ANALYSIS AND FEEDBACK CONTROL OF AIRCRAFT FLIGHT IN WIND SHEAR

Han, Chao

School of Astronautics
Beijing University of Aeronautics and astronautics
Beijing, 100083, P. R. China

ABSTRACT

One kind of Modal Aggregation Method (MAM) presented in this paper could retain the dominant of linear system in model reducing. Some approximate formulas of flight altitude or total energy response to windshear, which only containing the effect of phugoid mode and altitude mode of aircraft , have been derived by MAM . The relationship between altitude mode and total energy has been discussed. These formulas are useful to predict the hazard in windshear encounter.

INTRODUCTION

It is well known that low altitude microburst windshear can significantly affect aircraft performance during landing approach or taking off. Hazardous wind shear have caused several commercial airline accidents. In response to the windshear problem, numerous researches [1- 5] for the microdownburst encounter problem have been done. Now the studies of windshear problem are emphasized in area of windshear hazard detection and alerting system[6-8]. But the danger of aircraft affected windshear is not a directly measurable quantity. Some "Hazard criterion", such as F—factor, based on the impact of a microburst windfield on total energy of the aircraft, is only the point measurement of performance. It is not perfect because the danger of aircraft in windshear is accumulated with the time. So the foundation of windshear hazard detection should be the fully understanding of aircraft's performance in windshear.

The main objective of the paper is to obtain some simple analytical formulas which are useful to analyse the performance of aircraft in windshear. For this purpose, one kind of Modal Aggregation Method is presented first, which could handle the model reduction of linear system analytically. With MAM, the approximate transfer functions of aircraft's altitude response to windshear are derived. The relationship between altitude mode and total energy is discussed. A design method for improving aircraft performance of penetrating windshear is put forward.

MODAL AGGREGATION METHOD

Aggregation method [9] is one kind of model reduction method of large scale linear system modeling, which describes system with a coarser state variables and retains the key qualitative properties of the initial system. Modal Aggregation retains the dominant modes of the initial system.

Considering a linear dynamic system in the form as follows

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}\boldsymbol{u} \qquad (1)$$

where x is the n dimensional state vector and $\boldsymbol{u}$ is the m dimensional control vector. x_1 is the state vector interested. A_{11}, A_{12}, A_{21} and A_{22} are the block state matrix correspond to x_1 and x_2 respectively. B_1 and B_2 are block control matrix. By transformation of state variable

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}I & 0\\ -S & I\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (2)$$

the linear dynamic system equation (1) can be rewritten as

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix} = \begin{bmatrix}A_{11} + A_{12}S & A_{12}\\ S\,A_{21}\,S - A_{21} + S\,A_{11} - A_{22}\,S & A_{22} - S\,A_{22}\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}\boldsymbol{u} \qquad (3)$$

When the block matrix S satisfy the general Riccati equation

$$S\,A_{21}\,S + S\,A_{11} - A_{22}\,S - A_{21} = O \qquad (4)$$

the linear dynamic system will be uncoupled into two chained subsystems, i. e.

$$\begin{cases}\dot{x}_1 = (A_{11} + A_{12}\,S)\,x_1 + A_{12}\,y_2 + B_1\,\boldsymbol{u}\\ \dot{y}_2 = (A_{11} - S\,A_{12})\,y_2 + (B_1 - S\,B_2)\,\boldsymbol{u}\end{cases} \qquad (5)$$

If $A_{11} + A_{12}S$ and $A_{22} - SA_{21}$ correspond to the dominant modes and the nondominant modes of the linear dynamic system (1), the state variable y_2 is the relative faster modes, i. e. , it converges to its stable state faster than x_1 . From the second equation of Eq. (5), the stable state of y_2 is

$$y_2 = -(A_{22} - S\,A_{12})^{-1}(B_1 - S\,B_2)\,\boldsymbol{u} \qquad (6)$$

Substituding y_2 of (6) into the first equation of (5) leads to modal aggregation model

$$\dot{x}_1 = (A_{11} + A_{12}\,S)\,x_1 + [(B_1 - A_{12}(A_{22} - S\,A_{12})^{-1}(B_1 - S\,B_2)]\,\boldsymbol{u} \qquad (7)$$

x_1 is the aggregated state which retains the dominant modes of the initial system (1).

For uncoupling system (1) into two chained subsystems, the matrix S must satisfy the general matrix Riccati equation (4). But the matrix equation is a nonlinear equation group and it is difficult to be solved. On other hand, it could be proved, when

$$S = M_{21}M_{11}^{-1} \tag{8}$$

matrix S satisfies the Eq. (4), where M_{21} and M_{11} are the block matrix of the matrix M, the model matrix of linear system (1), having the form

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{9}$$

Matrix M consists of the eigenvectors of matrix A arranged in according to the dominant modes and the nondominant modes. It is seen from (9), the solution of matrix S is ununique. But when $|M_{21}| << |M_{11}|$ which means that the aggregated state is the primary state variable of dominant modes, the norm of matrix S is a smaller value. This means that the second order quantitative in Eq. (4) could be neglected. Thus the general Riccati matrix equation could be replaced approximately by a general Lypunov equation

$$S\ A_{11} - A_{22}\ S - A_{21} = O \tag{10}$$

and the matrix S can be solved approximately from Eq. (10). Eq. (10) is a linear equation group and in general it has a unique solution.

AIRCRAFT RESPONSE TO WIND SHEAR

The nonlinear equation of the motion of aircraft for symmetric flight in a variable wind field can be expressed in wind axis as follows

$$\dot{x} = f(x, u, w, \dot{w}) \tag{11}$$

where

$$x^T = (V_a, \alpha_a, q, \theta, h)$$
$$u^T = (\delta_e, \delta_p)$$
$$w^T = (w_h, w_v)$$

V_a, α_a, q, θ, and h are the air speed , angle attack, pitch velocity, pitch angle, and flight altitude of aircraft; δ_e is elevator angle; δ_p is thrust; w_h, w_v are the horizontal and vertical components of wind velocity respectively.

During the take — off or the approach, the aircraft is nominally trimmed at constant air speed (V_e) and flight angle (γ_e) . Linearized equation of motion could be derived from equation (11) for perturbation from the trim. The dynamic equation takes the form of

$$\Delta\dot{x} = A\Delta x + B\Delta u + C_1\Delta w + C_2\Delta\dot{w} \tag{12}$$

where

$$A = \frac{\partial f}{\partial x}; \quad B = \frac{\partial f}{\partial u}; \quad C_1 = \frac{\partial f}{\partial w}; \quad C_2 = \frac{\partial f}{\partial \dot{w}}; \tag{13}$$

Many researches[1,3,4] have proved that the phugoid mode and the altitude mode of aircraft determine the performance of aircraft penetrating a wind shear during take—off or approach. For the purpose of analyzing the penetrating wind shear performance of aircraft and emphaszing the effect of phugoid and altitude modes, we can take the phugoid and altitude modes as the dominant modes discussed in above section, and take the short period mode of aircraft as the nondominant mode because it changes faster than the phugoid, altitude modes, and the disturbance of windshear. And, it is known that air speed, pitch angle mainly present the movement of phugoid mode, attack angle and pitch velocity present the short period mode, and altitude mode only appeared in the altitude variables[10]. So V, q, and h are taken as the aggregated state

$$\begin{cases} \Delta x_1^T = (\Delta V_a, \Delta\theta, \Delta h) \\ \Delta x_2^T = (\Delta\alpha_a, q) \end{cases} \tag{14}$$

Using the Model Aggregation Method described in above section, the aggregated model of aircraft penetrating wind shear could be reduced. Then taking a Laplace transformation for the aggregated model and eliminating some little accounts, the approximate transfer functions of aircraft response to wind shear disturbance could be obtained

$$\begin{cases} G_{hw_h}(s) = -\dfrac{V_e}{g}\dfrac{\omega_{nph}^2}{s^2 + 2\zeta_{ph}\omega_{nph}s + \omega_{nph}^2} \\ G_{hw_v}(s) = \dfrac{1}{s} + \dfrac{1}{\omega_{nsp}^2}\dfrac{\omega_{nsp}^2{}^2\mu_z^q - \mu_z^\alpha s}{s^2 + 2\zeta_{ph}\omega_{nph}s + \omega_{nph}^2} \end{cases} \tag{15}$$

ω_{nph} and ζ_{ph} are the natural frequency and damper rate of the phugoid motion of aircraft, ω_{nsp} and ζ_{sp} are the of the short period motion. V_e is the airspeed in trim approach or take — off conditions. μ_z^q and μ_z^α are the partial derivatives of dynamic parameters with respect to pitch velocity and attack angle respectively. Figure (1) shows a comparison in Bode plot between the approximate formulas Eq. (15) with the exact transfer functions of dynamic equation (12) of a jet aircraft Airbus 300 during approach with $V_e = 72m/s$ trim airspeed and (γ_e) = — 3 *Deg* flight path angle, where the dash line is the results of Eq. (15) and the full line is the results of the exact solutions. Figure (1) shows that the formulas of Eq. (15) have a good precision relative to the exact transfer functions, especially at the low frequency, where the interested range of wind shear disturbance.

The first equation of Eq. (15) shows the relation of flight altitude of aircraft response to the horizontal wind input. It is a second order system

$$\frac{\omega_{nph}^2}{s^2 + 2\zeta_{ph}\omega_{nph}s + \omega_{nph}^2} \tag{16}$$

with the amplification of

$$-\frac{V_e}{g}$$

In microdownburst, the horizontal wind component varying from headwind to tailwind could be modeled simply as one period of sine wave. The second order system (16) response to a sine input can be found from

$$y(t) = \int_{t_0}^{t} \Phi(t-\tau)\ \sin\omega\tau\ d\tau \qquad (17)$$

where

$$\Phi(t) = e^{-2nt}\ \sin\omega t$$
$$n = \zeta_{ph}\,\omega_{nph}$$
$$\omega = \sqrt{1-\zeta_{ph}^2}\,\omega_{nph}$$

For estimating the maximum altitude response to horizontal windshear in form of sine wave, the frequency of sine windshear is supposed equal to ω_p, the resonant frequency of (16)

$$\omega_p = \sqrt{1-2\zeta_{ph}^2}\,\omega_{nph} \qquad 0 < \zeta_{ph} < \frac{\sqrt{2}}{2} \quad (18)$$

When $t_0 = 0$, the integration of (17) is

$$y(t) = \frac{\sqrt{1-2\zeta_{ph}^2}}{2\zeta_{ph}(1-\zeta_{ph}^2)} e^{-2nt}\ \cos\omega t + \frac{1}{2\zeta_{ph}\sqrt{(1-\zeta_{ph}^2)}}\sin(\omega_p t + \varphi_p) \qquad (19)$$

where

$$\varphi_p = -\ \mathrm{tg}^{-1}\frac{\sqrt{1-2\zeta_{ph}^2}}{\zeta_{ph}}$$

Figure (2) shows surface plot and contour map of equation (19) as $y = y(\zeta_{ph}, \omega_p t)$. For the case of $0 < \zeta_{ph} << 1$, expending (19) to a Taylor serious with some algebraic symbol system[11], REDUCE, the first order approximation expression is obtained

$$y(t) = -\frac{1}{2}(1 - \zeta_{ph}\,\omega_{nph}\,t)\,(\omega_{nph}\,t\ \cos\omega_{nph}\,t - \sin\omega_{nph}\,t) \qquad (20)$$

The transient response of altitude to horizontal windshear may be approximately expressed as

$$h(t) = \frac{V_e}{g} w_{hmax}\, y(t) \qquad (21)$$

Then the maximum altitude response to horizontal windshear can be estimated by setting t equal to 2π, namely

$$h_{max} = -\pi(1 - 2\pi\zeta_{ph})\,w_{hmax}\frac{V_e}{g} \qquad (22)$$

With the increasing of damping ratio, the heap response will be decreasing greatly.

The second equatuion in Eq. (15), altitude response to vertical windshear, consists of two parts, a integration term which presents the effect of aircraft floating with the airmass, and the effect of the phugoid mode as the second part. For the vertical windshear, the downdraft, the main reason of altitude loss is due to the integration effect for the reason of the low frequency of windshear disturbance. The effect of integration is proportion to the mean velocity of downdraft and the time in downdraft zone. Then, the altitude loss in downdraft can be estimated by

$$h_{max} = \overline{w}_v \frac{L}{V_e} \qquad (23)$$

where $\overline{w}_v$ is the mean velocity of downburst, L is the distance of aircraft flighting in downburst.

The horizontal and the vertical windshear have a different effect on aircraft altitude response. When the period of horizontal windshear near the aircraft phugoid period, it would produce a resonant response in altitude. It is necessary to increase the damping ratio of phugoid for decreasing the peak resonant must . While the downdraft causes aircraft altitude loss because of the integration effect. It means that we must eliminate the integration effect to decrease the altitude loss caused by downdraft .

TOTAL ENERGY AND ALTITUDE MODE

In Eq. (15), the integration called also as altitude mode is the main reason of the aircraft's altitude loss in downdraft. To eliminate the altitude mode is less easily achieved, because, the first, the movement of altitude mode should be able to be measured , the second, an effective controllability acting to altitude mode is needed. Many researches have showed that total energy of aircraft can improve the microburst penetration ability of aircraft. Total energy, h_e , the measurement of aircraft's mechanical energy is

$$h_e = h + \frac{1}{2g}V^2 \qquad (24)$$

Linearizing Eq. (24) for small perturbation, taking time derivative

$$\Delta\dot{h}_e = \Delta\dot{h} + \frac{V_e}{g}\Delta\dot{u} \qquad (25)$$

and replacing h in Eq. (12) with (25), some approximate transfer function of total energy could be obtained with the Modal Aggregation Method as follows

$$\begin{cases} G_{h_e w_x}(s) = -\frac{V_e}{g}\left(1 + x^V \frac{s - \bar{a}_{22}}{s^2 + 2\zeta_{ph}\omega_{nph}s + \omega_{nph}^2}\right) \\ G_{h_e w_y}(s) = \frac{s^2 + (2\zeta_{ph}\omega_{nph} - x^V \frac{\mu_z^a}{\omega_{nsp}^2})s + \omega_{nph}^2}{s(s^2 + 2\zeta_{ph}\omega_{nph}s + \omega_{nph}^2)} \\ G_{h_e \delta_p}(s) = \frac{V_e}{g} x^{\delta_p} \frac{s^2 + (2\zeta_{ph}\omega_{nph} - x^V)s + \omega_{nph}^2}{s(s^2 + 2\zeta_{ph}\omega_{nph}s + \omega_{nph}^2)} \end{cases} \quad (26)$$

Considering the value of x^V very small, the numerator and denominator of (26) can be canceled out each other approximately, thus Eq. (26) can be reduced further as

$$\begin{cases} G_{h_e w_x}(s) = -\frac{V_e}{g} \\ G_{h_e w_y}(s) = \frac{1}{s} \\ G_{h_e \delta_p}(s) = \frac{V_e}{g} x^{\delta_p} \frac{1}{s} \end{cases} \quad (27)$$

Figure (3) shows the comparison of Eq. (27) with the exact total energy transfer factions. It confirms that (27) present the main character of total energy response to windshear and thrust. The first formula of (27) means

$$\Delta h_e = -\frac{V_e}{g}\Delta w_h \quad (28)$$

Considering ( ), then obtained

$$\Delta h + \frac{V_e}{g}(\Delta V_a + \Delta w_h) = 0 \quad (29)$$

It is the energy conservative relation. The second and the third formula of Eq. (27) present that the change of total energy is in proportion to integratios of w_y and δ_p. From the comparison of Eq. (15) and Eq. (27), it is shown that total energy presents the movement of altitude mode. So, it would be more proper to call the integration term in Eqs. (15) and (27) a total energy mode. Eq. (27) can be also rewritten as

$$\Delta \dot{h}_e = \frac{V_e}{g} x^{\delta_p} \delta_p + w_y - \frac{V_e}{g}\dot{w}_h \quad (30)$$

Eq. (30) shows the simple relation of total energy with windshear and thrust. It also implies the weak correlation between total energy and phugoid, and the phugoid mode will not be affected greatly by the feedback of total energy to thrust. For jet aircraft, thrust is assumed to be independent to airspeed and there is a first — order lag between throttle setting, δ_{p_c}, and the thrust, δ_p

$$\dot{\delta}_p = \frac{1}{\tau_e}(\delta_{p_c} - \delta_p) \quad (31)$$

where τ_e is the lag time. By the feedback of total energy to throttle setting, i.e.

$$\delta_{p_c} = -K_1 \Delta h_e - K_2 \Delta \dot{h}_e \quad (32)$$

and considering (30) and (31), we obtain a second — order closed loop system

$$\begin{bmatrix} \Delta \dot{h}_e \\ \dot{\delta}_p \end{bmatrix} = \begin{bmatrix} 0 & a \\ -\frac{K}{\tau_e} & -\frac{1}{\tau_e}(1 + akK) \end{bmatrix} \begin{bmatrix} \Delta h_e \\ \delta_p \end{bmatrix} + \begin{bmatrix} 1 \\ -\frac{kK}{\tau_e} \end{bmatrix} f(t) \quad (33)$$

where

$$a = \frac{V_e}{g} x^{\delta_p}, \qquad k = \frac{K_2}{K_1}$$

$$K = K_1 \quad f(t) = w_y - \frac{V_e}{g}\dot{w}_x$$

Its characteristic function is

$$s^2 + \frac{1}{\tau_e}(1 + K_2 \frac{V_e}{g} x^{\delta_p})s + \frac{1}{\tau_e} K_1 \frac{V_e}{g} x^{\delta_p} = 0 \quad (34)$$

The natural frequency and damping ratio of system (33) depend on the parameters K_1 and K_2 respectively. For simplification, $f(t)$ is assumed as a white noise process. Then the mean square value of total energy response could be written as

$$E\{\Delta h_e^2(t)\} = \left[\frac{1}{2aK} + \frac{1}{2}\frac{\tau_e - k}{1 + akK}\right] E\{f^2(t)\} \quad (35)$$

It is also the function of parameters K_1, K_2, which can be obtained for the desired natural frequency and damping ratio. Eq (35) is used to calculate the mean square value of total energy response to windshear. In general, the value of (35) is not very large for a satisfied system.

EXAMPLE

Taking Airbus 300 as an example, with the basic parameters $\tau_e = 4s$, $x^{\delta_p} = 2.8282 ms^{-1}$, $V_e = 72 ms^{-1}$, and $a = 20.757 ms^{-1}$, when the desired closed loop pole is chosed as $(-1/\tau_e, 0.2/\tau_e)$, the beedback parameters it could obtained, $K_1 = 0.01253\ m^{-1}$, $K_2 = 0.0482\ sm^{-1}$ by Eq (34). And the mean square value of total energy reponse to windchear is $E\{h_e^2\} = 1.952 E\{f^2\}$. Thus the feedback of total energy to throttle setting is

$$\delta_{p_c} = -0.01253\Delta h_e - 0.0482\Delta \dot{h}_e \quad (36)$$

The desired closed loop pole is (−0.25, 0.05), while the actual closed loop pole with feedback of (36) is (−0.27, 0.046), the relative error is (7.4%, 8.7%). And, the phugoid pole has changed from (−0.0131, 0.167) in open loop to (0.0105, 0.167) in closed loop. It means that there is slight effect of total energy to phugoid mode. The improvemen of the phugoid performance can be easily achieved by feedback of pitch angle, θ, to elevator, δ_e

$$\Delta\delta_e = -1.5\Delta\theta \quad (37)$$

With the feedback strategies of (36) and (37), the phugoid pole of closed loop system is (−0.115, 0.1397) and the altitude — thrust pole is (−0.268, 0.0508), There is only a slight change in altitude — thrust pole. It means that even in the closed system, the correlation of total energy with phugoid model is also very weak.

Figure (4) shows the nonlinear simulation results of A300 jet aircraft penetrating a downburst during landing approach. Fig (4—a) is of open loop and Fig (4—b) is of closed loop with feedback of (36) and (37). Figure (5) is the downburst windshear encountered. In comparing Figs (4—a) with (4—b), it is apparent that the performance of penetrating windshear has been improved greatly, though the control strategies of (36) and (37) are very simple.

CONCLUSION

With the Modal Aggregation Method, the approximate transfer function of aircraft altitude and total energy response to windshear have been derived successfully. These formulas agree well with the exact transfer function. By these formulas, one can estimates the performance of aircraft in windshear.

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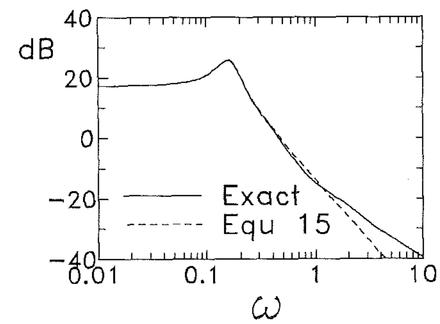
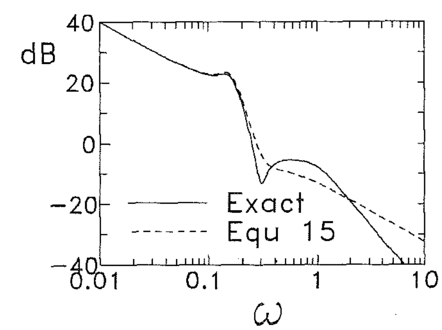
FIGURE



(a) $G_{h w_x}(j\omega)$



(b) $G_{h w_z}(j\omega)$

Fig 1 Bode Plot of Altitude Response

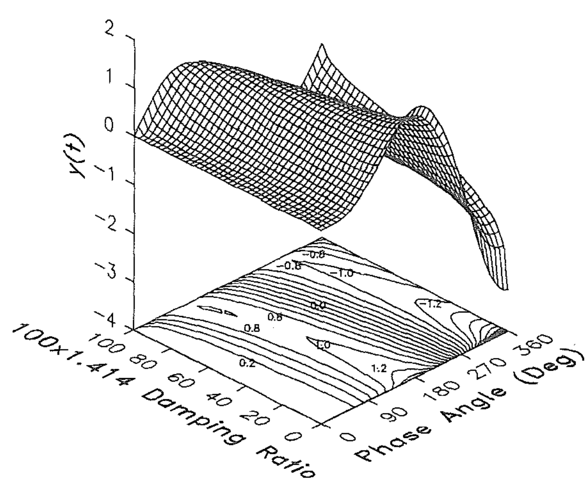
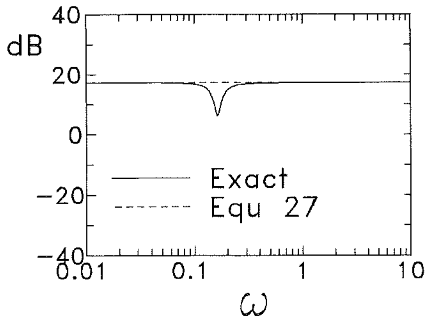
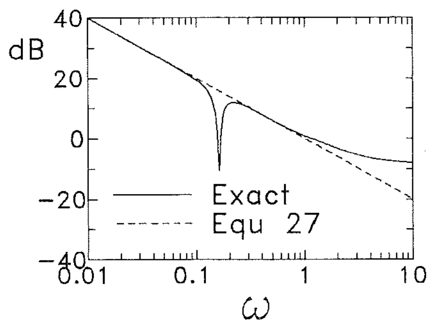


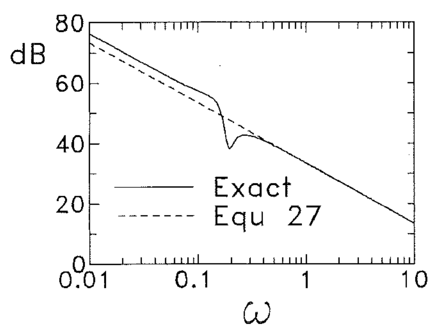
Fig 2 Surface plot and Contour Map of Eq(19)



(a) $G_{h_e w_x}(j\omega)$

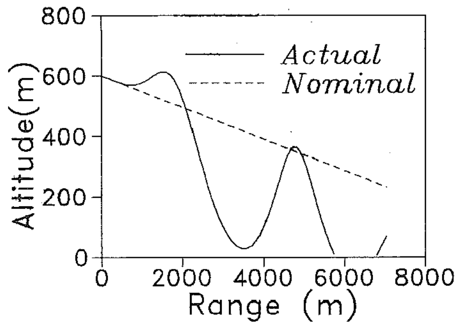


(b) $G_{h_e w_y}(j\omega)$

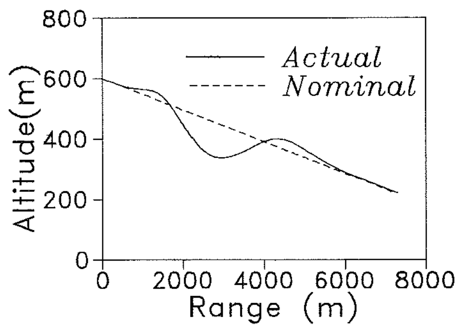


(c) $G_{h_e \delta_T}(j\omega)$

Fig 3 Bode Plot of Total Energy Response



(a) Open Loop



(b) Closed Loop with (36) and (37)

Fig 4 Transient Response Trajectory to Windshear

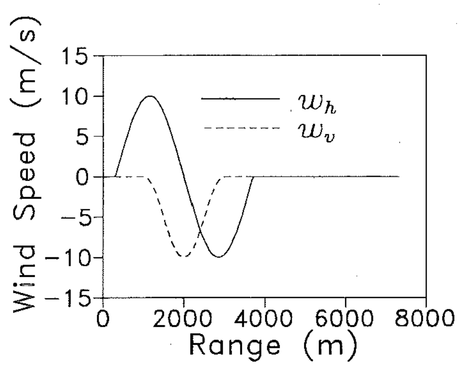


Fig 5 Wind Speed of Windshear