

# ANALYSIS AND DEVELOPMENT OF A TOTAL ENERGY CONTROL SYSTEM FOR A LARGE TRANSPORT AIRCRAFT\*

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## Abstract

A functionally integrated flight/propulsion control system, named total energy control system (TECS), is discussed in this paper. Based on the energy management point of view, the thrust is used to control the total energy amount and the elevator to control the desired energy distribution between flight speed and path. The mathematical model of aircraft point-mass energy movement dynamics is established as a two-input/two-output P-canonical system representation, with the transfer functions of pitch attitude control and engine control loops as its basic elements. Coordinated thrust and elevator control laws, providing decoupled flight path/speed maneuvering dynamics, are then developed with output feedback control strategy and fore-feed Vcanonical decoupling structure. Various operational requirements, control nonlinearities, and performance limits are considered and incorporated. Control priorities for speed/path are easily implemented by processing commanded signals when thrust reaches limiting values. Control laws for various path/speed modes, such as altitude/speed, flight path angle, vertical path segment transition, glide slope, automatic landing flare, go around, and optimum trajectory guidance, are analyzed and developed. All researches are demonstrated through digital simulations for a Boeing-707 transport aircraft model.

## Introduction

The conventional automatic flight control systems (AFCS) used in modern commercial aircrafts are generally developed based on the single-input/single-output control strategy, with the vertical flight path being controlled by the pitch autopilot through elevator and the flight speeds by the autothrottle through the engine throttles. Their analysis and design are commonly accomplished with the traditional but non-optimal bottom up design method of adding one control loop/control mode at a time based on the classical control theory of error-feedback and root locus analysis. Because both the elevator and throttle controls will produce coupled flight path and speed responses, there exists coupling effects between the flight path and speed controls, which makes

the design of AFCS quite complex and difficult. The analysis and design process relies heavily on experience, intuition, empirical approaches and tradition, as well as many times of improvements based on various kind of simulation tests. And the obtained results are still not satisfactory in most cases and bear lots of pilot complaints<sup>[1-2]</sup>.

How to reduce the coupling effects between path and speed dynamic responses, to get coordinated elevator and throttle responses for any path or speed control requirements, and to decouple the path and speed control functions, has always been one of the most difficult problems in the Integrated Flight/Propulsion Control (IFPC) techniques of aircrafts. This is because, in a large partition, that the flight path change is mainly decided by the short period attitude motion dynamics, the flight speed change is decided by the long term point-mass dynamics, and under current control strategy it is very difficult to efficiently integrate or combine them together. Many researches show that the present AFCS design have reached its development plateau, fundamental improvements can only be obtained by a multi-input/multi-output design approach, see ref. <sup>[1-3]</sup>.

In recent researches, many new control strategies for the IFPC have been set up and studied based on modern control theories. One very promising generalized design approach, named Total Energy Control System (TECS), is developed by Mr. Lambregts etc. of Boeing Commercial Aircraft Co., in which coordinated elevator and thrust commands are developed based on point-mass energy control considerations. The total energy of aircraft in flight is made up of the kinetic energy component related to flight speed and the potential energy component related to flight altitude. The change of total energy amount can cause the change of either flight speed or altitude, and also both of them. When keeping constant the total energy amount, the flight speed and altitude can also be exchanged each other simultaneously through the transformation of energy between kinetic and potential components. On the other hand, the variation of total energy amount is mainly caused by the thrust change, and the transformation of energy by the pitch attitude change. Thus, in the control strategy, if thrust (or au-

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tothrottle) is used to control the total energy amount, the pitch autopilot (through elevator) is used to rearrange the distribution of total energy between kinetic and potential components, and cross feedbacks are introduced to decouple the two control channels, then the resulted system could have a decoupled dynamic responses between the path and speed maneuvers. This forms the basic principle and control strategy of TECS, see reference [1] and [2].

A thorough study on the basic concept and principle, analysis and design methods, application to various path/speed control modes, and operational features of the TECS is conducted in this paper. The decoupling control law is developed mathematically with output-feedback control and forefeed V-canonical decoupling structure, system design criteria are determined, and a typical TECS is designed for an Boeing-707 transport aircraft model, satisfactory digital simulation results are finally obtained.

### Basic Concept and Principle

According to the point-mass flight dynamics of aircrafts, the required thrust during maneuvering flight is as follow:

$$T = W \cdot (\gamma + \dot{V}/g) + D \quad (1)$$

where  $\gamma$  and  $V$  are the longitudinal flight path angle (FPA) and true airspeed;  $T$ ,  $D$ , and  $W$  are the thrust, drag, and weight of aircraft respectively;  $g$  is the gravity constant.

The sum  $(\gamma + \dot{V}/g)$  is often referred to as "potential or total flight path angle" and has been used for thrust setting displays during manual control. Assuming the initial thrust is trimmed against drag, with initial FPA and acceleration being zero, then it follows that the incremental thrust requirement during maneuvering flight is proportional to the product of weight and the sum of FPA and longitudinal acceleration. The drag variation is generally slow, especially for commercial transports of which the maneuvering rate is very low and the speed change very slow, and can be taken care of by integral control in long term. Therefore, the requirement amount of thrust change in shot term maneuvering could be expressed as follows:

$$\Delta T \simeq W \cdot (\gamma + \frac{\dot{V}}{g}) \quad (2)$$

Considering the specific total energy and energy changing rate of aircrafts:

$$E = h + \frac{V^2}{2g} \quad (3)$$

$$\dot{E} = V(\gamma + \frac{\dot{V}}{g}) \quad (4)$$

Combining equation (2) and (4):

$$\frac{\dot{E}}{V} = \gamma + \frac{\dot{V}}{g} = \frac{\Delta T}{W} \quad (5)$$

Thus, the aircraft specific total energy rate is indicated by the potential FPA, and is mainly controlled by thrust changes. On the other hand, the variation of ele-

vator mainly generates pitch moment to change the longitudinal flight attitude, and in turn to change FPA, does not generate appreciable forces in the direct of the flight path. Hence, while at constant thrust, the elevator control can only change the distribution of total energy between its kinetic and potential components, i. e., between flight speed and altitude. Thus it comes that, from an energy management point of view, the thrust should be used to control the total energy amount and the elevator to control the desired energy distribution between flight speed and altitude, as shown in Fig. 1.

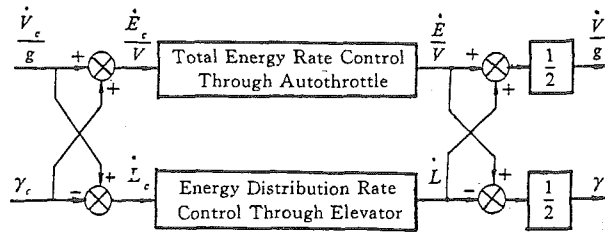


Fig. 1 Control Strategy of TECS

In order to maintain equal control weight between kinetic energy (flight speed) and potential energy (altitude), the energy distribution rate  $\dot{L}$  is employed as the control variable of elevator channel<sup>[1]</sup>, which is defined as the difference between kinetic and potential energy rates:

$$\dot{L} = \frac{\dot{V}}{g} - \gamma$$

Due to the decoupling control purpose, the following design requirements should be met in the analysis and design of TECS:

- There should have decoupled control responses between the total energy rate control channel (autothrottle channel) and the total energy distribution rate control channel (pitch autopilot).
- The two control channels should have matched dynamic and static characteristics, so as to enable both the total energy rate error and energy distribution rate error to go to zero simultaneously, and thereby to avoid undesirable energy exchange (control coupling) between path and speed.

### Control Law Development

Define the Specific net thrust parameter in normalized form for unit weight as  $\delta T = \Delta T/W$ , then, from equation (5) and (6) it follows:

$$\begin{bmatrix} \dot{E}/V \\ \dot{L} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \delta T \\ \gamma \end{bmatrix} \quad (7)$$

Independent engine thrust control system (autothrottle) and pitch attitude control loop (pitch pilot) forms the bases, i. e. inner control loops, of TECS, with the former producing the required specific net thrust in-

crements, the later stabilizing the short period dynamics and controlling the pitch attitude angle and in turn the FPA. Assuming their transfer functions are  $W_T(s)$  and  $W_P(s)$  with the following relations respectively:

$$\delta T = W_T \cdot \delta T_g \quad (8)$$

$$-2\gamma = W_P \cdot \delta \theta_g \quad (9)$$

where  $\delta T_g$  is the normalized command of the thrust control loop,  $\delta \theta_g$  is the pitch attitude angle command of the pitch control loop. Replacing equation (8) and (9) into equation (7), the mathematical model of the point mass energy state movement of aircraft, which contains its basic thrust and attitude control dynamics ( $W_T(s)$  and  $W_P(s)$ ), is obtained as follows:

$$\begin{bmatrix} \dot{E} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} W_T & 0 \\ W_T & W_P \end{bmatrix} \begin{bmatrix} \delta T_g \\ \delta \theta_g \end{bmatrix} \quad (10)$$

Equation (10) shows that, from the point-mass energy movement viewpoint, aircraft dynamics combined with its inner control loops can be modelled as a two-input/two-output P-canonical system. So with reference to the Mesarovic' decoupling criterion<sup>[5]</sup>, a V-canonical decoupling control structure is used for the purpose of simplifying the decoupling terms. And unit output feedback control and fore-feed decoupling strategy is further adopted to form the control structure of TECS, as shown in Fig. 2, where  $P_{11} = W_T$ ,  $P_{21} = W_T$ ,  $P_{12} = 0$ , and  $P_{22} = W_P$ . The decoupling terms  $R_{12}(s)$  and  $R_{21}(s)$  combined together with the controller terms  $R_{11}(s)$  and  $R_{22}(s)$  of the two main control channels forms the V-canonical decoupling system structure.

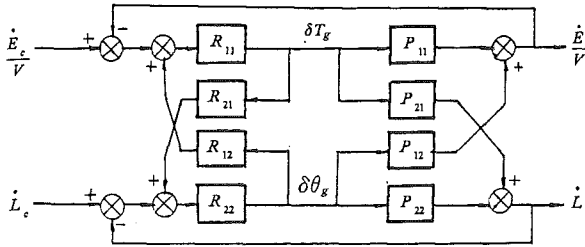


Fig. 2 Decoupling Control Structure of TECS

For this typical control system, the decoupling terms should have following forms<sup>[5]</sup>:

$$R_{12} = -\frac{P_{12}}{P_{11}} \cdot \frac{1}{R_{11}} = 0 \quad (11)$$

$$R_{21} = -\frac{P_{21}}{P_{22}} \cdot \frac{1}{R_{22}} = -\frac{W_T}{W_P} \cdot \frac{1}{R_{22}} \quad (12)$$

The closed-loop transfer matrix is as follows:

$$\Phi(s) = \begin{bmatrix} \frac{R_{11}W_T}{1 + R_{11}W_T} & 0 \\ 0 & \frac{R_{22}W_P}{1 + R_{22}W_P} \end{bmatrix} \quad (13)$$

According to the requirement for matched dynamics of the two main control channels, the diagonal terms in the transfer matrix should be equal:

$$R_{11}(s) = \frac{W_P}{W_T} R_{22}(s) \quad (14)$$

Equation (12) - (14) shows that the decoupling

terms  $R_{21}(s)$  and controller terms are all related to the inner-loop dynamics, which is evidently not acceptable. Hence, the controller  $R_{11}(s)$  and  $R_{22}(s)$  should be judiciously selected to simplify the decoupling terms and to reduce its dependence on  $W_P(s)$  and  $W_T(s)$ .

The proportional plus integral controller is basically selected as following forms:

$$\begin{cases} R_{11}(s) = K_{TP} + \frac{K_{TI}}{s} \\ R_{22}(s) = K_{EP} + \frac{K_{EI}}{s} \end{cases} \quad (15)$$

This will enable the total energy rate error to be reduced to zero with a first order time constant  $\tau_T = K_{TP}/K_{TI}$ , and similarly the energy distribution rate error with  $\tau_E = K_{EP}/K_{EI}$ . Let the proportional terms act only on the feedback signals, without relating to the command signals, so as to smooth the transient responses of the controller to command inputs, which do not affect the controller static performance<sup>[1]</sup>. Resulted system structure is shown in Fig. 3, where  $R_{X1}(s)$  and  $R_{X2}(s)$  together take the role of decoupling term  $R_{21}(s)$ .

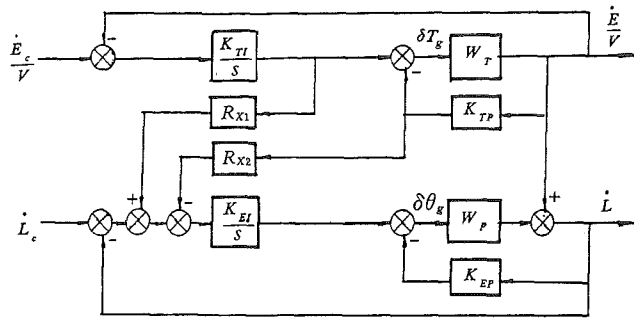


Fig. 3. Actual Control Structure of TECS

Its closed-loop transfer relation is as follows:

$$\begin{bmatrix} \dot{E} \\ \dot{L} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \dot{E}_c \\ \dot{L}_c \end{bmatrix} \quad (16)$$

where:  $\Phi_{11}(s) = \frac{K_{TI}}{s} \frac{\Phi_{KT}}{Q_T}$ ,  $\Phi_{12} = 0$

$$\Phi_{21}(s) = \frac{K_{TI}}{s} \left[ \frac{K_{EI}}{s} R_{X1} \Phi_{KP} - \frac{K_{TP} K_{EI}}{s} R_{X2} \Phi_{KP} \Phi_{KT} - K_{EP} \Phi_{KP} \Phi_{KT} + \Phi_{KT} \right] / [Q_T \cdot Q_P]$$

$$\Phi_{22}(s) = \frac{K_{EI}}{s} \frac{\Phi_{KP}}{Q_P}, \quad Q_T(s) = 1 + \frac{K_{TI}}{s} \Phi_{KT}(s),$$

$$Q_P(s) = 1 + \frac{K_{EI}}{s} \Phi_{KP}(s), \quad \Phi_{KT}(s) = \frac{W_T}{1 + K_{TP} W_T},$$

$$\Phi_{KP} = \frac{W_P}{1 + K_{EP} W_P}.$$

According to the decoupling conditions,  $\Phi_{12} = \Phi_{21} = 0$ , and assuming  $R_{X1} = 0$ , the decoupling term  $R_{X2}(s)$  should be:

$$R_{X2}(s) = \frac{s}{K_{TP} \cdot K_{EI}} \cdot \frac{1}{W_P} \quad (17)$$

The longitudinal attitude control loop ( $W_P(s)$ ) is mainly related to aircraft short period dynamics, with

relatively large bandwidth or rapid time responses. The energy control loops (of both  $\dot{E}/V$  and  $L$ ) are related to the long term point-mass dynamics, with relatively small bandwidth. So the inner control loop  $W_P(s)$  could be treated as a constant gain  $K_P$  in the analysis of outer energy control loops, which is especially true for commercial aircrafts, here  $K_P$  is the static gain of transfer function  $W_P(s)$ . Thus the decoupling term  $R_{X2}(s)$  could be expressed as following definite form:

$$R_{X2}(s) = \frac{s}{K_{TP} \cdot K_{EI} \cdot K_P} \quad (18)$$

And the total energy control law is :

$$\begin{aligned} \delta T_g &= \frac{K_{TI}}{s} \left( \frac{\dot{E}_c}{V} - \frac{\dot{E}}{V} \right) - K_{TP} \frac{\dot{E}}{V} \\ &= \frac{K_{TI}}{s} \left( \frac{\dot{V}_c}{g} - \frac{\dot{V}}{g} + \gamma_c - \gamma \right) - K_{TP} \left( \frac{\dot{V}}{g} + \gamma \right) \quad (19) \\ \delta \theta_g &= \frac{K_{EI}}{s} (L_c - L) - K_{EP} L - \frac{1}{K_P} \frac{\dot{E}}{V} \\ &= \frac{K_{EI}}{s} \left[ \frac{\dot{V}_c}{g} - \frac{\dot{V}}{g} - (\gamma_c - \gamma) \right] \\ &\quad - K_{EP} \left[ K_X \left( \frac{\dot{V}}{g} + \gamma \right) - 2\gamma \right] \quad (20) \end{aligned}$$

where  $K_X = 1 + 1/(K_{EP}K_P)$ .

Additionally,  $\Phi_{11}(s)$  should be equal to  $\Phi_{22}(s)$ , then it follows:

$$K_{TI}\Phi_{KT} = K_{EI}\Phi_{KP} \quad (21)$$

At the same time, the first order time constants of the two control channels should also be equal, i. e.,  $\tau_T = \tau_E$  or  $K_{TP}/K_{TI} = K_{EP}/K_{EI}$ . Considering the FPA change maneuver, for any command signal  $\Delta\gamma_c$  (assuming  $\dot{V}_c/g = 0$ ), it will come about that in steady state  $\delta T = \Delta\gamma_c$  and  $\Delta\theta = \Delta\gamma_c$ . So the proportional gains should have unit value, i. e.,  $K_{TP} = K_{EP} = 1$ , and in turn  $K_{TI} = K_{EI}$ . Thus from equation (21) it follows that  $\Phi_{KT} = \Phi_{KP}$ . This means that the closed-loop transfer function of thrust control loop  $W_T(s)$  encircled by a unit output feedback should have matched dynamic and static characteristics with that of pitch attitude control loop  $W_P(s)$ .

Conclusively, the design criteria which should be followed in the development of TECS can be summarized as follows:

- The dynamic characteristics of pitch attitude control loop ( $W_P(s)$ ) should be matched as close as possible to that of thrust control loop ( $W_T(s)$ ).
- The dynamic and static characteristics of  $\Phi_{KT}(s)$ , which is the closed-loop transfer function of  $W_P(s)$  enclosed through a unit output feedback, should be matched as close as possible to that of  $\Phi_{KP}(s)$ , the closed-loop transfer function of  $W_T(s)$ .
- The parameters of the controllers in the two main control channels should be selected as  $K_{TP} = K_{EP} = 1$  and  $K_{TI} = K_{EI}$ .

## System Design and Integration

Based upon the criteria discussed above, a typical TECS is analyzed and designed for a Boeing-707 aircraft mathematical model<sup>[8]</sup>. In the pitch attitude control loop, state feedback of pitch rate and pitch attitude angle are adopted to stabilize the short period dynamics and to get good attitude angle control and tracking properties, and the parameter space design method of robust control system<sup>[6]</sup> is applied in order to get good control dynamics for various flight conditions. In the thrust control system of JT3D-7 turbofan engine, the engine pressure ration (EPR) is used as feedback control variable to form a closed control structure, a simplified engine model is used in its analysis and simulation instead of sophisticated nonlinear high order models, and the control parameters are judiciously chosen to get a matched dynamic characteristics to the pitch attitude control loop. Detailed design process and results see reference [8].

Assuming the transfer function of thrust control loop is  $\Phi_T(s)$ , and pitch attitude control loop  $\Phi_\theta(s)$ , which has the transfer relation of  $\Delta\gamma = \Phi_\theta\delta\theta_g$ , then the TECS can be integrated according to Fig. 4. Where  $K_{T1}$  and  $K_{\theta1}$  are dynamic compensation gains for  $\Phi_T(s)$  and  $\Phi_\theta(s)$  to ensure the  $\Phi_{KT}(s)$  and  $\Phi_{KP}(s)$  be dynamically matched,  $K_{T2}$  is static compensation gain to match the static characteristics of the two loops.

Fig. 5 illustrates the simulation results of step response to FPA and acceleration respectively, in a flight condition of altitude = 26248 feet,  $W = 200655$  LB, and Mach = 0.6.

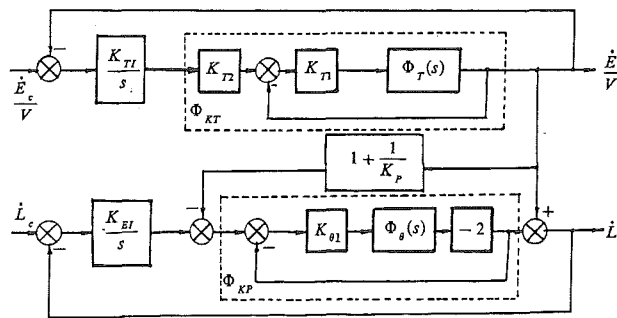
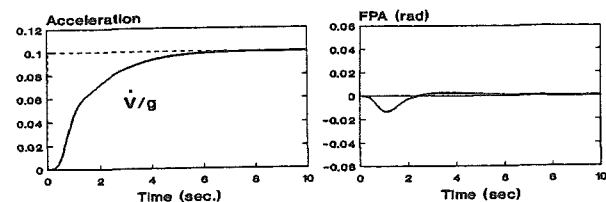
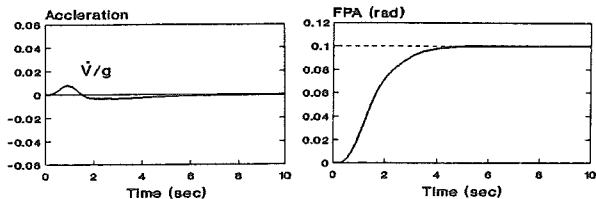


Fig. 4. TECS Design Structure



(a) Acceleration Step Response

Fig. 5. Step Responses of the Basic TECS



(b) FPA Step Response

Fig. 5. Step Responses of the Basic TECS

## Path/Speed Control and Operational Limitations

The TECS requires normalized commands  $\gamma_c$  and  $\dot{V}_c/g$ . Hence for various outer-loop path and speed control modes, all control errors should be converted to the normalized forms. In altitude and speed control modes, command signal is proportional to and counteracting to errors:

$$\frac{\dot{V}_c}{g} = K_V(V_C - V) \quad (23)$$

$$\gamma_c = \frac{\dot{h}_c}{V} = \frac{K_h}{V}(h_C - h) \quad (24)$$

where  $K_V$  and  $K_h$  are constant control gains. The linearized system block diagram is shown in Fig. 6.

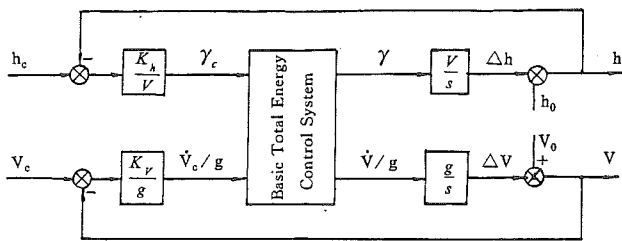


Fig. 6 Linear Altitude and Speed Control Modes

The above control laws call for an exponential reduction of the altitude and speed errors with a time constant  $\tau$  inversely proportional to  $K_h$  and  $K_V$ . Thus to preserve decoupling for simultaneous altitude and speed maneuvers and maintain the correct relative energy relationship between altitude and speed errors,  $K_h = K_V$  must be chosen. The outer-loop error of any speed or flight path control mode can thus be normalized into  $\dot{V}_c/g$  and  $\gamma_c$  signals for input to the generalized basic TECS. For the speed modes of Mach and CAS (Calibrated Airspeed), the command signal first be converted to true airspeed so as to preserve proper control law gains at all flight conditions. Because of the decoupling dynamics between FPA and acceleration controls, the analysis and design of the altitude and speed control loops in Fig. 6 can be conducted independently, and therefore become very simple<sup>[8]</sup>.

In addition to the linear analysis and design, many nonlinear operational limitations should be considered and implemented, such as normal acceleration limiting, speed limiting, and thrust limiting etc. Since the normal acceleration is related to FPA as  $a_n = V\dot{\gamma}$ , and the FPA responds to a command  $\gamma_c$  in a smooth overshoot free

way, the normal acceleration limiting is implemented simply by rate limiting the  $\gamma_c$  signal to  $\dot{\gamma}_c = a_{nLIMIT}/V$ . The same rate limit is also implemented to the  $\dot{V}_c/g$  signal as  $\dot{V}_{cLIMIT}/g = a_{nLIMIT}/V$ , so as to maintain the same processing in both signal channels. Rate limiting is realized through a software algorithm processing to the  $\gamma_c$  and  $\dot{V}_c/g$  signals in the simulation<sup>[8]</sup>.

For speed limiting, the allowable speed  $V_{MAX}$  and  $V_{MIN}$  are transferred into normalized signals at all times as follows:

$$\begin{cases} \dot{V}_{cMAX}/g = K_V(V_{MAX} - V)/g \\ \dot{V}_{cMIN}/g = K_V(V_{MIN} - V)/g \end{cases} \quad (25)$$

The actual command  $\dot{V}_c/g$  is continuously compared with  $\dot{V}_{cMAX}/g$  and  $\dot{V}_{cMIN}/g$ . If it does not fall in the allowable value range, the command  $\dot{V}_{cMAX}/g$  or  $\dot{V}_{cMIN}/g$  instead of  $\dot{V}_c/g$  is adopted automatically with absolute priority, as shown in Fig. 7.

Additionally, a limitation on the magnitude of FPA command  $\gamma_c$  is also implemented in the  $\gamma_c$  channel, and also in the  $\dot{V}_c/g$  channel for proper energy distribution relation, so as to avoid altitude overshoots/undershoots in altitude control cases. The magnitude of vertical speed or climbing/descent rate is actually limited to get a smooth and overshoot/undershoot free response of altitude.

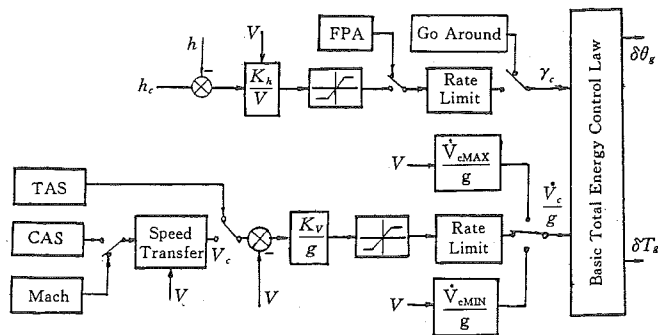


Fig. 7 Nonlinear TECS

Fig. 8 illustrates the simulation results of total energy climbing for a simultaneous climb/speed up commands. The altitude and CAS increase steadily and smoothly, the normal acceleration is limited to  $\pm 0.1g$ , the vertical speed  $\dot{H}$  is also limited to a certain value due to the FPA limitation, and the elevator ( $\Delta\delta e$ ) and throttle ( $\delta P$ ) changes steadily in its allowable range.

Fig. 9 illustrates the simulated energy transferring performance for simultaneous descent/speed up commands, with nearly zero total energy change. The throttle needs little variation in the control process, but slightly increase to trim the drag increase due to speeding up, and the elevator changes a lot to redistribute the total energy between path and speed.

## Control Priority With Limited Thrust

From equation (5), when thrust reaches its limitation, the total energy rate reaches its limited value, rep-

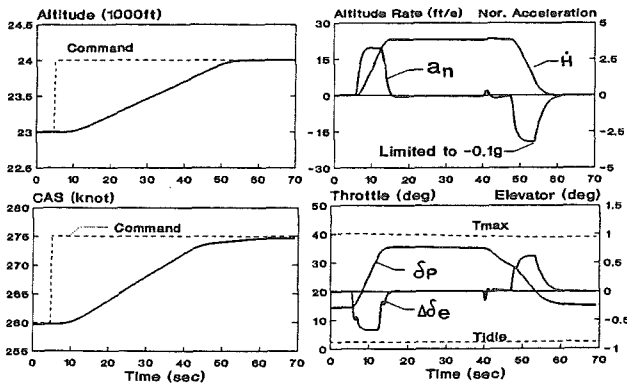


Fig. 8 Total Energy Climbing Simulation

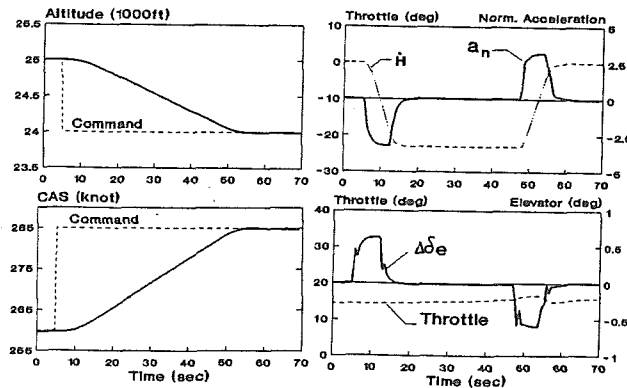


Fig. 9 Energy Transformation Simulation

resented by  $(\dot{V}/g + \gamma)$ , and only the elevator remains available to control the distribution of the available total energy rate between speed and path. For the case of maximum thrust, for instance, it follows that:

$$\delta T_{MAX} = \frac{\Delta T_{MAX}}{W} = \left[ \frac{\dot{E}}{V} \right]_{MAX} = \frac{\dot{V}}{g} + \gamma \quad (26)$$

In order to keep decoupling control and maintain correct relative energy relationship between altitude and speed errors in the elevator control channel, the command signals should be processed to restrict the total energy rate command to its available range. For the maximum thrust case, it follows:

$$\frac{\dot{E}_c}{V} = \frac{\dot{V}_c}{g} + \gamma_c \leq \left[ \frac{\dot{E}}{V} \right]_{MAX} = \frac{\dot{V}}{g} + \gamma \quad (27)$$

This means that the sum of accelerating and climbing abilities is limited to the available total energy climbing ability. How this available ability is distributed between acceleration and climb could be used to adjust the control priorities on speed or path. Defining a speed priority parameter  $p$ , which varies from 0.5 to 1.0, the speed control priority could be realized through processing the command signals as following for the maximum thrust case:

$$\begin{cases} \frac{\dot{V}_c}{g} \leq p \cdot \left( \frac{\dot{V}}{g} + \gamma \right) \\ \gamma_c = \left( \frac{\dot{V}}{g} + \gamma \right) - \frac{\dot{V}_c}{g} \end{cases} \quad (28)$$

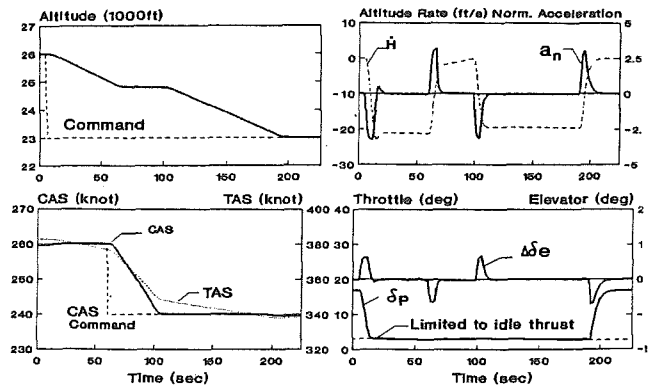
This means that the portion of the total available energy rate used for speed control is at most  $100p\%$ .

When  $p = 1$ , it corresponds to the absolute priority of speed control.

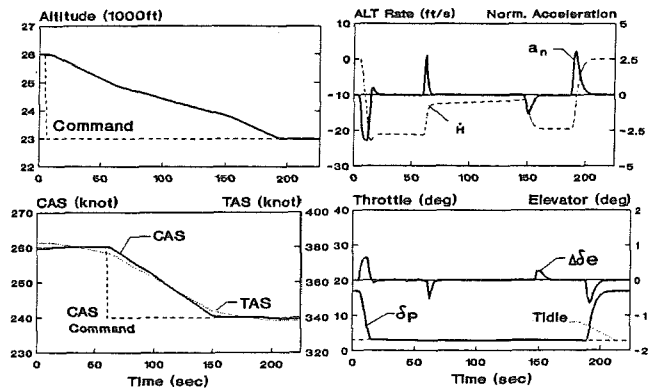
Similarly, path control priority can also be realized through the processing on  $\dot{V}_c/g$  and  $\gamma_c$  according to equation (27). Defining path priority parameter  $q$ , which varies also from 0.5 to 1.0, the command signals should be processed as follows for the maximum thrust case:

$$\begin{cases} \gamma_c \leq q \cdot \left( \frac{\dot{V}}{g} + \gamma \right) \\ \frac{\dot{V}_c}{g} = \left( \frac{\dot{V}}{g} + \gamma \right) - \gamma_c \end{cases} \quad (29)$$

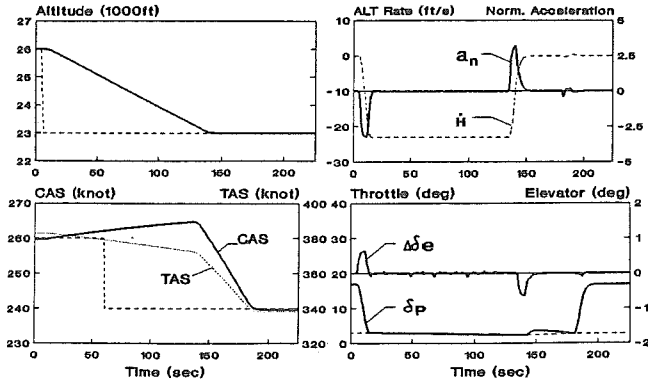
For the minimum thrust case, same processing principle is used, with slightly different processing equations<sup>[3]</sup>. Fig. 10 illustrates three sets of simulation results for different speed/path control priorities,  $p = 1.0$ ,  $p = 0.5$ , and  $q = 1.0$ , at same flight conditions and control requirements. As shown, the absolute speed priority can provide the operational flexibility to change speed during descent/climb at minimum/maximum thrust without the need for the pilot to manually reconfigure the modes. Here, for the  $p = 1.0$  case, after thrust limits at idle due to a large descent command in the altitude control mode, a command is given to decelerate from 260 kt to 240 kt. This is accomplished through the elevator control by temporarily reducing the descent rate until the speed command is captured. Finally, linear control is resumed automatically when the altitude target is approached and a net thrust rate command is developed to



(a) Speed Priority ( $p=1.0$ )



(b) Speed/Path Priority ( $p=0.5$ )



(c) Path Priority ( $g=1.0$ )

Fig. 10. Speed/Path Control Priority Simulations

drive the thrust back into the linear range. For the case of  $p=0.5$ , the speed and altitude controls are actually at same priorities. For the path priority case ( $q=1.0$ ), CAS slightly increase after thrust reaches idle instead of decreasing, but the true airspeed does not increase which means that no inverse energy exchange from speed to path occurred.

### FPA Control and Go Around Control Modes

The flight path angle (FPA) control is simply implemented in the  $\gamma_c$  control channel of the basic TECS, as shown in Fig. 8. The allowable FPA is mainly decided by maximum/minimum thrust ability, or potential FPA, so the saturation limiting on FPA magnitude is not used in FPA control mode. The performances of climbing at maximum thrust and of descent at idle thrust with constant speed could be checked with the FPA control mode, in which speed control priority ( $p=1.0$ ) is set up for limited thrust cases.

The go around control mode is also implemented in the  $\gamma_c$  control channel, as shown in Fig. 8. The FPA command is selected to provide the desired initial pull up and final climb gradient, and it is not rate limited in order to minimize altitude loss. Speed control priority is set up when thrust reaches maximum, so as to ensure speed safety. For a heavy aircraft situation, more than the available thrust may be required to satisfy  $\gamma_c$ . Then full thrust climb at available FPA is obtained with speed controlled by the elevator. Fig. 11 illustrates the digital simulation results for this situation (initial aircraft weight = 250000LB), which actually represents the maximum climbing ability. For light weight conditions, only partial thrust will satisfy  $\gamma_c$ , which avoids excessive climb gradients and pitch attitudes.

### Vertical Flight Path Segment Transition

The transition flight from one straight flight segment to another is needed in many control cases, such as vertical navigation, glide slope etc. Fig. 12 shows its geometric relationship. The transition should be started at

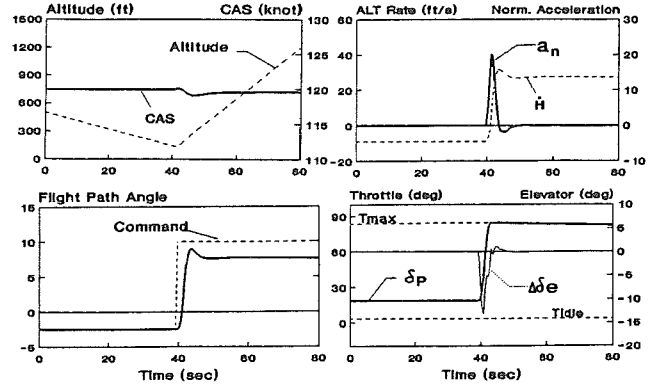


Fig. 11. Go Around Control at Maximum Thrust

point D earlier than the crossing point, in order to get a path overshoot free response. The FPA difference between the current flight direction and the upcoming path segment could be obtained as following<sup>[8]</sup>:

$$\Delta\gamma = \gamma_2 - \gamma = \frac{sh_\epsilon - \tau_{vp}s \cdot \dot{h}}{\tau_{vp}s + 1} \cdot \frac{1}{V} \quad (30)$$

where  $\tau_{vp}$  is a time constant.

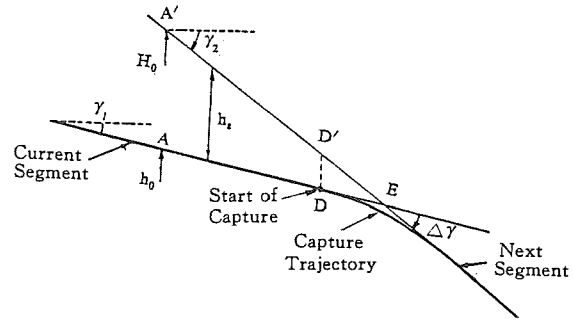


Fig. 12 Segment Transition Geometry

The transition control law is then developed as following, based mainly on the FPA control mode<sup>[1][8]</sup>:

$$\gamma_c = \begin{cases} \gamma_1 & \text{when } \text{sign}(\gamma_c) \cdot \text{sign}(h_\epsilon) > 0 \\ \gamma_1 + \gamma_c & \text{when } \text{sign}(\gamma_c) \cdot \text{sign}(h_\epsilon) \leq 0 \end{cases} \quad (31)$$

where:

$$\gamma_c = \gamma_2 - \gamma_1 + \left( K_h h_\epsilon + \frac{h_\epsilon s - \tau_{vp} \dot{h} s}{\tau_{vp} s + 1} \right) \frac{1}{V}$$

$$\text{sign}(x) \triangleq \begin{cases} +1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$$

The acceleration command  $\dot{V}_c/g$  is directly decided by speed control requirement, no special processing is needed. Fig. 13 illustrates a climbing transition simulation results, where aircraft is commanded to capture a fixed climb segment at  $FPA=+3$  (deg.).

An outstanding characteristics of the control law in (31) is its automatic adaptation to speed. With increasing speed, it takes an increasing  $h_\epsilon$  to make  $\gamma_c$  change its sign, hence capture will start sooner.

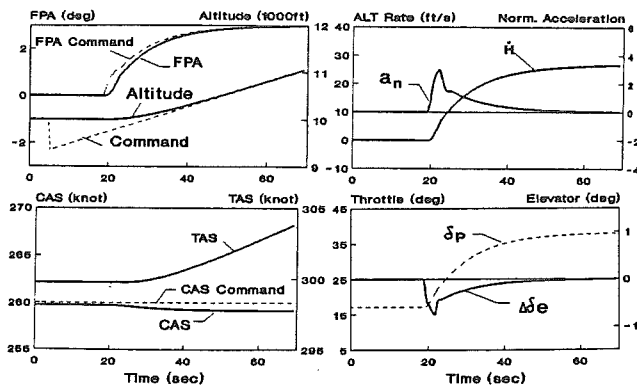


Fig. 13. Flight Segments Transition Simulation

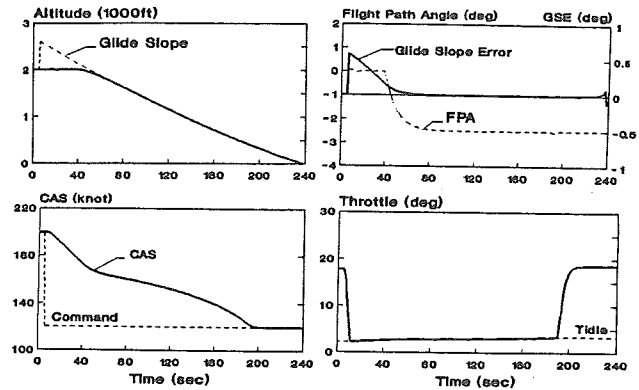


Fig. 14. Glide Slope Mode Simulation

### Special Landing Control Modes

In the automatic landing procedure, two final consecutive flight modes, Glide Slope mode and Flare mode, are needed. The glide slope mode is implemented mainly based on the segment flight path transition mode, but the linear altitude error  $h_e$  is derived from the glide slope error (GSE) angle  $\Gamma$ ?

$$h_e = \frac{h\Gamma}{\alpha_a - \Gamma} \quad (32)$$

where  $\alpha_a$  is the glide slope angle. Then the control law of FPA channel for the glide slope mode is as following, similar to the path transition mode:

$$\gamma_c = \begin{cases} 0 & \text{when } \gamma_e > 0 \\ \gamma_e & \text{when } \gamma_e \leq 0 \end{cases} \quad (33)$$

where:

$$\gamma_e = \left[ \frac{K_h \cdot h \cdot \Gamma}{\alpha_a - \Gamma} + \frac{h\Gamma s}{\alpha_a - \Gamma - \tau_{vp}s h} \right] \frac{1}{V} - \alpha_a$$

The acceleration control signal  $\dot{V}_c/g$  is derived from the deceleration requirement. And when thrust reaches idle state, a path control priority ( $q = 1.0$ ) is adopted to ensure aircraft to glide down along the G. S. accurately. Fig. 14 illustrates a simulation results for glide slope mode.

The flare trajectory is generally chosen as a exponential curve, with its asymptote being set lower than the runway plane by a constant height  $H_{bias}$ . Its altitude has following function relative to flight time:

$$h(t) + H_{bias} = H_0 \cdot e^{-\frac{t}{\tau}} \quad (34)$$

where  $H_0$  is the initial altitude of exponential flare path,  $\tau$  is its time constant which determines the touchdown time. In order to decrease the dispersion of touchdown points, the time constant  $\tau$  is set to be varied inversely to the ground speed  $V_G$  as  $\tau = K_0/V_G$ , where  $K_0$  is a constant gain used to determine the initial value of  $\tau$ . From equation (8), the FPA command signal for flare mode is as:

$$\gamma_c = \begin{cases} -\alpha_a & \text{when } -\frac{V_G(h + H_{bias})}{K_0 V} < -\alpha_a \\ -\frac{V_G(h + H_{bias})}{K_0 V} & \text{when } -\frac{V_G(h + H_{bias})}{K_0 V} \geq -\alpha_a \end{cases} \quad (35)$$

The normalized acceleration is  $\dot{V}_c/g = a_c/g$ , where  $a_c$  is the required deceleration. Fig. 15 illustrates a simulation results for automatic landing flare, beginning from glide slope flight mode. Table 1 gives the simulated results at different wind speeds, with tail wind being positive and head wind being negative.

### Optimum Trajectory Guidance

One major function of flight management system (FMS) onboard modern commercial aircrafts is to control aircraft to fly at its optimum performance (or along its optimum flight trajectory) with minimum cost and fulfill flight missions perfectly and automatically. TECS can also be able to track the optimized flight trajectories, especially for the flight paths optimized based on the point-mass energy state approximation approaches<sup>[8-10]</sup>. In this

Table 1. Flare Performance at Different Wind Conditions

Wind Speed (ft/sec)	-30	-20	-10	0	10	20	30	40	50
Flare Distance (ft)	1690	1674	1663	1657	1656	1656.5	1666	1675	1690
Flare Time (sec)	10.20	9.50	8.96	8.46	8.10	7.66	7.36	7.10	6.90
$H_0$ (ft)	53	50	46	43.2	41	38.8	36.5	34.8	33



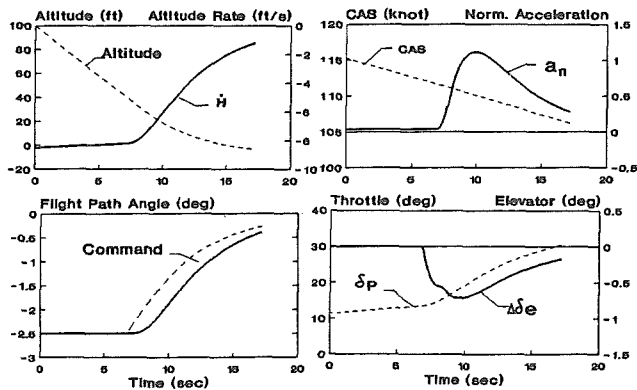


Fig. 15. Automatic Flare Simulation

method, the point-mass energy state approximation model is used as the state equations, the specific total energy state is used as the independent variable or time-like variable during the optimization process, the direct operating cost (DOC), defined as the weighted sum of fuel cost and time cost, is used as the performance index function. The trajectory optimization problem is finally simplified into pointwise extremization problems of several algebraic functions<sup>[9]</sup>. The optimization algorithm can give out not only the optimal altitude and speed profiles, but also the optimal total energy rate profile and FPA profile<sup>[8-9]</sup>, which are the basic controlled variables in TECS. Hence the guidance law, as shown in Fig. 16, is developed to directly give out the total energy rate and distribution rate command signals, instead of the  $\gamma_c$  and  $\dot{V}_c/g$  signals, as following forms:

$$\left(\frac{\dot{E}}{V}\right)_c = \frac{\dot{E}_{opt}}{V_{opt}} + \frac{K_V}{g}(V_{opt} - V) + \frac{K_h}{V}(h_{opt} - h) \quad (36)$$

$$\dot{L}_c = \frac{\dot{E}_{opt}}{V_{opt}} - 2\gamma_{opt} + \frac{K_V}{g}(V_{opt} - V) - \frac{K_h}{V}(h_{opt} - h) \quad (37)$$

where subscriptive *opt* represents the optimal profile information. Its distinguishing characteristic is the application of the optimal total energy rate and energy distribution rate signals, which represents the requirement on the changing rate of both altitude and speed. Hence, there actually exist the derivative signals of optimal altitude and speed profiles in the guidance law, which enables it to control aircraft through TECS to follow the optimal flight profile accurately and precisely.

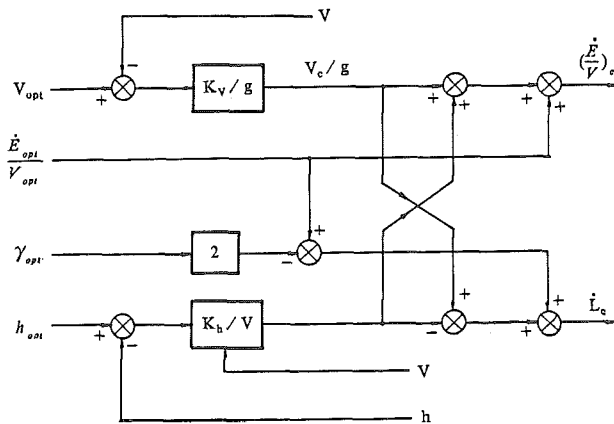


Fig. 16. Optimal Trajectory Guidance Law

Because of the basic assumption made in the point-mass energy state approximation model that the kinetic energy and potential energy could be exchanged at zero time during flight, there exist large step changes and high frequency vibrations in the resulted optimal FPA and acceleration profiles, which can not be directly followed or realized by actual aircrafts due to nonlinear limitations on normal acceleration etc. Hence some adaptive measures are introduced to the trajectory optimization algorithm and aircraft flight simulation, and several pre-processing methods, such as the lead-compensation and smooth filtering etc., are applied to the ideal optimal flight profiles, see ref. [8] for details.

The guidance law relates the performance optimization subsystem with the flight control system TECS, forming a simple flight management system in vertical profile. Digital simulation tests show its satisfactory control performance. The TECS can control the aircraft to track and realize the optimal profile accurately with acceptable elevator and throttle operations, and acceptable normal acceleration. Comparing the actual simulated flight profile with the pre-processed optimal profile, it can be found that in a steady climb, cruise and descent flight, the altitude error is less than 5 feet, speed error less than 0.1 knot; and in the transition flight (climb to cruise, cruise to descent etc.), the maximum altitude error is less than 30 feet, speed error less than 2 knots. Table 2 summarizes the simulation results for each flight

Table 2. Comparison of Simulation Results

Index	FUEL in clim (LB)	FUEL in cruise (LB)	FUEL in descent (LB)	FUEL in total (LB)	total flight TIME (s)	total COST (dollar)
Ideal Profile	8006.25	1677.46	616.25	10299.81	3142.51	1081.2
Simulated Profile	8050.19	1751.97	681.82	10483.97	3138.54	1092.2
Relative Error(%)	0.549%	4.44%	10.64%	1.788%	-0.126%	1.017%

segments as well as the whole performance index. Fig. 17 gives a typical portion of the whole profile simulation result, which is the most difficult portion to be tracked.

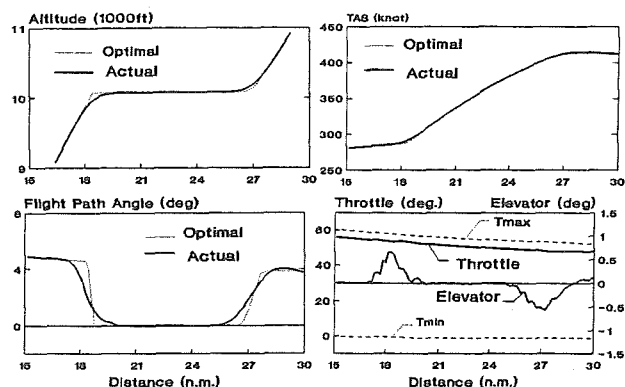


Fig. 17 Simulated Tracking Results.

### Conclusions

The concept of total energy control discussed is a very promising new design approach with many outstanding features. It absolutely changed the control strategy and system structure of conventional AFCS, provided decoupled path/speed maneuver controls, and had vast flexibility for various flight control requirements.

- The long term point-mass dynamics of aircraft is efficiently related to the short period attitude movement dynamics through controlling the amount of variation and transformation of total energy state, forming the basis for integrated flight/propulsion control system development.
- A multi-input/multi-output control structure is adopted to provide a generalized design foundation for various control requirements, normalized control and feedback signals are introduced to provide flexibility for various out-loop control modes.
- Modern decoupling control theory for multi-variable system is used to analyze and design TECS, instead of the conventional frequency domain design methods. Thus cross-feed decoupling functions can be directly constructed.

The total energy control system (TECS) discussed and developed in this paper meets all the operational requirements of the traditional autopilot and autothrottle. The normalization of all control signal feedbacks into energy related quantities makes the design highly generic and directly transportable to other commercial aircrafts.

### References

- [1] Lambregts, A. A. " Vertical Flight Path and Speed Control Autopilot Design Using Total Energy Principles " AIAA paper 83-2239cp
- [2] Lambregts, A. A. " Operational Aspects of the Integrated Vertical Flight Path and Speed Control System " SAE 831420
- [3] Kelly, J. R. and Bruce, K. R. " Flight Testing TECS—The Total Energy Control System " SAE 861803
- [4] Kaminer, I. and O'shaughnessy, R. " 4D-TECS Integration for NASA TCV Airplane " AIAA 88-4067cp.
- [5] Liu, Chen-hui " General Decoupling Theory of Multivariable Process Control Systems " Xinhua Book Company, 1984.
- [6] Ackermann, J. " Parameter Space Design of Robust Control Systems " IEEE Trans. on Auto. Contr. Vol. AC-25, No. 6, 1980.
- [7] Kaminer, I. and Benson, R. A. " Design of Integrated Autopilot/Autothrottle for NASA TSRV Airplane Using Integral LQG Methodology " AIAA89-3595cp.
- [8] Wu, Shufan " Studies on The Flight Performance Management and Integrated Control Techniques Based on The Total Energy Principles " Ph. D. Dissertation, Nanjing Aeronautical Institute, PRC, Sept. 1990
- [9] Lee, H. Q. and Erzberger, H. " Algorithm For Fixed-Range Optimal Trajectories " NASA TP 1565, July, 1980
- [10] Sorensen, J. A. " Computer Program For Generation and Evaluation of Optimal Vertical Profiles " NASA CR 3688, 1983