THE APPLICATIONS OF ACTIVE CONTROL TECHNOLOGY FOR VIBRATION CONTROL IN AERONAUTICAL AND ASTRONAUTICAL ENGINEERING

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Abstract

This paper summarizes our analytical and experimental studies on active control technology for control of vibration response and dynamic stability of mechanical system in aeronautical and astronautical engineering in recent years. There are two kinds of meaning for application of active control technology. As usual one is the application of closed loop feedback control. The other one is 'open loop' design for vibration control using the ideas of modern control theory. The studies mentioned are briefly reported here.

I . Introduction

The goal of vibration study is to control vibrations. It is necessary and possible to find out more efficient vibration control modes and design methods according to the developments of modern sciences and technologies, and more and more stringent requirements for the products and structures in the field of aeronautical and astronautical engineering. Active control technology, because of its potential features superior to passive control technology, is now becoming a kind of powerful means for solving more complicated and difficult problems of vibration control, to which more and more dynamics and control specialists highly pay attention, especially in the field of aeronautical and astronautical engineering.

Modern control theory provides powerful means for control design. But until now many topics worthy to be further studied for vibration control remain. This paper summarizes our analysis and test researches on active vibration controls which are important for their applications in aeronuatical and astronautical engineering, and tries to open a new approach of design for passive vibration control, i. e. using the ideas of modern control theory for optimization design. The former is dealt with active flutter suppression for aircraft, active vibration isolation for helicopter and active vibration reduction for flexible structures. The latter is dealt with "Ground Resonance" suppression for helicopter and optimization design of vibration reduction for complicated system.

II . Active Flutter Suppression of the Wing

The unstable modes become stable using closed loop control, and critical flutter speed can be increased[1,2]. The state and output equations of the servoaeroelastic system can be written as

$$\dot{X}_S = F_S X_S + G_U U + G_\xi \zeta \tag{1}$$

$$Y_S = H_S X_S + Y_0 \tag{2}$$

The state and output equations of dynamic output feedback controller can be written as

$$\dot{X}_C = A X_C + B Y_S \tag{3}$$

$$U = C X_C \tag{4}$$

By combining equations (1)-(4), the equation of closed loop system (see Fig. 1) can be obtained.

$$\dot{X}_a = F_a X_a + G_a \eta_a \tag{5}$$

The meaning of the notations in mentioned equations is shown in reference[2]. It is ideal method for determining matrices A, B and C to use suboptimal output feedback methodology. But usually there exist two kinds of problem, i.e. difficulty for choosing initial values of iteration for controller and poor convergence of iteration. According to the characteristics of aeroelastic system and the idea of pole placement we presented the gradually changing parameter design method. Thus the problems mentioned above can be solved. The optimization with unequality constraints is constructed as follows.

Determine the design variables in matrices A, B and C to minimize the following performance index J_S:

$$J_S(A,B,C) = \frac{1}{2} E\left[\int_0^\infty (Y_S^T Q_1 Y_S + U^T Q_2 U) dt\right] \tag{6}$$

with least stability margin

$$g_S(A,B,C) = \mathrm{Max} Re[\lambda_i(F_a)] < -\sigma \tag{7}$$

in which $\lambda_i(F_a)$ is i-th eigenvalue of matrix F_a and E is ensemble averaging.

Table 1 shows the calculated and measured values of critical flutter speed and frequency for delta wing test model. The critical speed of controlled system (26.7 m/s) is increased by 37%, compared to that of uncontrolled system (19.5 m/s).

Table 1 Comparisons for Calculation and Test

calculation (test)	uncontrolled	controlled
critical speed	19.8 (19.5 m/s)	27 (26.7 m/s)
flutter frequency	6.72 (7.0 Hz)	6.58 (7.125 Hz)

Ⅲ. Active Dynamic Antiresonant Vibration Isolator for Helicopter

Dynamic Antiresonant Vibration Isolator (DAVI) has been used for vibration isolation of helicopter rotor and equipments because of its very low transmissibility in specified low frequency ranges and at the same time not too low stiffness of the supported spring. In order to further improve its performance it is necessary to use feedback control to obtain lower transmissibility in several specified frequency ranges and more satisfactory isolation in wider frequency ranges, thus a kind of active DAVI (A-DAVI) is constructed the mechanical model of which is shown in Figure 2.

The closed loop transfer function of active isolation system can be written as

$$\Phi(s) = X(s)/Y(s) = \frac{Bs^2 + Cs + K}{As^2 + Cs + K - G(s)} \tag{8}$$

The transfer function of the displacement, velocity and acceleration feedback consisted of second order oscillator with zero can be expressed as

$$G(s) = -\left[K_1 + K_2 s + K_3 s^2 + \frac{\lambda s^3 + a s^2}{s^2 + \beta s + \gamma}\right] \tag{9}$$

The parameters of A nad B in equation (8) are ones related to DAVI. By analysing equation (8) the influences of the feedback parameters on the transmissibility and stability can be known[3]. The feedback parameters K_1, K_2, K_3, α and λ are determined by optimization method. The objective function J is

$$J = \int_0^{w_0} |\Phi(\omega)| d\omega \tag{10}$$

in which ω_0 is upper frequency limit. There are two kinds of constraint conditions: (1) the requirement of stability: corresponding to the roots with negative real part for characteristic polynomial for equation (8). (2) the limit of control force: in the case of second order oscillator with zero the ratio of control force to the acceleration amplitude T_F at two frequencies should be controlled. One is the value $T_F(\omega_{R_1})$ at resonant frequency ω_{R_1}, and other one is the value $T_F(\omega_{A_2})$ at antiresonant frequency ω_{A_2} produced by active feedback, i. e. the following conditions should be satisfied.

$$T_F(\omega_{R_1}) \leqslant T_1, T_F(\omega_{A_2}) \leqslant T_2 \qquad (11)$$

in which T_1 and T_2 are selected according to specified vibration environments. Figure 3 shows the measured transmissibility curve of ADAVI with velocity, acceleration feedback and second order oscillator. it is known from the results of analyses and tests that the vibration isolation of ADAVI is superior to that of DAVI in reduction of resonant frequency and its amplitude, increase of effective frequency width of isolation, formation of several antivibration ranges, reduction of transmissibility at higher frequency range and increase of stability margin.

Ⅳ. Active Vibration Control for Flexible Structures

The flexible structures essentially belong to infinitive degrees of freedom systems, which still are high order system after discretion. How to design a low order controllers and simplify the design for controllers are important practical issues. Besides, the selection of position for sensors and actuators give direct influences on the performances of closed loop system.

Optimization Design for Low Order Controllers[4]

Here we presented an improvement on optimization design method for the controllers to meet the requirements of both modal damping ratio and small control energy. Furthermore, the control law designed for reduced order model can satisfy the design requirements for high order design model after local modifications.

The state and output equations for high order design model are expressed as

$$\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX \end{aligned} \qquad (12)$$

The state and output equations for dynamic output feedback controllers can be written as

$$\begin{aligned} \dot{S} &= DS + FY \\ U &= GS + HY \end{aligned} \qquad (13)$$

in which

$$P = \begin{bmatrix} H & G \\ F & D \end{bmatrix}$$

Equation (12) can be reduced as

$$\begin{aligned} \dot{X}_R &= A_R X_R + B_R U \\ Y &= C_R X_R \end{aligned} \qquad (14)$$

Consider the objective function J as

$$J = E\left[\int_0^\infty U^T R U dt\right] \qquad (15)$$

in which R is positive definite weighting matrix and E is mathematical expectation.

The objective function J can be transformed into the following expression

$$J = Tr(K\bar{Z}_0) \qquad (16)$$

in which K is the solution of Liapunov equation

$$A_r^T K + K A_r = -Q_r \qquad (17)$$

The notations of equation (17) are shown in Ref. [4].

Now the question is to find matrix P in order to minimize J and meet the requirement for modal damping ratio, i. e.

$$G_j = \delta_j - \delta_{j0} \geqslant 0 \ (j = 1, \cdots, r) \qquad (18)$$

in which δ_{j0} is required i-th modal damping ratio and r is its number.

$$\delta_j = -a_j / \sqrt{a_j^2 + b_j^2} \tag{19}$$

Here a_j and b_j are the real and imaginal parts of closed loop eigenvalues respectively.

Usually the controller parameter matrix P obtained for the reduced order system (equation (14)) does not satisfy the design criterion mentioned above for high order design model and needs local modifications. The criterion for modification is the same as that mentioned above.

Decentralized Control of Structural Vibration[5]

The decentralized control method is specially suitable for the structures which need to be controlled partly. Thus the computation amounts of controller design can be reduced.

The global system is devided into N subsystems in physical space. Suppose that the controls between the subsystems can be decoupled, i. e.

$$\dot{X}_i = A_i X_i + B_i U_i + C_i Z_i$$
$$Z_i = \sum_{j=1, j\neq i}^{N} L_{ij} X_j \tag{20}$$

in which C_i is interconnection matrix and Z_i is interconnection vector. The second expression of equation (20) is called interconnection equation. Suppose (A_i, B_i) is controllable completely.

The optimal control of the global system is expressed as

$$MinJ = Min\left[\sum_{i=1}^{N} \frac{1}{2}\int_0^{\infty} (X_i^T Q_i X_i + U_i^T R_i U_i) dt\right]$$
$$= Min \sum_{i=1}^{N} J_i \tag{21}$$

The constraint conditions are equation (20). Assuming the interconnection vector Z_i can be represented as the following dynamical model

$$\dot{Z}_i = A_{Zi} Z_i \tag{22}$$

If $(\widetilde{A}_i, \widetilde{B}_i)$ is controllable completely, according to optimal control there exists

$$U_i^* = -R_i^{-1} B_i^T P_i W_i = -G_{i1} X_i - G_{i2} Z_i \tag{23}$$

in which P_i satisfies

$$\widetilde{A}_i^T P_i + \widetilde{A}_i P_i - P_i \widetilde{B}_i R_i^{-1} \widetilde{B}_i^T P_i + \widetilde{Q}_i = 0 \tag{24}$$

and

$$\widetilde{A}_i = \begin{bmatrix} A_i & C_i \\ 0 & A_{Zi} \end{bmatrix}, \widetilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \widetilde{Q}_i = \begin{bmatrix} Q_i & 0 \\ 0 & 0 \end{bmatrix}$$

In order to improve the interconnection trajectories on-line a reference model is introduced the input of which is provided from equation (22) and $\hat{Z}_i$ is changed on-line.

The dynamical equation of subsystem model is expressed as

$$\dot{X}_i = \hat{A}_i X_i + C_i Z_i$$
$$- B_i G_{i2}[\hat{Z}_i - S_i^{-1} C_i^T P_i'(X_i - \hat{X}_i)]$$
$$\dot{\hat{X}}_i = \hat{A}_i \hat{X}_i + (C_i - B_i G_{i1}) \hat{Z}_i \tag{25}$$

and equation (22). The block diagram is shown in Fig. 4.

Optimal Selection of the Positions for Sensors and Actuators[6]

For equation (12) the more general form can be written as

$$\dot{X} = AX + BU$$
$$Y = CX + DU \tag{26}$$

In order to use full state feedback the state observer is needed the form of which is

$$\dot{\hat{X}} = (A - B_C C)\hat{X} + B_C Y + (B - B_C D) U \tag{27}$$

in which $\hat{X}$ is the observation value of X and B_C is the gain matrix of observer.

$$U = -K\hat{X} \tag{28}$$

Thus equations (26)-(28) form a closed loop system. If (A, B, C) is both controllable and observable, the poles of state feedback system and observer can be placed respectively and arbitrarily according to seperation principle. The characteristic equation of observer is

$$|I - C(\lambda I - A)^{-1} Bc| = 0 \tag{29}$$

The characteristic equation of feedback control system of controlled structure is

$$|I - K(\lambda I - A)^{-1}B| = 0 \qquad (30)$$

Therefore the combined optimization design for the poles of system and the positions of sensors/actuators consists of the following two parts:

(1) the optimization design for the positions of sensors and the poles of observers

By placing the poles of observers s_0 in a specified complex plane region optimally select the positions of sensors (reflecting in matrix C) to minimize the objective function J_1.

$$J_1 = Tr(B_C B_C^T) \qquad (31)$$

Here constraint conditions are equation (29) in which λ is replaced by s_0, and Real $(s_0) \leqslant -\sigma_1$, $\zeta_2 \leqslant |\text{Real}(s_0)|/|s_0| \leqslant \zeta_1$

(2) the optimization design for the positions of actuators and the poles of feedback control system of controlled structure

By placing the poles of feedback control system s_s in a specified complex plane region optimally select the positions of actuators (reflecting in matrix B) to minimize the objective function J_2.

$$J_2 = Tr(K^T K) \qquad (32)$$

Here constraint conditions are equation (30) in which λ is replaced by s_s, and Real $(s_s) \leqslant -\sigma_2$, $\zeta_4 \leqslant |\text{Real}(s_s)|/|s_s| \leqslant \zeta_3$

V. Active Vibration Control Based on Tolerance Index and Robustness

For practical applications the vibration response of controlled structures is restricted within a specified value, and the parameters of the system (reflecting in system matrix), external disturbances and controls are different from those for design according to all kinds of uncertain factors. These can be expressed as bounded uncertainties.

In the case of state feedback $U = -KX$, If there exist the uncertainties of system parameters and control inputs, the state equation of closed loop system can be expressed as

$$\dot{X} = (A - BK)X + Dd(t) + (\Delta A)X + Bv(t) \qquad (33)$$

in which ΔA is system increment matrix with uncertainty, d(t) is external disturbances the uncertainty of which is expressed as bound constant β_d, and v(t) is uncertainty imposed on effective control which is expressed as bound constant β_v too.

Therefore the unequality for feedback matrix can be deduced in the case of satisfying the tolerance index of vibration response (which is expressed as β_0), and bounded uncertainties and controls[7]:

$$\|QLQ^{-1}\| \leqslant \frac{\beta_0}{\beta_0\|Q(\Delta A)Q^{-1}\| + \beta_d\|QD\| + \beta_v\|QB\|} = U_b^* \qquad (34)$$

in which Q is nonsingular weighting matrix, L is linear operator, $(Lx)(t) = \int_0^t e^{R(t-\tau)}x(\tau)d\tau$, $R = A - BK$, $\|\cdot\|$ is the norm of $(\cdot)$. In order to satisfy equation (34) the eigenvalues of linear operator L should be placed reasonably. Additional constraints, such as the placements for some eigenvectors, may be added to avoid the nonuniqueness of feedback matrix. Ref. [7] shows the active vibration isolation for one degree of freedom system, in which the effectiveness for low frequency vibration isolation is illustrated from the test results (see Fig. 5).

VI. Optimization Design for Passive Vibration Control Using the Ideas of Modern Control Theory

Optimization Design for Suppressing "Ground Resonance" of Helicopter[8]

For the helicopter with hinged rotor (or equivalent hinged rotor) the occurrence of the mechanical instability accounts for the coupling of fuselage-landing gear system and rotor system under certain conditions. Such kind of instability is called "Ground Resonance".

The optimization design based on pole-region placement for suppressing "Ground Resonance" of helicopter can be transformed into the following constrained optimization problem. For the specified ranges of rotor speed, parameters of system stiffness and damping, optimally select the parameters of the latter to make the necessary damping parameters minimum. At the same time the poles of the whole dynamic system are all within the specified sector zone. In such case the whole system is dynamically stable in a given rotor speed range with certain stable margin.

For a specified rotor speed range $[\Omega_L, \Omega_H]$ the state equation for helicopter "Ground Resonance" is

$$A\dot{X} = B(\{c\}, \{k\}, \Omega)X$$

Find $\{c\}$ and $\{k\}$ to minimize the following objective function J and satisfy the following constrained conditions

$$J = \sum_{i=1}^{n_c} W_i c_i^2$$

$$\delta_j \leqslant \delta_{j0}, \{c\}_L \leqslant \{c\} \leqslant \{c\}_H,$$

$$\{k\}_L \leqslant \{k\} \leqslant \{k\}_H$$

in which the meanings of δ_j and δ_{j0} are the same as those in equation (19).

Because the requirements mentioned above should be satisfied in a given range of rotor speed, the selection for design rotor speeds is essential. The more the design rotor speeds, the larger the amount of calculation. Based on the principle of helicopter "Ground Resonance" we presented four criteria for reducing the amounts of calculation. Furthermore, the optimization design method in different layers is provided for $\{c\}$ and $\{k\}$ which give different influences on the objective function in optimization. Fig. 6 shows the characteristic root locus for space model system before and after the optimization.

Optimization Design of Vibration Reduction Devices for Complex Structures

From the view point of control the action of passive vibration reduction devices for complex controlled system corresponds to the input of control. Here we study the following optimization design problem for vibration reduction, i. e. determine the optimal positions and parameters of vibration reduction devices to make the maximum response of given points within a specified range. In the case of stationary external excitations F(t) the state equation of controlled system with vibration reduction devices is

$$\dot{X} = AX + BU + DF \qquad (35)$$

in which U is control forces, i. e. actions of passive vibration reduction devices.

The objective function with quadratic performance index J can be formed.

$$J = \frac{1}{2}\int_0^{t_f}(X^TQX + U^TRU)dt$$

(1) the determination of optimal positions and control forces of vibration reduction devices

Based on the Maximum Principle and satisfaction of minimum J and equation (35) a nonhomogeneous matrix equation can be deduced.

$$\begin{Bmatrix}\dot{X}\\ \dot{P}\end{Bmatrix} = \begin{bmatrix}A & -BR^{-1}B^T\\ -Q & -A^T\end{bmatrix}\begin{Bmatrix}X\\ P\end{Bmatrix} + \begin{bmatrix}D\\ O\end{bmatrix}F \qquad (36)$$

in which P is costate vector. Equation (36) is a kind of first order nonhomogeneous differential equations with 4n dimension in which $X \in R^n$. Knowing F(t), X(t) and P(t) can be solved from equation (36). The control forces are obtained according to the expression $U = -R^{-1}$

B^TP.

In order to determine the optimal positions and control forces of vibration reduction devices two iterations are needed.

A. The maximum responses of given points (say X_c) should be limited in a specified range by adjusting the weighting matrices R and Q.

B. Determine the control forces U, and retain the larger components of U to decrease the number of vibration reduction devices.

Usually these two iterations are repeated several times. Thus more effective and less number of vibration reduction devices can be obtained. At the same time their positions are determined and the requirements for maximum responses are satisfied.

(2) the optimization for the parameters of vibration reduction devices

Based on the principle that the forces acted on the controlled system by passive vibration reduction devices are as closer as possible to the optimal forces, the parameters of vibration reduction devices can be determined. The method of curve fitting is one of the useful methods. Ref. [9] shows an example for a three degrees of freedom system. In the case of harmonic excitation with frequencies covered two modal frequencies of the system and given limits for maximum response at specified point, the optimal number of dynamic absorbers, its positions and parameters can be determined by using minimum norm method[9] (see Fig. 7).

Ⅶ. Conclusions

Our analytical and experimental studies show that by using active control technology the performances of controlled system are superior to those by using passive vibration control. Some are impossible for passive techniques. Furthermore, the idea of active control technology opens a new way for optimization design of passive control.

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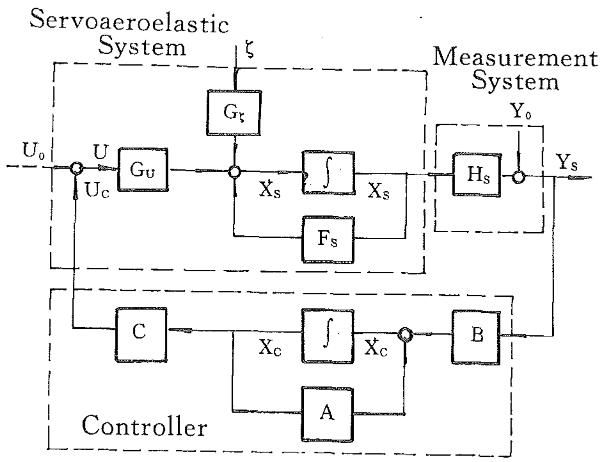

Figure 1 Block Diagram for AFS

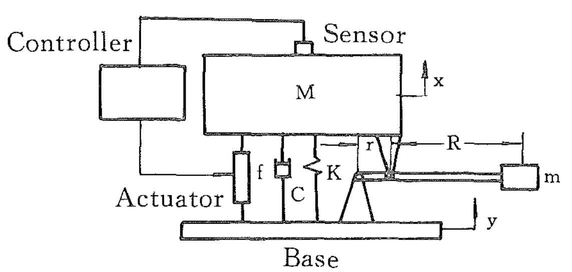

Figure 2 Active Dynamic Antiresonant Vibration Isolator

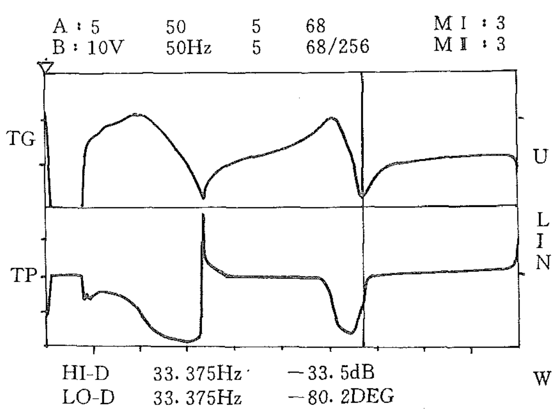

Figure 3 Transmissibility for ADAVI

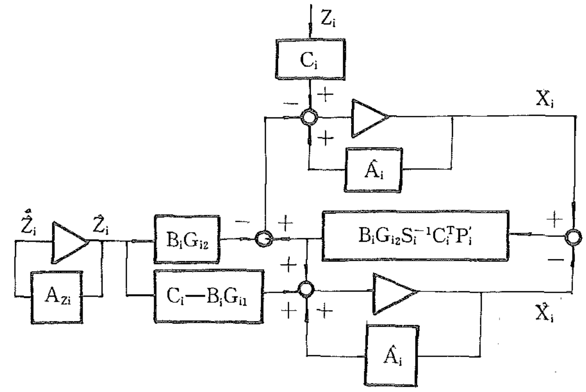

Figure 4 Block Diagram for Decentralized Control

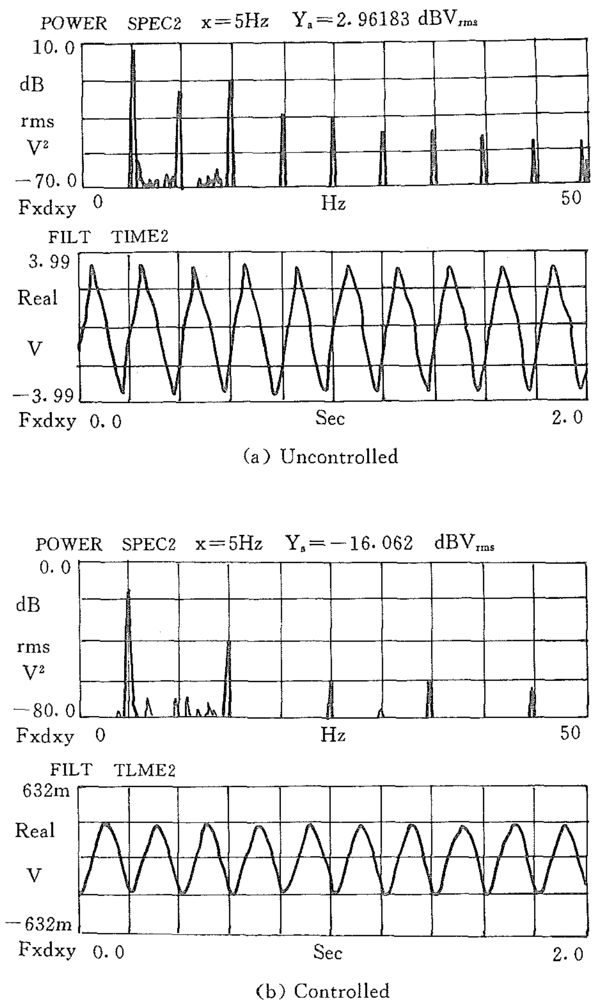

Figure 5 Test Results for One Degree of Freedom Uncontrolled and Controlled System

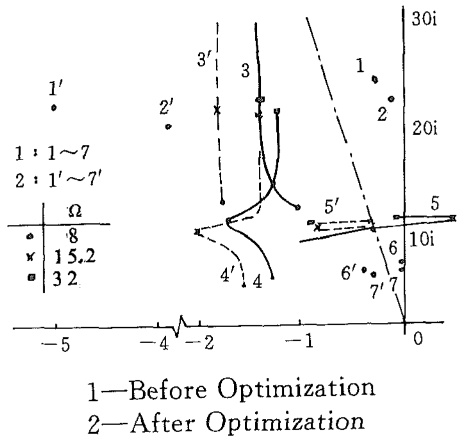

Figure 6 Pole Locus for Space Model of 'Ground Resonance'

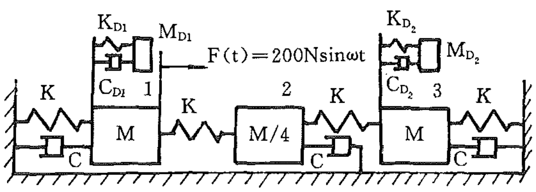

Figure 7 Dynamic Vibration Absorbers for 3 Degrees of Freedom System