH RATE. 6 STATE FIT WITH ACTUATOR RATE LIMIT.

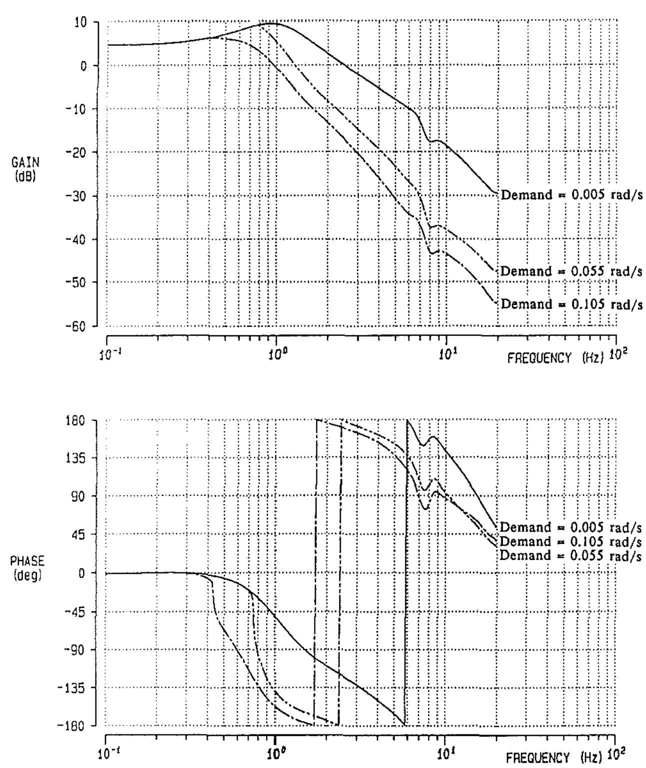

FIG. 16 FREQUENCY RESPONSE OF NONLINEAR SYSTEM FOR VARYING INPUT AMPLITUDES. AIRCRAFT PITCH RATE / DEMANDED PITCH RATE.