#### A FUZZY DYNAMIC ANALYSIS METHOD FOR AEROMAINTENANCE SYSTEM

Li Xueren

Yao Hong

Air Force College of Engineering 3rd Department Xi'an, Shaanxi, 710038, P.R. of CHINA

#### Abstract

In this paper the authors first proposes the approach of a fuzzy system dynamic modeling for a large system by applying the technique of the fuzzy optimum estimation on the basis of traditional system dynamic and the fuzzy mathmatical and morder control theory. The method of modeling is more convenent quantum to analysis the interation with many factors on aeromaintenance system which has been vertified through the stimulating the special aeromaintenance system there force the new way has been opened up for realizing the system optimum runing.

Keyword: system dynamic, fuzzy mathmatic mathmatic model, aeromaintenance

#### 1. Introduction

Aeromaintenance system can be considered as a large system, in which many factors for example the quality of maintenance people and maintenance cost or maintenance magnament and maintenance providing and so on, are included. In order to make the system optimum running it is urgent need to search the method by analying the mechanism of system runing in aeromaintenance field. With developing of control engineering and fuzzy mathmatics and system dynamic in recent years a new way may be provided for quantum analying aeromaintenance system by synthesizing the fuzzy mathmatics theory and states space theory or system dynamic method, the method of fuzzy system dynamic modeling can be presented in this paper, then the traditional system dynamic is developed forwards a large step.

## 2. Basic Principle

The thought of fuzzy system dynamic is described as firstly, the dynamic equation of system to be analyzed is set up by using the method of traditional system dynamic, then the experment of the stimulation is corried out based on historical date and input information scondly, based on output information of the stimulation of system dynamic, the fuzzy state equation of system is set up by using some proper fuzzy transform and the relation analysis. Thirdly, the fuzzy state equation of system is lineareel and optimum decopled, at last, for the fuzzy state equation of system which has been simplified the matrix of weighted modification coefficient which can be obtained by analyzing

Copyright © 1992 by ICAS and AIAA. All rights reserved.

and calculating with the weighted matrix, is used to modify the system dynamic equation so F.S.D.E (ie the fuzzy system dynamic equation) can be obtained then the analysis for aeromaintenance system can be carried out by using F.S.D.E. The system principle is shown in the block diagram of Fig 1.

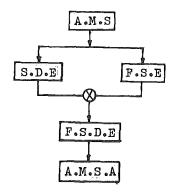


Figure 1. The block diagram of the system principle

The map of system causality and system flow diagram are given out respectively in Figure 2. and Figure 3.

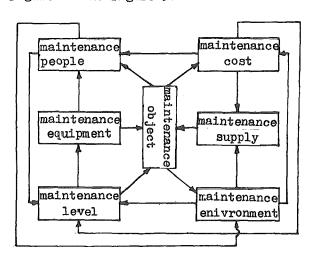


Figure 2. system causality map

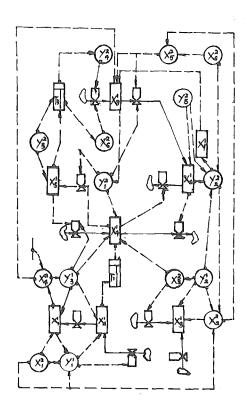


Figure 3. system flow diagram

#### 3. Mathmatical model

Based on system causality map and system flow diagram, the system dynamic equation can by set up, where  $X'_{m\times 1}$  is the rector of flow level variable,  $X^2_{n\times 1}$  is the vector of other variable, but also two type of fuzzy state equation are set up respectively as below

$$\dot{X}^{1} = A_{1}X^{1} + B_{1}U_{1} + F_{1}$$
 (1)

$$\dot{X}^2 = A_2 X^2 + B_2 U_2 + F_2$$
 (2)

where  $F_i$  ,  $F_z$  is the function of interacting of fuzzy factors.

#### 3.1 Determining Cognate matrix and coupling evaluation matrix

Firstly, the cognate matrix of main system can be determined as below Let  $X_i^t(kl) = X_{kl}$ ,  $X_j^t = (kl) = y_{kl}$ 

where 
$$l=1$$
, ... N,  $X_i^1(kl)$ ,  $X_i^1(kl)$ 

are the output of the stimulation of two flow level variable in the 1 time, on which are influnced by the corgnate factor K, then the relation analysis of two variable can be carried out as below

$$\rho_{k} = \frac{N \circ \sum_{l=1}^{N} x_{k \, l} \circ y_{k \, l} - \sum_{l=1}^{N} x_{k \, l} \circ y_{k \, l}}{N \circ \sum_{l=1}^{N} x_{k \, l}^{2} - \sum_{l=1}^{N} (y_{k \, l})^{2}}$$
(3)

let 
$$\rho_{ij} = \sum_{k=1}^{t \ i \ j} \rho_k \tag{4}$$

where  $\rho_{i,j}$  is subordinated to fuzzy normal distribution thereforce the congnate matrix T is defined as

$$T = \left(T_{ij}\right)_{m \times n} = \left(\frac{1}{\rho_{ii}}\right)_{m \times n} \tag{5}$$

Similarty, the coupling evaluation matrix of other variable in subsystem can be get as below

$$V_{ij} = \frac{1}{m} \sum_{k=1}^{m} \sum_{\substack{j=1 \ i \neq i}}^{n} Q_{ij}^{2} Y_{ijk}$$
 (6)

so 
$$V = (V_{i,i})_{n \times n}$$
 (7)

where  $Y_{ijk}$  is the fuzzy functional of whighten between the i variable and j variable on condication of the thinking of k target,  $Q_{ijk}$  is the fuzzy relation coefficient between the i variable and the j variable, it can be caried out in the same way as Equ. (3) and (4).

## 3.2 Linearel the fuzzy state equations Let the fuzzy transform matrix be

as below respectively

$$P_{1m \times m} = T^{t} \cdot (A_{1^{o}} F_{1}) \cdot T \tag{8}$$

$$P_{2n\times n} = V^{t} (A_{2} \circ F_{2}) \circ Vt \, r(P_{1})$$
(9)

then let 
$$x^1 = x^{-1} \circ P_1$$
,  $x^2 = x^2 \circ P_2$  (10)

Taking Equ.(10) into Equ.(1) and Equ.(2) we can get:

$$\dot{X}^{1} = \ddot{A}_{1} \dot{x}^{1} + \ddot{B}_{1} \ddot{U}_{1}$$
 (11)

$$\dot{\overline{X}}^2 = \overline{A}_2 \overline{x}^2 + \overline{B}_2 \overline{U}_2 \tag{12}$$

where o is the fuzzy operator

$$\bar{\mathbf{A}}_{1}$$
=diag( $\bar{a}\mathbf{1}_{11}$ ,  $\bar{a}\mathbf{1}_{22}$ , ...  $\bar{a}\mathbf{1}_{mn}$ ) (13)

$$\bar{A}_z = \text{diag}(\bar{a}_{2_{11}}, \bar{a}_{2_{22}}, \dots \bar{a}_{2_{nn}})$$
 (14)

Because of some parameters of the matrix  $A_1$  and  $A_2$  being unknown, we must estimate these parameters by using the method of optimum estimation based on the stimulation output information of S.D.E thus two vectors of unknown parametrers  $\hat{\theta}_1$ ,  $\hat{\theta}_2$  can be estimated according to the below formulation

$$\hat{\theta}_{1} = \left( \phi_{1}^{T} \circ \vec{P}_{1}^{-1} \phi_{1} \right)^{T} \phi_{1}^{T} \cdot \vec{P}_{1}^{T} \circ y^{1}$$

$$(15)$$

$$\hat{\theta}_{2} = (\phi_{2}^{T} \circ P_{2}^{-1} \phi_{2})^{-1} \phi_{2}^{T} \circ P_{2}^{-1} y^{2}$$
(16)

# 3.3 Determining the optimum modified Weighten matrix

Fistly, determining the weighten matrix of information on system architecture. when taking  $\hat{\theta}$  and  $\hat{\theta}_2$  into Equ (13) and (14), we get two equations as below

$$\hat{\hat{x}}^1 = \hat{A} \cdot \hat{x}^1 + \hat{B} \cdot \hat{U}$$
 (17)

$$\hat{\hat{x}}^2 = \hat{A}, \hat{x}, + \hat{B}, \hat{U}, \tag{18}$$

Let characteristic equation of above two equation be as below respectively

$$\left| \mathbf{S}_{1} - \hat{\mathbf{A}}_{1} \right| = 0 \tag{19}$$

$$\left| \mathbf{S}\mathbf{1} - \hat{\mathbf{A}}_{z} \right| = 0 \tag{20}$$

Sloving the two equations (19) and (20) we can obtained some characteristic roots as  $S_1$ , ...  $S_p$  or  $\lambda_1$ , ...  $\lambda_e$ . where  $P \le m$ ,  $1 \le n$   $S_i + S_j$ ,  $\lambda_i + \lambda_j$  i, j = 1, ... m or i, j = 1, ... m thus the weighten matrix of information of system construction can be defined as below

$$\overline{W}_1 = \left( \frac{1}{S}, \frac{1}{S}, \frac{1}{S} \right)_{P \times 1}$$
 (21)

$$\overline{W}_{z} = \left(e^{-\frac{1}{\lambda_{1}}}, \dots e^{-\frac{1}{\lambda_{I}}}\right)_{I \times 1} \tag{22}$$

Secondly, Determing the optimum targets. The optimum targets can be defined as below

$$J_{n} = min\left\{ \left( \widehat{x}^{1} - x^{1} \right) \left( \widehat{x}^{1} - x^{1} \right)^{T} \wedge F\left( \overline{W}_{1} \right) \right\}$$
 (23)

$$J_{2}=min\left\{\left(\hat{x}^{2}-x^{2}\right)\left(\hat{x}^{2}-x^{2}\right)^{T} \otimes F\left(\overline{W}_{2}\right)\right\}$$
 (24)

where  $x^1$ ,  $x^2$  are the output of stimulation of S.D.E,  $\hat{x}$  and  $\hat{x}^2$  can be computed from Equ (17) and (18),  $\triangleq$  is fuzzy operator,  $F(w_1)$ ,  $F(w_2)$  is the synthetic fuzzy function respectively, however  $F(w_1)$  and  $F(w_2)$  are defined as below

$$F(\overline{W}_1) = \sum_{i=1}^{p} u(\overline{W}_{1i})$$
 (25)

$$F(\overline{W}_2) = \sum_{i=1}^{l} u(\overline{W}_{2i})$$
 (26)

where u( w, i) is the satisfying fuzzy function of i flow-variable, u( w, i) is the satisfying fuzzy function of i other variable Note: different variable obey unlike the fuzzy distribution for example, maintemance cost or investment variable and so on obey the distribution as below

$$u(W_{i}) = \begin{cases} 1 & 1 \leq W_{i} \leq \theta_{1} \\ 1 - \frac{W_{i} - \theta_{1}}{\theta_{2} - \theta_{1}} & \theta_{1} \leq W_{i} < \theta_{2} \end{cases}$$
 (27)

however the variable of the construction of knowledge or degree and age or experience of aeromaintenance people obey the distrubition as below

$$u(W_i) = \begin{cases} 0 & 0 \leqslant W_i \leqslant \theta_1 \\ 1 - \frac{\theta_2 - W_i}{\theta_2 - \theta_1} & \theta_1 \leqslant W_1 \leqslant \theta_2 \\ 1 & \theta_2 \leqslant W_i \leqslant \theta_3 \\ 0 & W_i > \theta_3 \end{cases}$$
(28)

Thirdly determining the optimum madified weighten matrix.

let 
$$R_1 = J_{1min} \hat{U}_1 \hat{U}_1^t = (R_{1ij})_{m \times m}$$
 (29)

$$R_z = J_{2min} \hat{U}_z \circ \hat{U}_z^{t} = (R_{2ij})_{n \times n}$$
 (30)

so 
$$\widetilde{x}_1 = x^1 \otimes R_1$$
 (31)

$$\widetilde{x}_2 = x^2 \otimes R_2 \tag{32}$$

where  $R_1$ ,  $R_2$  is the optimum madified weignten matrix,  $\otimes$  is the special fuzzy operator it is defined as below

$$x \otimes R = (\widetilde{x}_1, \cdots \widetilde{x}_n)^t$$

where

$$\widetilde{x}_{i} = \begin{cases} 0 & \left| \frac{x_{i}(t)}{x_{i}(t + \Delta t) - x_{i}(t)} \right| < \sum_{j=1}^{m} R_{ij} \\ x_{i} \sum_{j=1}^{m} R_{ij} < \left| \frac{x_{i}(t)}{x_{i}(t + \Delta t) - x_{i}(t)} \right| > \sum_{j=1}^{m} R_{ij} \end{cases}$$
(33)

# 3.4 Fuzzy system Dynamic equations

Based on above the equations, the fuzzy system dynamic equations can be determined as below

$$\overline{x}_{1}(k+1) = \overline{x}^{1}(k) + (\widetilde{x}^{1}(k) - \widehat{x}^{1}(k))$$

$$(34)$$

$$\bar{x}^{2}(k+1) = \bar{x}^{2}(k) + (\bar{x}^{2}(k) - \hat{x}^{2}(k))$$
 (35)

where  $\overline{x}_1$ ,  $\overline{x}_2$  are the ouput state variable vector of fuzzy system dynamic equations the whole process of computing in detile referred to the scheme of F. S.D.E in Figure 4.

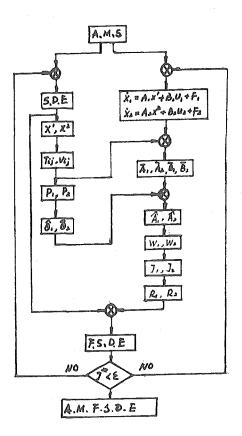


Figure 4. the scheme of F.S.D.e modeling

## 4. Example

Taking the aeromainenance system of the special equipment which are mounted on the some type of aircraft as an example to verify the F.S.D.E. In the process of modeling the aeromaintenance system of special equipment, six flow-level variable are selected, including the quality of aeromaintenance plople and unrepaired equipment or the ability of maintenance's supplying and the cost of maintenance and so on, the number of other variable selected is 102.

F.S.D.E has been stimulated by using the method of Adams estimatecorrection let the interuption of stimulations  $\Delta t =$ 1 unit, the results of the system runing have been stimulated during thirty years. Under the condition of the investment with constrait and the degree of useful equipment being 95%, the analysis of stimulation can be done thereforce it is achieved that the optimum mumber of matintenance people and the optimum rate of construstion of academic attainments or knowledge or ability and the optimum quantity of stock of equipment and the optimum provided of stock of equipment and the optimum period of mainenance, however we have discussed other problems that these factors affected the performance of aeromainenance system in this paper for example the mainenance people adjusting and replacement of generation of test devics of new type and the renewal of special equipment and the change of enivovmment of aeromainenance and so on. The whole process of computing and the analysis in detile is not given out because of the paper space limitations, but the results states that it is key to the efficiency of aeromainenance system that the quelity of aeromainenance people and mainenance regime or the finance of mainenance and so on.

#### 5. Conclusion

In this paper the traditional wystem dynamic have been developed a large step by applying the fuzzy mathematics and morder control theory. The method of fuzzy system dynamic modeling which has been presented is convenient to consider the indefinite complexity influence of factors on the largy system, paticularly the thought of modeling proposed is very adaptable to set up the mathmatic model of large system which includs the man or mangment or socity and so on.

This model has three advantage the one is to analyze the dynamic property of aeromainenance system and the interation relationship with many factors of aeromaintence system easily, the second is convenient to determine the rate of main element which consits of aeromainenance system, the another is convenient to search

the optimum scheme of aeromainenance system runing.

This research proved that the fuzzy transform is effective for dealing with the problem of noliner in the dynamic state equations of large system. The applicability and advantages of this method have been verified through the simulating the mainenance system of the special aero-equipment.

The result states that the quatum mathmatics models of aeromainenance system can be set up by using this method easily thereforce. In order to realize the scientific aeromainterce and mangment, the new way has been presented this paper.

## References

- (1) SUE Xiao kang., (The principle of system Dynamic and application), shanhai university of traffic, 1988.
- (2) R. Ceofferey Coyle, "The use of optimization methods for Policy design in a system dynamics model", « System Dynamic Review », 1985 No1, pp81-91.
  (3) Yong Qidong, "Application of the Eval-
- (3) Yong Qidong, "Application of the Evaluation of Integration Schems to Natural Gas Network system", «System Engineering—Theory and Practice», Vol. No.2, p43-p50.
- (4) He ZhongXiong, «Fuzzy mathmeitics and application», Tianjing, 1982.