INLET DESIGN USING A BLEND OF EXPERIMENTAL AND COMPUTATIONAL TECHNIQUES

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Abstract

The objective of this study was to use a designed experiment to optimize an aircraft gas turbine inlet for all critical operating conditions. The designed experiment achieved an optimum inlet design through the use of an orthogonal array that dictated how the shape variables were combined to arrive at a set of experimental inlets. These inlet shapes were then analyzed by a computational fluid dynamics (CFD) code and rated as to their merit. The "criterion of goodness," also called the response, was the peak Mach number occurring on the inlet surface predicted by the CFD code. The minimization of peak Mach number in the inlet maximizes the pressure recovery at the engine fan by minimizing shock losses and diffusion rates. Following the analysis, the responses were used to develop a polynomial model of the CFD code results using least squares. This polynomial was then optimized to find the inlet shape predicted to have the best peak Mach number without developing additional inlet geometries or running the CFD code.

Nomenclature

Superellipse describing the inlet lip: $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^m=1$

n_c = n exponent at the crown
m_c = m exponent at the crown
n_h = n exponent at the half-breadth (90°)
m_h = m exponent at the half-breadth (90°)
n_k = n exponent at the keel
m_k = m exponent at the keel
a/b_c = ratio of superellipse axes at the crown
a/b_k = ratio of superellipse axes at the keel
CR_c = contraction ratio crown
CR_k = contraction ratio keel
X-ref = distance of drooping
d = offset of hilite and throat centerlines due to droop angle
β = coefficient of linear term of polynomial model
μ = coefficient of quadratic term of polynomial model
M_∞ = freestream Mach
R_h = hilite radius
R_t = throat radius
Y = response (criterion of goodness)

Subscripts

mx = maximum cruise
to = takeoff
cw = crosswind
st = static
rt = rolling takeoff

Introduction

In this study, a designed experiment was investigated to improve on the traditional approach to the design of engine inlets for commercial transport aircraft, a tedious process that ends with a less-than-optimum design. The designed experiment technique of optimization has been most commonly applied to optimize industrial processes that cannot be described mathematically. More recently, it has been applied to problems for which the cost of calculating a computer solution prohibits analysis of numerous designs. In the case of inlet design, the advantage of using a designed experiment comes from minimizing the time spent developing surface geometries and preparing for a CFD analysis. The CFD code in use at The Boeing Company for developing inlets(1) does not allow geometry perturbations to be easily automated, and thus standard optimization packages are not easily applied. However, a designed experiment can minimize the number of inlet designs required to find an optimum without additional tools or code.

Method

Inlet Shape Considerations

The inlet contour governs the delivery of air to the engine fan and thus plays a significant role in engine performance. Aspects of the inlet contour such as lip shape, contraction ratio, and drooping can be optimized to achieve the desired flow conditions at the fan. (Refer to Figure 1 for inlet geometry terminology used in this discussion.)

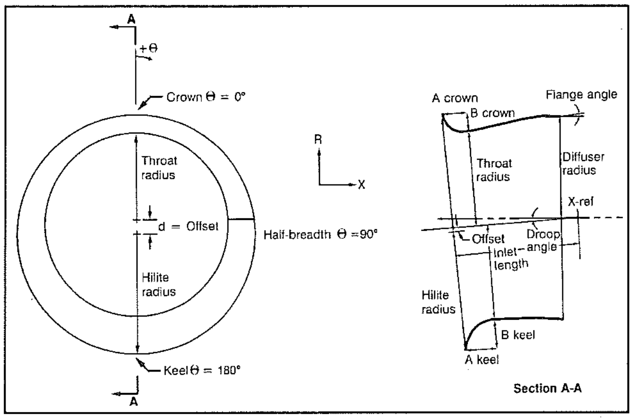
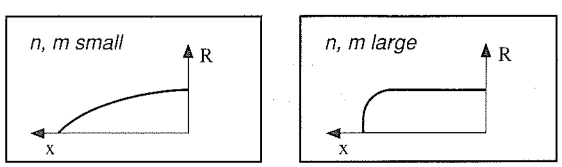


Figure 1. Inlet Cross Section

The inlet lip shape—the contour from the hilite to the throat—can be described by a superelliptical curve, which provides a high degree of flexibility through variation of four parameters. A superellipse has the form:

$$\left(\frac{x}{a}\right)^{n}+\left(\frac{y}{b}\right)^{m}=1$$

where x is measured in the axial direction, y is measured in the radial direction, and a and b are the lengths of the major and minor axes, respectively. Varying the exponents of this equation allows the shape to vary for a given axis length from flat to square, as shown below:



The length of the axes can also be varied to give the lip a long and low profile or a short and wide profile. Since the inlet lip shape must perform well at multiple operating conditions such as high angle of attack, yaw, and cruise, optimization requires that the lip shape vary in the θ or angular direction from the inlet crown to the keel.

Another important factor in inlet performance is the contraction ratio, a term defining the "necking down" or contraction of the inlet duct. It is a function of the ratio of the hilite radius squared to the throat radius squared:

$$CR_c=\left(\frac{R_h-d}{R_t}\right)^2 \qquad CR_k=\left(\frac{R_h-d}{R_t}\right)^2$$

where R_h is hilite radius, R_t is throat radius, and d is the offset of the hilite and throat centerlines due to droop angle. Installation drag considerations define the maximum acceptable hilite radius and thus constrain the upper bound of the contraction ratio. In addition, an average throat Mach number greater than 0.73 during cruise flight conditions leads to poor pressure recovery at the fan. Therefore, the throat radius was fixed to provide an average throat Mach number of 0.685 for cruise airflow, and the contraction ratio was varied by changing the hilite radius.

To minimize installation drag, an inlet is drooped or oriented so that the hilite plane is perpendicular to the oncoming flow. Thus efficient turning of the air flow must be accomplished by the inlet between the hilite plane and the inlet attachment flange. The turning or "drooping" may take place all at once in a bend in the inlet duct, or gradually over the entire distance from the throat to the attachment flange. In this study, the variable name X-ref represents the axial location where the drooping terminates. A range of X-ref locations was explored to discover the influence of drooping in the optimum inlet shape.

The Designed Experiment Methodology

A flexible lip description (superellipse), various flight condition constraints, contraction ratio, and drag effects provided a list of inlet shape parameters important in the optimization of an inlet design. For simplicity, a linear model was selected to define the relationship between the variables and the response. The linear model could be tested for adequacy and improved by including quadratic and interaction terms if necessary. This approach is known as the "central composite designed experiment."(2). The linear model is of the form:

$$Y = ß0 + \sum_{i=1}^{11} ßiXi$$

where Y is the response, Xi is the ith variable, and ßi is the coefficient of the ith variable.

To build a linear model for each variable, the response was determined for two different values of each variable. The variable range chosen defined the limit of the design space being modeled by the linear function of the variables. The range chosen for each variable was derived from past inlet designs and extended to include exploration of more of the design space. The variables and their range of values are shown in Table 1.

Table 1. Variables Chosen for Optimization and Range To Be Explored

Variable	Low value	High value
n_c	2.0	2.6
m_c	1.6	2.4
n_h	2.0	2.6
m_h	1.6	2.4
n_k	2.0	2.6
m_k	1.6	2.4
a/b_c	2.5	3.0
a/b_k	2.0	2.5
CR_c	22.0	26.0
CR_k	26.0	30.0
X-ref	123.0	134.0

A designed experiment uses orthogonal arrays to define the series of experimental inlet shapes that determine a set of responses, which allows the effect of each variable on the response to be determined. The orthogonal array chosen for the linear model of the 11 variables of this experiment is shown in Table 2.

Table 2. Plackett and Burman Screening Design Orthogonal Array

	n_c	m_c	n_h	m_h	n_k	m_k	a/b_c	a/b_k	CR_c	CR_k	X-ref
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
2	-1	-1	-1	-1	-1	+1	+1	+1	+1	+1	+1
3	-1	-1	+1	+1	+1	-1	-1	-1	+1	+1	+1
4	-1	+1	-1	+1	+1	-1	+1	+1	-1	-1	+1
5	-1	+1	+1	-1	+1	+1	-1	+1	-1	+1	-1
6	-1	+1	+1	+1	-1	+1	+1	-1	+1	-1	-1
7	+1	-1	+1	+1	-1	-1	+1	+1	-1	+1	-1
8	+1	-1	+1	-1	+1	+1	+1	-1	-1	-1	+1
9	+1	-1	-1	+1	+1	+1	-1	+1	+1	-1	-1
10	+1	+1	+1	-1	-1	-1	-1	+1	+1	-1	+1
11	+1	+1	-1	+1	-1	+1	-1	-1	-1	+1	+1
12	+1	+1	-1	-1	+1	-1	+1	-1	+1	+1	-1

This orthogonal array was developed by Plackett and Burman for screening significant variables. Below each variable name is a column of -1's and +1's, which represent the normalized low and high value of each variable. Each row defines the combination of the variables for an experimental inlet design. Once each inlet design is developed, the flow analysis completed, and a response determined, a least squares technique is used to fit a linear model to the 12 data points. The orthogonality of each of the column vectors in this matrix allows the effect of each of the variables on the response function to be determined even though more than one variable changes in each experiment.[3] The "main effect" of a variable is defined as the average difference in the level of response as the variable changes from its lowest value to its highest value.[3] For example, referring to Table 2, the main effect of the variable n_c is the difference between the average of the response for all the -1's, where n_c is at the lowest value (the first six runs), and the average of all the +1's, where n_c is at the highest value (the last six runs). Note that the entries in the first six rows under any other variable sum to zero and therefore cancel each other out. Likewise, the last six rows cancel each other out, so the effect of n_c is isolated. The same procedure is used to calculate the main effect of each of the other variables. A linear graph showing a line between the average of the responses at the lowest value of the variable and the average of the responses at the highest value of the variable is called a main effect plot, as shown in Figure 2. The slope of the line on the main effect plot is an indicator of the sensitivity of the response to a change in the design parameter.

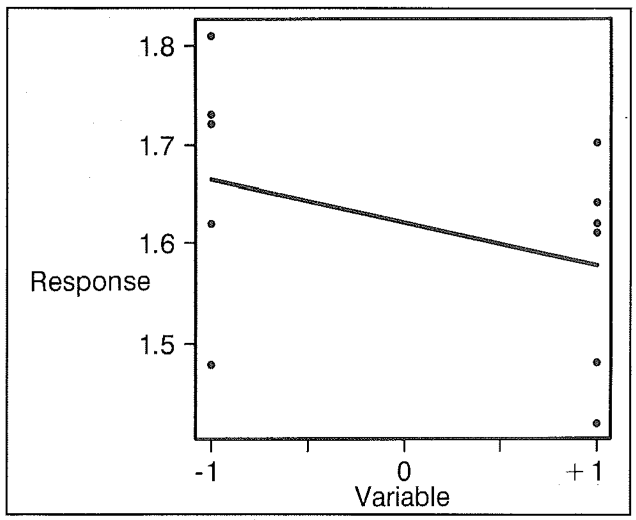


Figure 2. Example Main Effect Plot

A center point is an inlet design in which the values of the variables are at the mean of the high and low values used in the orthogonal array. The use of a main effect plot in conjunction with a center point allowed a check of whether a linear model is accurate over the whole range of the variables tested. The response generated by this center point can be located on the main effect plot and compared to the response predicted by the linear model (the center of the line on the main effect plot). If the observed response for the center point is close to the predicted response, then the linear model may be adequate. If not, it may be necessary to add quadratic terms to the model.

For each quadratic term of a variable to be modeled, two more inlet designs were developed and analyzed. Once confidence in the model was achieved by agreement of the model with data, it was used to find the optimum inlet design. This inlet design was then analyzed as another check of the accuracy of the model.

An inlet design is customarily developed by considering the most extreme operating conditions the inlet will encounter in flight and designing an inlet that will perform acceptably for these conditions. In this study, this was accomplished by analyzing the performance of each inlet at five operating conditions and developing a model of inlet performance for each condition. The operating conditions considered were:

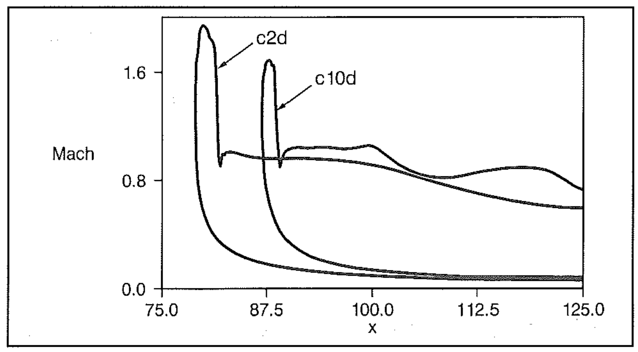
a. Maximum (max) cruise (angle of attack (AOA) = 4°, yaw = 0°, freestream Mach (M_∞) = 0.84).
b. Takeoff (AOA = 26.4°, yaw = 8.83°, M_∞ = 0.274).
c. Crosswind (AOA = 0°, yaw = 90°, M_∞ = 0.023).
d. Static (AOA = 0°, yaw = 0°, M_∞ = 0.01).
e. Rolling takeoff (AOA = 0°, yaw = 26.5°, M_∞ = 0.106).

Experience shows that for peak Mach numbers beyond some upper limit, the performance of an inlet is degraded excessively. Thus it is desirable to optimize efficiency at max cruise without compromising the performance at the other conditions. Once linear models for each condition were developed, the optimum inlet was found using standard linear programming software.

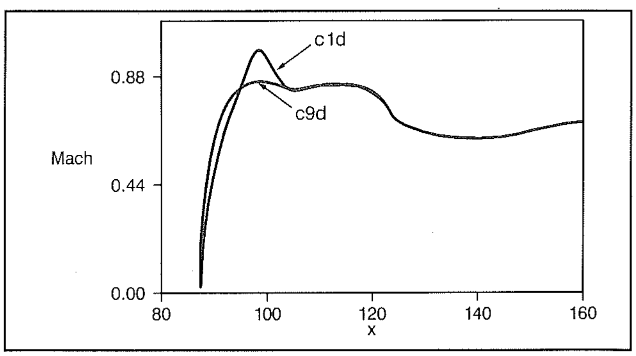
Results

I. Twelve-Experiment Orthogonal Array and Center Point

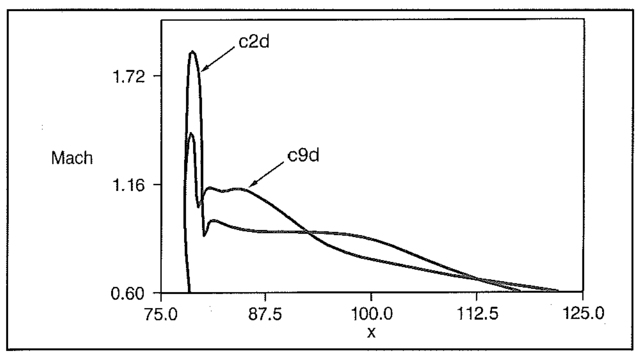
Mach Number Contours and Responses. The results of the flow analysis were plotted as planar cuts through the inlet at 45° angular intervals. Mach number was plotted against axial distance, x, and the overall peak chosen as the response. A sample of the Mach number contours that gave the maximum and the minimum response for each operating condition is shown in Figure 3.



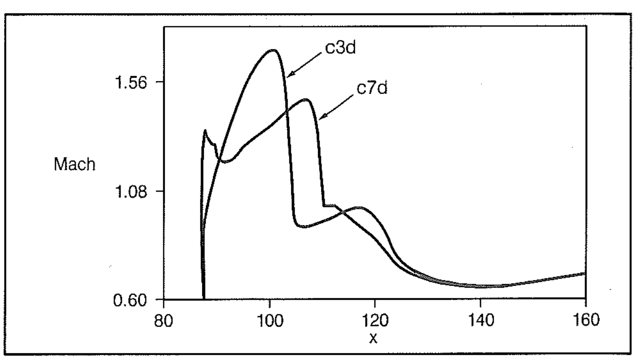
(c) Crosswind, θ = 180°, Inlet c10d; θ = 315°, Inlet c2d



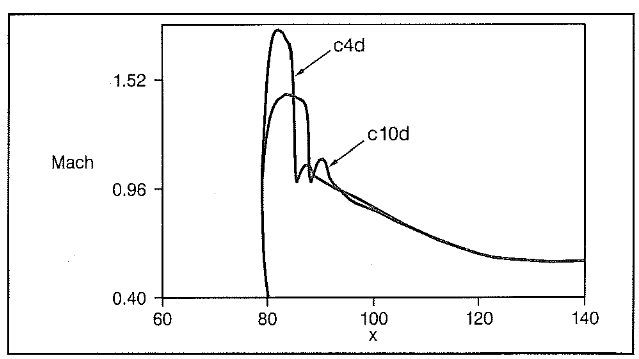
(a) Max Cruise, θ = 180°, Inlets c9d, c1d



(d) Static, θ = 0°, Inlets c9d, c2d



(b) Takeoff, θ = 180°, Inlets c7d, c3d



(e) Rolling Takeoff, θ = 315°, Inlets c10d, c4d

Figure 3. Mach Number Contours

The responses for the first 12 experiments and the center point at each operating condition are given in Table 3.

Table 3. Responses Y (Peak Mach Number) at Max Cruise (mx), Takeoff (to), Crosswind (cw), Static (st), and Rolling Takeoff (rt) Conditions for 12-Experiment Plackett and Burman Orthogonal Array (c1d–c12d) and Center Point

	Y_{mx}	Y_{to}	Y_{cw}	Y_{st}	Y_{rt}
c1d	0.980	1.635	1.840	1.750	1.620
c2d	0.910	1.540	1.950	1.850	1.670
c3d	0.925	1.695	1.780	1.640	1.550
c4d	0.905	1.620	1.930	1.820	1.770
c5d	0.900	1.640	1.760	1.740	1.550
c6d	0.915	1.730	1.720	1.640	1.670
c7d	0.975	1.475	1.780	1.740	1.630
c8d	0.890	1.820	1.830	1.750	1.580
c9d	0.860	1.725	1.740	1.420	1.650
c10d	0.905	1.495	1.680	1.525	1.440
c11d	0.905	1.615	1.810	1.735	1.700
c12d	0.950	1.650	1.730	1.520	1.540
center	0.905	1.610	1.675	1.550	1.520

Main Effects. Main effect plots were constructed for each operating condition. Figure 4 shows max cruise and takeoff conditions side by side. The relative distance between the center point and the prediction was the criterion used to determine whether a quadratic effect exists in at least one variable.

II. Quadratic Effects

In an effort to improve the model so that the center point was better predicted, a pure quadratic term was added for the following variables: n_h, m_h, m_k, a/b_k, and CR_k. The model with the quadratic terms included had the form:

$$\overbrace{Y = ß0 + ß1(n_c) + ß2(m_c) + \ldots + ß10(CR_k) + ß11(X\text{-ref})}^{\text{Linear model for all variables}} + \mu1(n_h)^2 + \mu2(m_h)^2 + \mu3(m_k)^2 + \mu4(a/b_k)^2 + \mu5(CR_k)^2$$

where μ is the coefficient of the quadratic terms. Two more inlet designs were analyzed for each quadratic term of the new model. For each quadratic variable, an inlet design having the quadratic variable at a high value with all other variables zero (or at the center of the range in the orthogonal array) was evaluated. A second design having the same quadratic variable at a low value with all others zero was also evaluated. These new designs are known as the "star points" in the central composite design(2), equivalent to a classical "vary one at a time" approach. These designs are summarized in Table 4.

Table 4. Inlet Designs Developed To Add a Quadratic Term to the Model

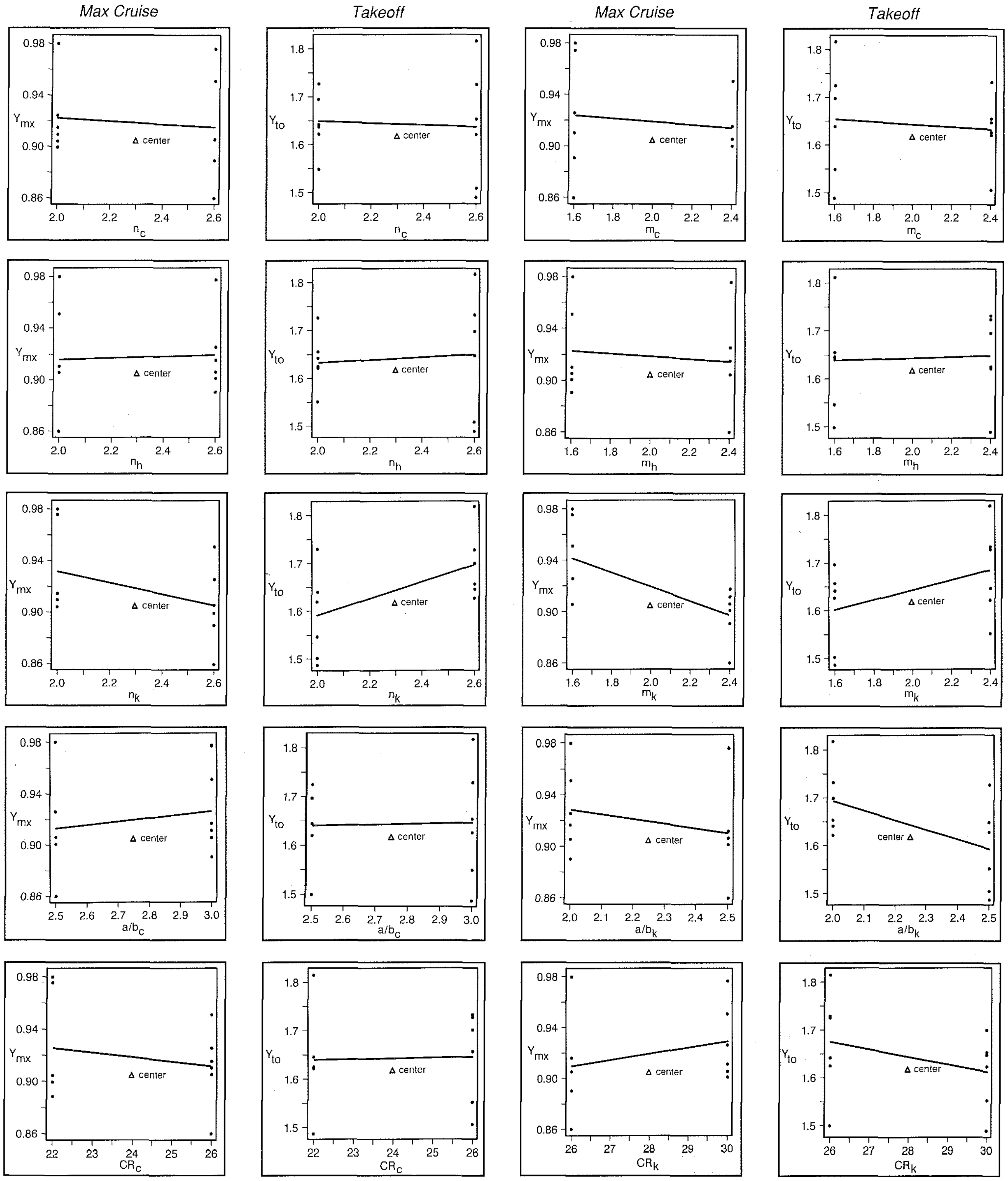
Conditions:
Inlets c13d and c14d: m_k is varied
Inlets c15d and c16d: a/b_k is varied
Inlets c18d and c19d: CR_k is varied
Inlets c20d and c21d: m_h is varied
Inlets ccd and c22d: n_h is varied

	n_c	m_c	n_h	m_h	n_k	m_k	a/b_c	a/b_k	CR_c	CR_k	X-ref
c13d	0	0	0	0	0	1.125	0	0	0	0	0
c14d	0	0	0	0	0	-1.125	0	0	0	0	0
c15d	0	0	0	0	0	0	0	2	0	0	0
c16d	0	0	0	0	0	0	0	-2	0	0	0
c18d	0	0	0	0	0	0	0	0	0	-1.5	0
c19d	0	0	0	0	0	0	0	0	0	1.5	0
c20d	0	0	0	-1.25	0	0	0	0	0	0	0
c21d	0	0	0	1.25	0	0	0	0	0	0	0
ccd	0	0	-1	0	0	0	0	0	0	0	0
c22d	0	0	1	0	0	0	0	0	0	0	0

The responses calculated for these inlets are summarized in Table 5.

Table 5. Responses Y (peak Mach number) at Max Cruise (mx), Takeoff (to), Crosswind (cw), Static (st), and Rolling Takeoff (rt) Conditions for Inlets Developed To Model Quadratic Effects

	Y_{mx}	Y_{to}	Y_{cw}	Y_{st}	Y_{rt}
c13d	0.880	1.650	1.670	1.500	1.535
c14d	0.980	1.620	1.700	1.540	1.570
c15d	0.945	1.525	1.730	1.580	1.580
c16d	1.020	1.780	1.680	1.590	1.540
c18d	0.925	1.710	1.730	1.580	1.600
c19d	0.930	1.560	1.680	1.590	1.520
c20d	0.904	1.577	1.768	1.536	1.571
c21d	0.914	1.628	1.746	1.560	1.671
ccd	0.895	1.600	1.700	1.535	1.550
c22d	0.910	1.616	1.582	1.556	1.511



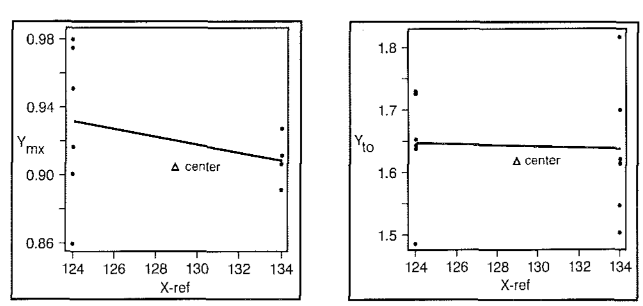


Figure 4. Max Cruise and Takeoff Main Effect Plots

III. Linear Versus Quadratic Comparison

Both linear and quadratic models were fit to the responses for all the data at each operating condition. These are summarized in Table 6. Next to each variable name is the coefficient for that variable in the model. The coefficients for the linear terms in a purely linear model do not change when the quadratic terms are added to the model because of the normalization of the variables and the structure of the central composite design. The absolute size of a coefficient is an indicator of the corresponding variable's effect on the response. In each model the variables that have a substantial effect on the response are shown boxed in Table 6. Coefficients were deemed "substantial" when the slope of the main effect plot was greater than 1.15 times the root mean square deviation of the predicted responses from the observed responses.

Table 6. Coefficients of the Quadratic Model for All Operating Conditions

The quadratic model is:

$$Y = \beta 0 + \beta 1(n_c) + \beta 2(m_c) + \beta 3(n_h) + \beta 4(m_h) + \beta 5(n_k) + \beta 6(m_k) + \beta 7(a/b_c) + \beta 8(a/b_k) + \beta 9(CR_c) + \beta 10(CR_k) + \beta 11(X\text{-ref}) + \mu 1(n_h)^2 + \mu 2(m_h)^2 + \mu 3(m_k)^2 + \mu 4(a/b_k)^2 + \mu 5(CR_k)^2$$

	Max cruise	Takeoff	Cross-wind	Static	Rolling takeoff
ß0	0.9069	1.6045	1.6383	1.4942	1.5225
ß1	-0.0041	-0.0066	-0.0341	-0.0625	-0.0241
ß2	-0.0500	-0.0116	-0.0241	-0.0141	-0.0025
ß3	0.0011	0.0061	-0.0405	-0.0027	-0.0406
ß4	0.0025	0.0094	-0.0038	-0.0072	0.0459
ß5	-0.0133	0.0550	-0.0008	-0.0291	-0.0075
ß6	-0.0224	0.0385	0.0048	0.0060	0.0197
ß7	0.0058	0.0025	0.0275	0.0425	0.0291
ß8	-0.0126	-0.0587	-0.0105	-0.0010	-0.0077
ß9	-0.0075	0.0025	-0.0291	-0.0783	-0.0275
ß10	0.0080	-0.0389	0.0003	0.0191	-0.0150
ß11	-0.0116	-0.0058	0.0341	0.0425	0.0041
µ1	-0.0046	0.0046	0.0114	0.0643	0.0075
µ2	0.0012	-0.0005	0.0794	0.0395	0.0627
µ3	0.0056	0.0157	0.0343	0.0308	0.0035
µ4	0.0077	0.0086	0.0107	0.0159	0.0043
µ5	0.0014	0.0033	0.0200	0.0302	0.0136

The improvement between the linear model and the quadratic model can be seen in a graph of the observed response versus the response predicted by the model (Figure 5). The diagonal line drawn on these graphs shows where the data would fall if the model predicted exactly what was observed. Figure 6 shows an example comparison of the residuals (observed response minus predicted response) of a linear model and a quadratic model for the cross-wind condition. In all conditions, the residuals for the linear model were more than three times the size of the residuals for the quadratic model.

IV. The Optimum

The analysis work described previously produced five response functions for predicting the peak Mach number of inlet designs at five different operating conditions. No one design is optimum at all five operating conditions. What is desired is an inlet design that is optimum at cruise condition and does not seriously degrade inlet performance at the other conditions. Therefore, an optimum inlet design was determined that maximizes inlet performance during cruise and satisfies a set of empirically determined constraints for the other four conditions. These constraints, determined from previous inlet design experience, were:

a. Peak Mach number at takeoff conditions < 1.55.
b. Peak Mach number at crosswind conditions < 1.60.
c. Peak Mach number at static conditions < 1.60.
d. Peak Mach number at rolling takeoff < 1.45.

In addition to the constraints posed by the operating conditions, the optimum cruise inlet shape was constrained to fall within the normalized range of the variables shown in Table 7.

Table 7. Normalized Range of Variables

	Low	High
n_{cr}	-1	+1
m_{cr}	-1	+1.25
n_h	-1	+1
m_h	-1.25	+1.25
n_k	-1	+1
m_k	-1.125	+1.125
a/b_c	-1	+1
a/b_k	-2	+3
Crc	-1	+1
Cr_k	-1.5	+1.5
X-ref	-1.0	+1.0

The linear model was improved by including the responses from the new inlets and was then used to predict the optimum. A packaged linear programming tool was used to solve the problem of minimization of a linear function subject to a set of linear constraints. The performance of the (linearly chosen) optimum was also predicted using the quadratic model.

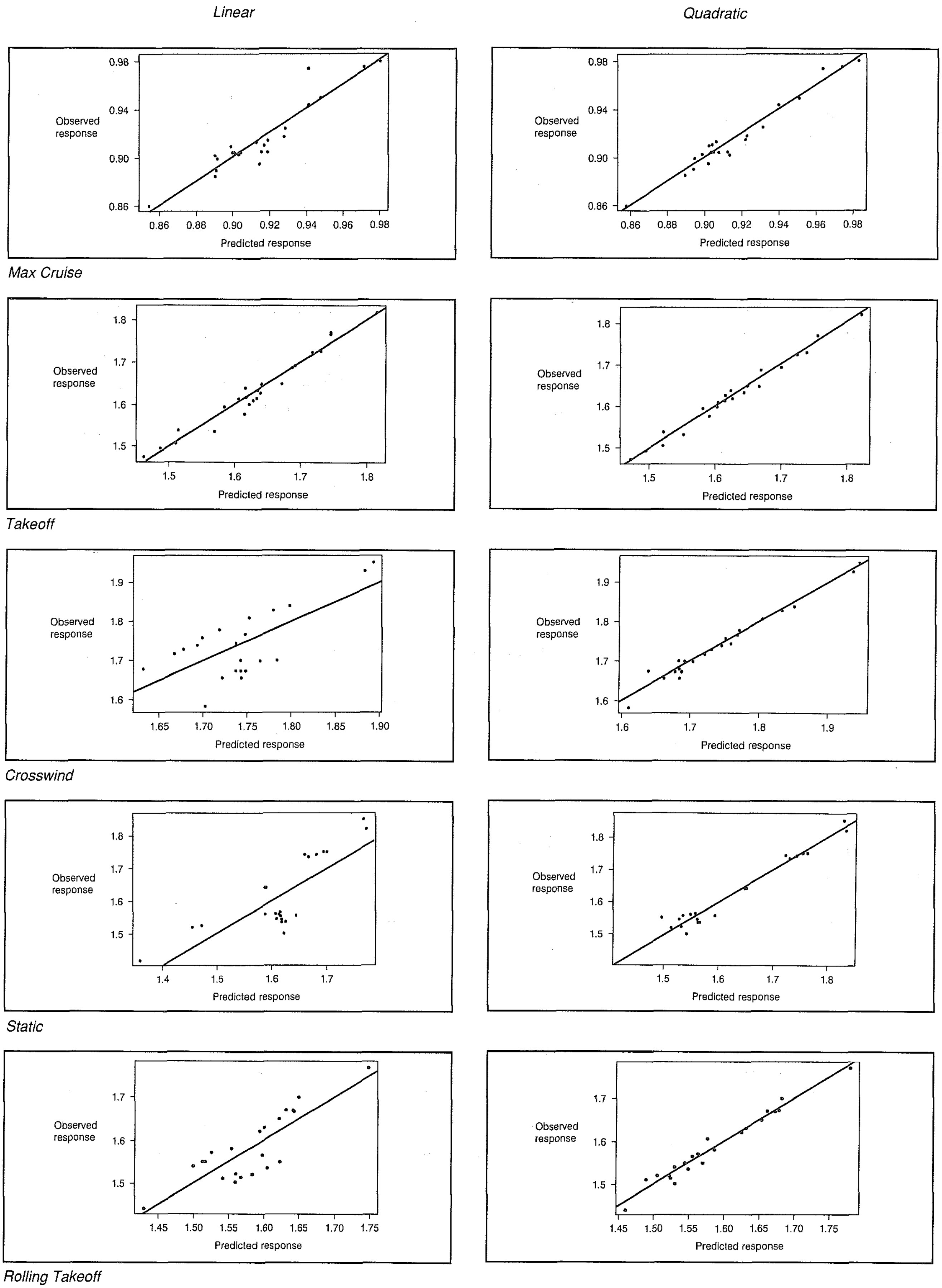


Figure 5. Observed Versus Predicted Response for Both Linear and Quadratic Models

Note: Vertical scale is larger on graph of linear model residuals.

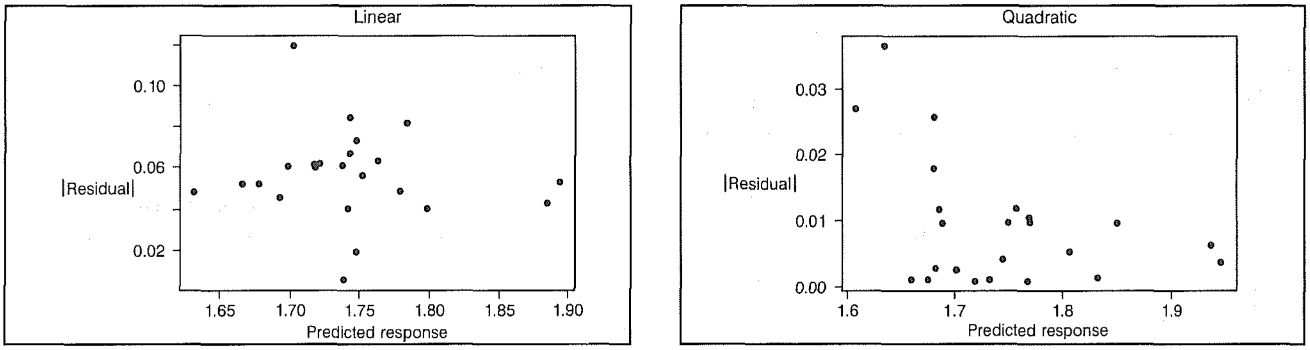


Figure 6. Residuals for Linear and Quadratic Models for Crosswind Condition

The optimum cruise design was found to have the combination of variables shown in Table 8, column (a). A "confirmation" inlet design, given the name OPTA, was chosen as having a shape close to that of the optimum. The variables for OPTA are shown in Table 8, column (b). The prediction of the linear model, the quadratic model, and the observed responses are shown in Table 9.

Table 8. Unnormalized Variables for Optimum and OPTA

	(a) Optimum	(b) OPTA
n_c	2.6	2.6
m_c	2.5	2.5
n_h	2.6	2.6
m_h	1.53	1.6
n_k	2.6	2.6
m_k	2.45	2.4
a/b_c	2.5	2.5
a/b_k	3.0	3.0
CR_c	26.0	26.0
CR_k	26.8	26.0
X-ref	124.95	126.0

Table 9. Responses for Optimum and OPTA

	Linear optimum	Quadratic prediction of linear optimum	Linear OPTA	Quadratic OPTA	Observed OPTA
Y_{mx}	0.8227	0.89	0.82	0.88	0.90
Y_{to}	1.55	1.63	1.56	1.64	1.69
Y_{cw}	1.60	1.76	1.61	1.74	1.67
Y_{st}	1.36	1.55	1.36	1.56	1.55
Y_{rt}	1.45	1.52	1.46	1.53	1.49

Discussion

Without prior knowledge of the relationship between the inlet shape variables and the Mach number response of the CFD code, the sequential central composite design approach to this problem was valuable. A minimal number of experiments were performed before the data were analyzed, and the direction of study refocused on the remaining aspects of the problem to be solved. The following summary describes the value of the results of the study.

Completing analysis of the Plackett and Burman array of experimental inlet designs allowed the first opportunity to evaluate the designed experiment approach to inlet design. Main effect plots were the only tool relied on to assess the applicability of a linear model to the CFD code. On the positive side, the use of the main effect plot in predicting the presence of quadractic effects proved reliable. The criterion used to decide whether there was a quadratic effect present in one of the variables was the proximity of the center point to the main effect line. In retrospect, the lack of improvement with the addition of quadratic terms to the takeoff model can be correlated to the relative closeness of the center point and the main effect line in the takeoff condition main effect plots. On the negative side, the main effect plot can be misleading, and any conclusions must be qualified with the potential presence of variable interactions.

The Plackett and Burman orthogonal array did not provide any insight about the presence of variable interactions or lack thereof. The Plackett and Burman design does not allow the isolation of the main effects of a variable from variable interactions. Therefore, characterization of interactions would require analysis of additional inlet designs. Interactions present in the physical relationship being modeled would show up in the main effect plot as a change in slope of the main effect line. Without further study, any conclusions about the variable drawn from the slope of the main effect plot must be suspect.

While the distance of the center point from the main effect line may be a reliable indicator of quadratic effects, no information is provided to resolve which variable or variables have the quadratic effect. This is because the presence of a quadratic effect in one variable will draw the center point away from the main effect line for all variables. In this study, an effort to investigate quadratic variables that might improve the model at max cruise was attempted first. Three variables relating to the inlet contour at the keel were chosen, and inlet designs were developed to explore quadratic effects in those variables: m_k, a/b_k, and CR_k.

The improvement of the model with the addition of the quadratic terms was evaluated using predicted versus observed response graphs, shown in Figure 5, and an evaluation of the size of the residual as shown in Figure 6. Statistical tools were not applied to aid in the evaluation of the model because the experiments were done using a computer and were repeatable; therefore, they had no noise. Evaluation of experimental noise in industrial settings is traditionally used to characterize the significance of a variable on the response. In this case, statistical techniques that rely on randomness of the data are not applicable.

As expected, while the performance of the max cruise model did improve based on the peak Mach number criterion, the addition of quadratic keel variables did not significantly improve the models for crosswind, rolling takeoff, and static. An improvement in the model at crosswind and rolling takeoff was achieved by the addition of quadratic terms to the variables affecting the half-breadth contour n_h and m_h.

The graphs of predicted versus observed response and of the residuals are a good visual display of the impact of the quadratic terms on the model. A comparison of the linear and quadratic predicted versus observed response plots shows the data clustered more closely to the diagonal line in the quadratic model for all conditions. This means that the observed value is better predicted in the quadratic case. This improvement can also be seen in the residual plots of Figure 6. Here, the size of the largest residual in the quadratic model is approximately one-third that of the linear model, which was true for all conditions.

The final test of the utility of a designed experiment for inlet design was to develop a predicted optimum inlet shape and determine if the CFD code responses are correctly predicted. The optimum prediction was made using the linear models only. As shown in Figure 4, the significant variables (those with a steep slope) for max cruise had the opposite sign as those for takeoff condition. Therefore, optimization of max cruise required compromising takeoff performance. This was not true for a/b_k; increasing a/b_k had a beneficial effect on both max cruise and takeoff conditions. To take advantage of this benefit, the range of a/b_k was increased to 3.0. The inlet shape, given the name OPTA, was constructed and analyzed as the confirmation of the optimum prediction. As shown in Table 9, the quadratic model better predicted the responses observed for inlet OPTA. The residual for the quadratic model for OPTA for max cruise, takeoff, crosswind, and rolling takeoff conditions was larger than the biggest response measured for any of the preceding inlet designs. This may indicate that the quadratic model is not close enough to the observed response of the CFD code over the extent of the range studied, possibly due to variable interactions.

Conclusion

As a practical tool, the linear model developed with only 12 experiments did not produce a design that was satisfactory. The value of this work is the progress made toward characterizing the response of the CFD code to inlet shape variables. Further study involving the investigation of interactions and of more quadratic variables would allow the form of the CFD code response to be discovered. Then an efficient designed experiment involving only the necessary variables could be developed. This new experiment could be put to practical use in streamlining the inlet development process.

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