# DESIGN AND IMPLEMENTATION OF AN INVERTER USING WALSH-FUNCTION-METHOD

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## Abstract

In this paper, an inverter using Walsh-function-method is proposed, the inverter consists of three power switching groups without polarity changer circuit, and forms a stepped sinewave output. The power switching groups are controlled by a Walsh function generator with three Walsh-function-signals: Wal(1, t), Wal(5, t) and Wal(13, t). This Walsh generator is realized by three standard integrated circuits with low cost.

The proposed inverter has a simple topology, constant internal resistance and low distortion. The paper introduces the method to synthesize a given sinewave and to analyze and eliminate harmonic content. Two concepts of the inverter, design method of the control circuit (a Walsh generator) with some examples and prototype circuits are introduced. Simulation results of the designed inverter compared with measured results of the prototype circuits are given.

## Introduction

Walsh functions and Walsh transforms are important analytical tools for signal processing and have found wide applications in digital communications as well as in digital image processing. The Walsh- method is also a new technique to analyze power electronic circuits or to synthesize sinusoidal waveform[1]. To achieve harmonic analysis and elimination in a PWM waveform using Walsh functions, a group of linear equations is used replacing the nonlinear equations in Fourier analysis [2].

Stepped sinewave inverters have been applied in space systems and some terrestrial systems [3] and in AC drive systems [4]. The inverter proposed by [3] is not suitable to variable loads, since it has a time varying internal resistance. The inverter by [4] can be used to reduce the low-order harmonic content in the output voltage significantly, however it requires a more complex inverter circuit with additional power semiconductor devices.

This paper presents a method for design and implementation of a stepped sinewave inverter using Walsh functions. The inverter has low distortion and a constant internal resistance. The power circuit of the inverter has a simple topology and is controlled by a Walsh function generator. A set of linear algebraic equations is proposed for analysis and elimination of harmonics.

## Walsh Functions and Walsh Transforms [5]

In order to present the proposed method, two orthogonal functions (Rademacher and Walsh functions), and Walsh- Hadamard transforms are introduced first.

Rademacher functions, R(n, t), have 2n- 1 periods of square-wave over a normalized time base $0 \leqslant t \leqslant 1$. The amplitudes of the functions are - 1 and + 1. Rademacher functions can be derived from sinusoidal functions which have identical zero crossing positions. Thus,

$$R(n, t) = \mathrm{sign}[\sin(2^n \pi t)] \qquad (1)$$

The first six Rademacher functions are shown in Fig. 1.

Walsh functions form a complete set of two-valued orthogonal functions with values +1 or -1 over a normalized time base $0 \leqslant t \leqslant 1$. The set of Walsh functions in Walsh ordering is denoted by

$$S_w = \{wal_w(n, t), n=0, 1, \ldots, N-1\} \qquad (2)$$
$$\text{where } N=2^p, p=1, 2, 3, \ldots$$

The subscript 'w' denotes Walsh ordering, n denotes the n-th member of $S_w$, and t is the normalized time. The first eight functions of $S_w$ are shown in Fig. 2.

fast Walsh-Hadamard transform ( the computation for the transform can be achieved manually or by a computer automatically) . The obtained Walsh functions and their coefficients form a presupposition for design of the inverter circuits. For the sequence {X( m) } above given, the obtained Walsh coefficients in Hadamard-ordering only six terms are not zero: $A_h(16)=6.5$, $A_h(26)=-1.5$, $A_h(28)=-2.5$, and $A_h(21)=A_h(22)=A_h(25)=-0.5$

With the Eq. (8) the corresponding Walsh -ordered Walsh coefficients can be obtained. For example, in the case for $A_h(28)$, $n=(28)_d=(11100)_b$, $<n>=00111$, $b<n>=(00101)_b=(5)_d$. (Where the subscript ' d ' and ' b ' denote decimal and binary number respectively). It means, $A_h(28) = A_w(5)$ . By the analogy calculation we have:

$A_w(1)=A_h(16)$, $A_w(5)=A_h(28)$,
$A_w(13)=A_h(26)$, $A_w(9)=A_h(22)$,
$A_w(25)=A_h(21)$, $A_w(29)=A_h(25)$

Three Walsh coefficients of significance are $A_w(1) = 6.5$, $A_w(5) = -2.5$, $A_w(13) =-1.5$

Where $A_w(1)$, $A_w(5)$, and $A_w(13)$ are the coefficients of the first, 5-th and 13- th Walsh functions in Walsh- ordering. The coefficients of the 9-th, 25-th and 29- th Walsh functions equal -0.5, The other 26 coefficients are zero. Hence, to form a approximate stepped sinewave only three Walsh functions are required.

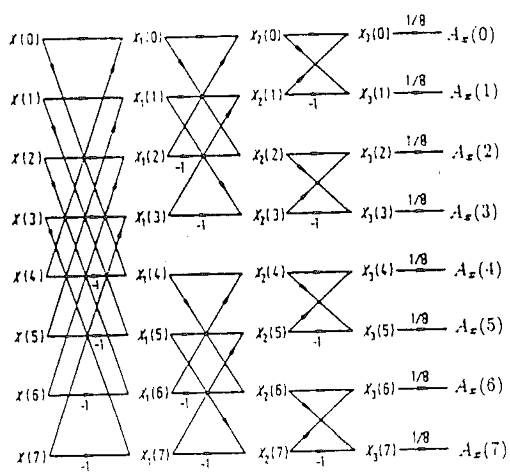


Fig. 3 $(FWHT)_h$ signal graph for $N = 8$

## Implementation of a Stepped Inverter

Two circuit concepts can be used to realize a stepped inverter. A serial circuit concept is shown in Fig. 5, and a parallel concept in Fig. 6. The power circuits in both concepts contain three groups of power switches. The power switches are controlled by corresponding output Walsh functions (signals) from a Walsh function generator. The Walsh coefficients obtained by fast Walsh transform determine the amplitudes of DC voltages (in the case of the serial concept ) or the transformer ratio(in the case of the parallel concept). The control circuit of the inverter is a Walsh function generator. To implement this circuit, a method for design of the Walsh generator will be used.

The essentiality of the method is to establish a transformation from Rademacher functions to the corresponding Walsh functions [ 6] . The transformation representation is,

$$wal_w(j, t) = \sum_{i=1}^{m} g_{i-1} R(i, t) \qquad (9)$$

The summations are expressed as Modulo- 2 addition, i.e. binary sums without carry. $wal_w(j, t)$ denotes the j-th Walsh function in Walsh- ordering, $R(i, t)$ is i- th Rademacher function, $g_{i-1}$ (=0 or 1) denotes (i-1)-th bit in the Gray code expression of the original binary number j. For example, to obtain the Walsh function $wal_w(5, t)$, we have $j=(5)_d=(00101)_b$.

The Gray code expression of $(00101)_b$ is $(00111)_g$. It means, $g_0=g_1=g_2=1$, and $g_3=g_4=0$ Using the Eq. (9), then

$$wal_w R(5, t) = R(1, t) \oplus R(2, t) \oplus R(3, t) \qquad (10)$$

The Eq. (10) demonstrates that through a 'exclusive or' operation of the three Rademacher functions,-R(1, t), R(2, t) and R(3, t)-, the $wal_w(5, t)$ function can be generated.

The Rademacher functions may be easyly obtained by using a binary counter [ 6] . Samely, we have:

$wal_w(1, t)=R(1, t)$ and

$wal_w(13, t)=R(1, t) \oplus R(2, t) \oplus R(4, t)$

With two 'exclusive or gates' and a binary counter, the three Walsh functions (signals) may be generated (shown Fig. 7).

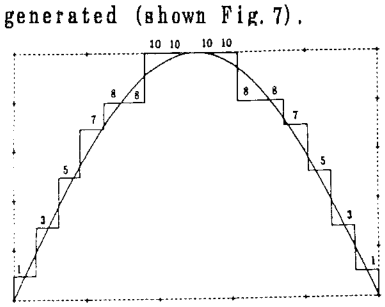


Fig. 4 A sampled sinewave for $N = 32$

Walsh-Hadamard transforms (WHT) are used to establish the conversion relationship between a time function (signal) and the Walsh-coefficients. Let {X(m)} denote an N-periodic sequence X(m), m=0, 1, ..., N-1.

$$\{X(m)\} = \{X(0)\,X(1)\ldots X(N-1)\} \qquad (3)$$

{X(m)} is represented by means of an N-vector $\mathbf{X}(n)$ to obtain

$$\mathbf{X}(n)' = [X(0)\,X(1)\ldots X(N-1)] \qquad (4)$$

where $n=\log_2 N$, and $\mathbf{X}(n)'$ is the transpose of $\mathbf{X}(n)$. The Hadamard-ordered Walsh-Hadamard transform $(WHT)_h$ of {X(m)} is defined as

$$\mathbf{A}_x(n) = \frac{1}{N}\,\mathbf{H}_h(n)\,\mathbf{X}(n) \qquad (5)$$

Where $\mathbf{H}_h(n)$ are the Hadamard matrices, $\mathbf{A}_x(n)$ are the *coefficients of Hadamard- ordered* Walsh functions. The Hadamard matrices $\mathbf{H}_h(n)$ *can be generated using the following* recurrence relation:

$$\mathbf{H}_h(k) = \begin{bmatrix} \mathbf{H}_h(k-1) & \mathbf{H}_h(k-1) \\ \mathbf{H}_h(k-1) & -\mathbf{H}_h(k-1) \end{bmatrix} \qquad (6)$$

$$k=1, 2, \ldots, n$$

$$\mathbf{H}_h(1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{H}_h(2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \qquad (7)$$

An algorithm for fast Hadamard-ordered Walsh-Hadamard *transform* $(FWHT)_h$ is introduced by [5] and the algorithm of this transform for the case N=8 is shown in Fig. 3.

Using the Eq. (5) or Fig. 3 the Hadamard-ordered Walsh coefficients can be computed. In order to obtain the Walsh-ordered Walsh coefficients, the following equation can be used:

$$A_h(n) = A_w[b\langle n\rangle] \qquad (8)$$

Where <n> is obtained by the bit-reversal of n, b< n> is the Gray code- to- binary conversion of <n>, $A_h(n)$ and $A_w[b(\langle n\rangle)]$ are the Hadamard-ordered Walsh-coefficients and Walsh- ordered Walsh- coefficients respectively.

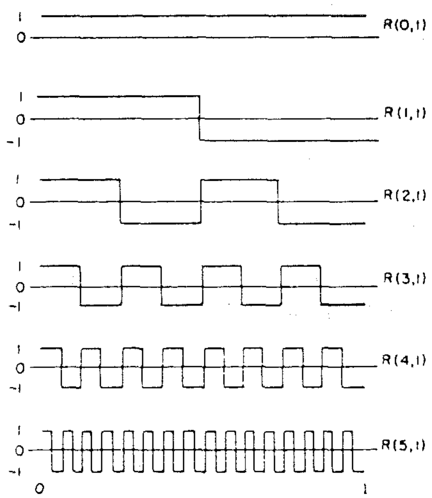



Fig. 1 The first six Rademacher functions

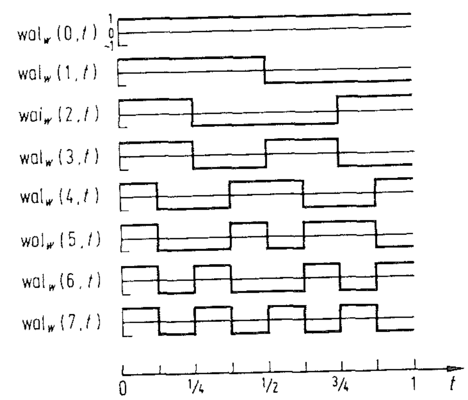



Fig. 2 The first eight Walsh functions

## Synthesis of a Sinewave Using Walsh-Function-Method

A sinewave may be first sampled symmetrically to generate a discrete N-periodic sequence for the Walsh- Hadamard transform (Fig. 4).

For a sinewave with amplitude = 10, the sampled discrete N-periodic sequence for N=32 is

{X(m)} =
{1, 3, 5, 7, 8, 8, 10, 10, 10, 10, 8, 8, 7, 5, 3, 1, -1, -3, -5, -7, -8, -8, -10, -10, -10, -10, -8, -8, -7, -5, -3, -1}

The corresponding Walsh functions and their coeffcients may be computed by means of the

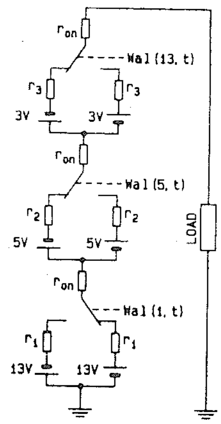



Fig. 5 A serial circiut concept of the inverter

## Harmonic Analysis and Elimination

The Walsh coefficients generated by Walsh transforms of a signal have a fixed conversion relation with the Fourier coefficients by the Fourier transform of the same signal. A Walsh to Fourier method for harmonic analysis has been introduced by [7]. By means of a conversion- factor -table, a set of linear algebraic equations is established;

For using three Walsh functions, we have:

$$
\begin{aligned}
a_1&=S(1,1)A(1)+S(1,2)A(5)+S(1,4)A(13)\\
a_3&=S(3,1)A(1)+S(3,2)A(5)+S(3,4)A(13)\\
a_5&=S(5,1)A(1)+S(5,2)A(5)+S(5,4)A(13) \qquad (11)\\
a_7&=S(7,1)A(1)+S(7,2)A(5)+S(7,4)A(13)\\
a_9&=S(9,1)A(1)+S(9,2)A(5)+S(9,4)A(13)\\
&\ldots\ldots
\end{aligned}
$$

For using six Walsh functions;

$$
\begin{aligned}
a_1&=S(1,1)A(1)+S(1,2)A(5)+S(1,3)A(9)\\
&\quad +S(1,4)A(13)+S(1,7)A(25)+S(1,8)A(29)\\
a_3&=S(3,1)A(1)+S(3,2)A(5)+S(3,3)A(9)\\
&\quad +S(3,4)A(13)+S(3,7)A(25)+S(3,8)A(29)\\
a_5&=S(5,1)A(1)+S(5,2)A(5)+S(5,3)A(9) \qquad (12)\\
&\quad +S(5,4)A(13)+S(5,7)A(25)+S(5,8)A(29)\\
a_7&=S(7,1)A(1)+S(7,2)A(5)+S(7,3)A(9)\\
&\quad +S(7,4)A(13)+S(7,7)A(25)+S(7,8)A(29)\\
a_9&=S(9,1)A(1)+S(9,2)A(5)+S(9,3)A(9)\\
&\quad +S(9,4)A(13)+S(9,7)A(25)+S(9,8)A(29)\\
&\ldots\ldots
\end{aligned}
$$

Where $a_k$ is the amplitude of k-th harmonic, A(n) is the coefficient of n- th Walsh function, S(k, n) are factors for Walsh to Fourier conversion, the factors are known and have been given by [7] with a table and a set of equations.

A set of the factors shown in Table.1 is useful for synthesis and analysis of a sinewave.

Looking up the Table.1 for above case we have

$S(1,1)=1.27375$,
$S(1,2)=-0.52760$,
$S(1,4)=-0.25336$,

These algebraic equations (11) and (12) may be used not only for harmonic analysis but also for harmonic elimination. For the serial concept (Fig.5) with $V_1=A(1)=13V$, $V_2=A(5)=-5V$, $V_3=A(13)=-3V$, the 1-15 harmonics can be calculated;

$a_1=1.274(13)-0.528(-5)-0.253(-3)=20$ (V)
$a_3=0.426(13)+1.028(-5)+0.285(-3)=-0.46$ (V)
......

All 1-15 -th harmonics computed by Eq. (11) and the simulation results by Fourier analysis are following;

| | computed results | | simulation results | |
|---|---|---|---|---|
| $a_1$ | 20.00 | 100% | 19.99 | 100% |
| $a_3$ | 0.46 | 2.30% | 0.44 | 2.20% |
| $a_5$ | 1.39 | 6.95% | 1.39 | 6.95% |
| $a_7$ | 0.00 | 0.00% | 0.00 | 0.00% |
| $a_9$ | 0.00 | 0.00% | 0.00 | 0.00% |
| $a_{11}$ | 0.66 | 3.30% | 0.63 | 3.15% |
| $a_{13}$ | 0.11 | 0.55% | 0.11 | 0.55% |
| $a_{15}$ | 1.46 | 7.30% | 1.33 | 6.65% |

For different application, the Eq. (11) may be used with different forms. For example, to obtain a 20V fundamental amplitude and to eliminate the 5-and 7- th harmonics, we have

$$
\begin{aligned}
1.274A(1)-0.528A(5)-0.253A(13)&=20\\
0.257A(1)+0.621A(5)-0.385A(13)&=0 \qquad (13)\\
0.186A(1)-0.077A(5)+0.933A(13)&=0
\end{aligned}
$$

and obtained;

$A(1)=12.2$ (V), $A(5)=-6.9$ (V), $A(13)=-3.0$ (V)

Using the circuits Fig.5 and Fig.6 with this three values, the experimental results have been obtained by means of simulation and Fourier analysis (Table. 2). The calculated and simulation results show good agreement.

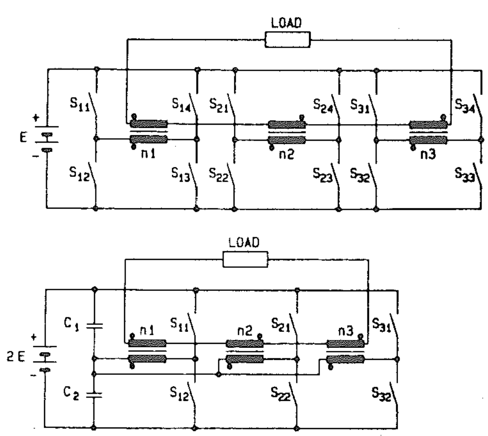



Fig. 6 A parallel circuit concept of the inverter

For three phase applications, the 3n- th harminics of the line- to- line output waveform may be removed. In this case only the influencing of 13- th harmonic is significant (about 3%).

## Distortion in Case of Variable Loads

It is shown in Fig. 5 and Fig. 6 obviously, that the inverter has a constant internal resistance, the load voltage characteristic is linear and the total harmonic distortion (THD) which is defined as

$$THD = \sqrt{\sum_{i=2,3...} \left(\frac{v(i)}{i}\right)^2} \quad (14)$$

where $v(i) = a_i / a_1$

is not vary with variable loads. The serial circuit in Fig. 5 with $r_{on}=0.4\,\Omega$, $r_1=0.01\,\Omega$, and $r_2=0.02\,\Omega$, $r_3=0.04\,\Omega$ have been simulated for different loads and analyzed by a Fourier method. The results are shown in Fig. 8.

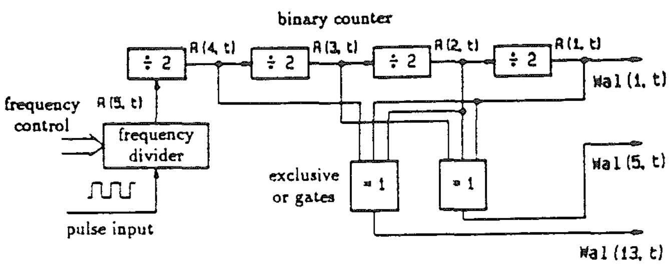
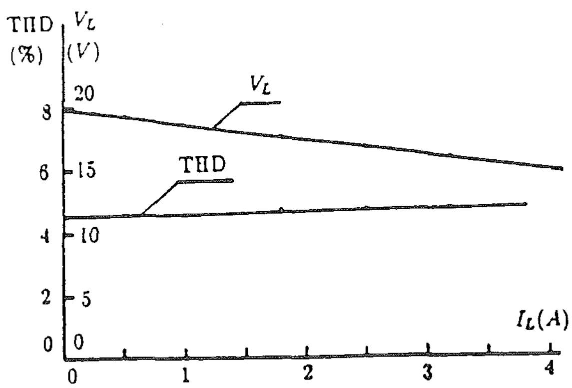



Fig. 7 The control circuit of the inverter




THD–the total harmonic distortion
$V_L$–the load valtage, $I_L$–the load current

Fig. 8 The load characteristic and harmonic distortion of the inverter

## Prototype of the Inverter and Experimental Results

A prototype of the inverter in Fig. 9 has been built. The main switching circuit consists of 12 MOS-FET. The Walsh generator is built by a counter and two integrated circuits of 'exclusive or' gates. A driver supplies sufficient power to control the switches and achieves potential isolation. By varying the pulse frequency of pulse generator, the frequency of the sinewave can be varied from 2 Hz to 5 KHz ( in case of the serial concept) and the waveshape of the output voltage remains a stepped sineshape as shown in Fig. 10.

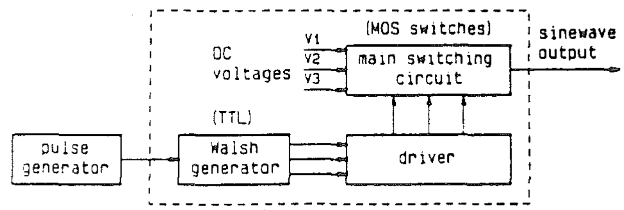



Fig. 9 The block graph of the prototype

## Conclusion

Some properties of the proposed method are:
- The synthesis method is based on a simple computation process
- The control circuit is simple and of low cost
- A set of linear algebraic equations may be used for analysis and elimination of harmonics
- Since the inverter has a constant internal resistance, it is applicable for variable loads

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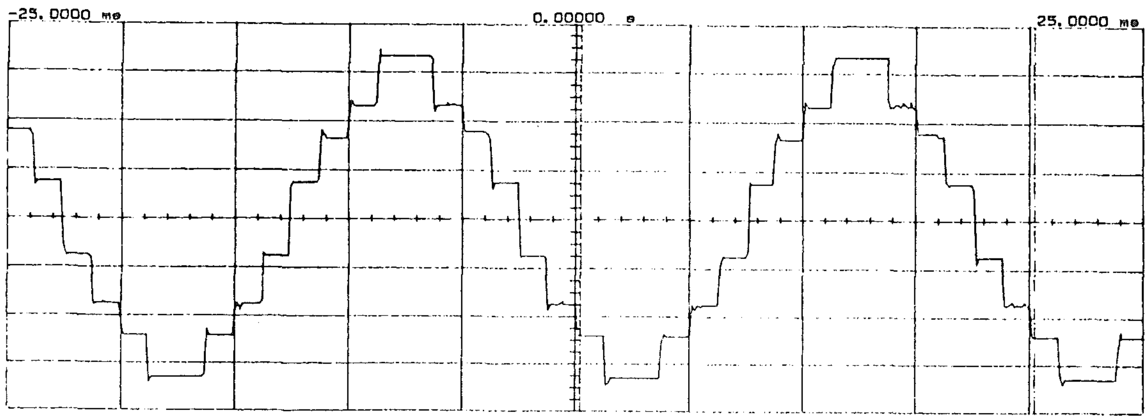


scales: 6V/div 5ms/div

Fig. 10 Waveshape of the output voltage

Table.1: The factors $S(k,n)$ for Walsh to Fourier conversion

| $k \backslash n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.27375 | -0.52760 | -0.10495 | -0.25336 | -0.02495 | 0.01034 | -0.05196 | -0.12545 |
| 3 | 0.42595 | 1.02834 | -0.68711 | 0.28461 | -0.08634 | -0.20843 | -0.31194 | 0.12921 |
| 5 | 0.25722 | 0.62099 | 0.92938 | -0.38496 | -0.20577 | -0.49676 | 0.33193 | -0.13749 |
| 7 | 0.18552 | -0.07685 | 0.38633 | 0.93267 | -0.76543 | 0.31705 | 0.06307 | 0.15225 |
| 9 | 0.14618 | -0.06055 | 0.30440 | 0.73490 | 0.89547 | -0.37092 | -0.07378 | -0.17812 |
| 11 | 0.12157 | 0.29350 | 0.43925 | -0.18194 | 0.34039 | 0.82178 | -0.54910 | 0.22744 |
| 13 | 0.10492 | 0.25330 | -0.16925 | 0.07010 | 0.23110 | 0.55793 | 0.83501 | -0.34587 |
| 15 | 0.09307 | -0.03855 | -0.00767 | -0.01851 | 0.18796 | -0.07785 | 0.39140 | 0.94493 |

Table.2: The results for eliminating of 5, 7–th harmonics

| | results by Eq.(11) | | results by simul. and Four. analysis | | | |
|---|---|---|---|---|---|---|
| | | | serial circuit | | paralell circuit | |
| | ampl. (V) | rel. ampl.(%) | ampl. (V) | rel. ampl.(%) | ampl. (V) | rel. ampl.(%) |
| $a_1$ | 20.00 | 100.0 | 20.03 | 100.0 | 20.97 | 100.0 |
| $a_3$ | 2.77 | 13.8 | 2.77 | 13.8 | 2.80 | 13.3 |
| $a_5$ | 0.00 | 0.0 | 0.00 | 0.0 | 0.00 | 0.0 |
| $a_7$ | 0.00 | 0.0 | 0.00 | 0.0 | 0.04 | 0.0 |
| $a_9$ | 0.00 | 0.0 | 0.00 | 0.0 | 0.06 | 0.0 |
| $a_{11}$ | 0.01 | 0.1 | 0.00 | 0.0 | 0.00 | 0.0 |
| $a_{13}$ | 0.68 | 3.4 | 0.64 | 3.2 | 0.65 | 3.1 |
| $a_{15}$ | 1.45 | 7.2 | 1.34 | 6.7 | 1.40 | 6.7 |