OPTIMAL NONLINEAR GUIDANCE FOR A REENTRY VEHICLE

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Abstract

Using the exact nonlinear equations of motion an optimal guidance law for a reentry vehicle to achieve at impact a zero miss and a predefined flight path angle is derived. The application of the optimal guidance law in feedback form is based on the on-line solution of a nonlinear algebraic equation. Numerical results are presented.

1. Introduction

Terminal guidance schemes for reentry vehicles are in general based on either the classical approach using proportional navigation(1) or on modern control theory. Kim and Grider(2) obtained an optimum guidance law based on linearizing the problem about a nominal trajectory, time was assumed fixed and a quadratic index was employed in order to minimize a weighted combination of final miss, final relative velocity direction with respect to the vertical and the integral of the squared vehicle lateral acceleration along the trajectory. The angle of attack of the reentry vehicle was neglected and the autopilot response was assumed to be either instantaneous, i.e., with no lag time attributed to the transfer of input commands to output reaction, or represented by a single time constant. The optimal guidance law was obtained in terms of time varying feedback gains. The final time was determined using an off-line approximation and the results showed great sensitivity to the final time value. In order to overcome this problem a suboptimal guidance law was derived based on proportional navigation plus an additional term proportional to the reentry angle error.

York and Pastrick(3) gave a formulation for a system that has a finite time delay and the guidance law time varying coefficients were approximated by piecewise linear functions. Still, performance turned out to be too sensitive to the approximation error. In this same work an attempt was made to analyze the problem including the angle of attack; however, no results were presented besides derivation of the system equations.

In Reference (4) an optimal guidance law in the plane was derived using the exact nonlinear equations of motion. The guidance law minimizes a weighted linear combination of the time of capture and the expended maneuvering energy. Final miss was imposed.

In the present work the reentry problem is considered employing the exact nonlinear equations of motion. Final miss and reentry angle at impact are imposed. An optimal guidance law with free final time is derived that minimizes the expended maneuvering energy. A closed form solution is obtained for the equations of motion in terms of elliptic integrals. This enables to obtain a nonlinear guidance law in feedback form. Since no approximations are required, this guidance law has no sensitivity problems of the kind appearing in previous results. A numerical example will be presented.

2. Problem Definition

A vehicle P flying at constant velocity V_P able to control its normal acceleratuion U is depicted in Figure 1. With coordinates centered at the constant velocity target T, and axis X along its velocity vector V_T, the equations of motion in terms of the nondimensional quantities, $x = X/R_0$, $z = Z/R_0$, $t' = tV_P/R_0$, $v = V_T/V_P$ and $u = UR_0^2/V_P$, where R_0 is the initial vehicle to target range, are

$$\dot{x} = \cos\gamma - v \tag{1}$$

$$\dot{z} = \sin\gamma \tag{2}$$

$$\dot{\gamma} = u \tag{3}$$

The problem to be solved is to find a control u such that the vehicle P reaches the target $(x(t_f)=z(t_f)=0)$, in a finite time t_f at a predefined impact angle $\gamma(t_f)=\gamma_f$, while the performance index J,

$$J = \frac{1}{2}\int_{t_0}^{t_f} u^2 dt \tag{4}$$

is minimized.

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3. The Optimal Control

The maximum principle will be employed to solve this problem[5]. Let us write the Hamiltonian

$$H = p_x(\cos\gamma - v) + p_z \sin\gamma + p_\gamma u - (1/2)u^2 \tag{5}$$

where p_x, p_z, p_γ the components of the adjoint vector are defined by,

$$\dot{p}_x = 0, \qquad p_{xf} \text{ free} \tag{6}$$

$$\dot{p}_z = 0, \qquad p_{zf} \text{ free} \tag{7}$$

$$\dot{p}_\gamma = p_x \sin\gamma - p_z \cos\gamma, \; p_{\gamma f} \text{ free} \tag{8}$$

The Hamiltonian is maximized for

$$u^* = p_\gamma \tag{9}$$

In order to obtain the optimal control the adjoint system (6)-(8) is to be solved along an optimal trajectory.

From (6) and (7) follows directly that,

$$p_x \equiv p_{xf} = p_s \cos\phi_s \tag{10}$$

$$p_z \equiv p_{zf} = p_s \sin\phi_s \tag{11}$$

where $p_s = (p_{xf}^2 + p_{zf}^2)^{1/2}$ and $\phi_s = \tan^{-1}(p_{zf}/p_{xf})$.

Since this is a free end time problem and the Hamiltonian is not an explicit function of time,

$$H(u^*) = 0 \tag{12}$$

Substituting Eqs. (9)-(11) into H as defined in Eq. (5), equating to zero and rearranging,

$$p_\gamma^2 = 2p_s[v\cos\phi_s - \cos(\gamma - \phi_s)] \tag{13}$$

This completes the integration of the adjoint equations.

The optimal control u^* is now obtained substituting p_γ from Eq. (13) into Eq. (9),

$$u^* = \pm\sqrt{2p_s[v\cos\phi_s - \cos(\gamma - \phi_s)]} \tag{14}$$

The control u^* is a function of the flight path angle and the unknown constants p_s, ϕ_s. These constants are to be determined as functions of the given boundary conditions of the problem. For this purpose the system differential equations (1)-(3) are integrated along the optimal trajectories.

Dividing Eqs. (1) and (2) by Eq. (3), with u^* as defined in Eq. (14),

$$\frac{dx}{d\gamma} = \pm \frac{\cos\gamma - v}{\sqrt{2p_s[v\cos\phi_s - \cos(\gamma - \phi_s)]}} \tag{15}$$

$$\frac{dz}{d\gamma} = \pm \frac{\sin\gamma}{\sqrt{2p_s[v\cos\phi_s - \cos(\gamma - \phi_s)]}} \tag{16}$$

Rearranging and integrating on both sides from the initial to the final conditions it is obtained:

$$-x_0 = \pm 1/\sqrt{2p_s} \int_{\gamma_0}^{\gamma_f} \frac{\cos\gamma - v}{\sqrt{v\cos\phi_s - \cos(\gamma - \phi_s)]}} \tag{17}$$

$$-z_0 = \pm 1/\sqrt{2p_s} \int_{\gamma_0}^{\gamma_f} \frac{\sin\gamma}{\sqrt{v\cos\phi_s - \cos(\gamma - \phi_s)]}} \tag{18}$$

These two equations (17) and (18) are to be solved for p_s and ϕ_s as functions of the given boundary conditions x_0, z_0, γ_0, γ_f.

Dividing Eq. (18) by Eq. (17) and rearranging

$$\int_{\gamma_0}^{\gamma_f} \frac{\sin(\gamma - \phi_0) - v}{\sqrt{v\cos\phi_s - \cos(\gamma - \phi_s)]}} d\gamma = 0 \tag{19}$$

where $\phi_0 = \tan^{-1}(z_0/x_0)$ is the angular direction of the initial radius vector PT.

Equation (19) is a nonlinear algebraic equation for ϕ_s. This equation can be numerically solved and p_s is then obtained from,

$$p_s = \frac{1}{2z_0^2} \left(\int_{\gamma_0}^{\gamma_f} \frac{\sin\gamma}{\sqrt{v\cos\phi_s - \cos(\gamma - \phi_s)]}} d\gamma \right)^2 \tag{20}$$

A feedback solution for the optimal control u^* is readily implemented introducting into Equations (19) and (20), at each time step, the present values of γ, x, z instead of the initial values γ_0, x_0, z_0.

4. Numerical Results

To illustrate the use of the nonlinear reentry guidance law, an application example was numerically solved.

The initial conditions and parameters of the application example are,

$$\gamma_0 = -30°,\ x_0 = 0.707,\ z_0 = 0.707,\ v = 1/50$$

and the required impact angle $\gamma_f = 90°$.

Fig. 2 shows the vehicle relative trajectory and Fig. 3 the vehicle acceleration as a function of time. As expected, the vehicle achieves impact at the required flight path angle with a zero miss.

5. Summary

A nonlinear optimal guidance law for a vehicle with a constrained attitude angle at impact is derived using the nonlinear equations of motion. Numerical results are presented of a representative case. Since the nonlinear guidance can be implemented in feedback form, further work will be directed to numerically analyze the behaviour of the system under different disturbances.

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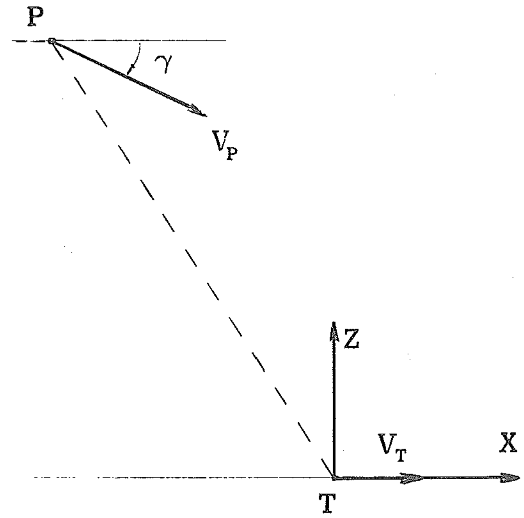

Fig. 1 : Reentry geometry

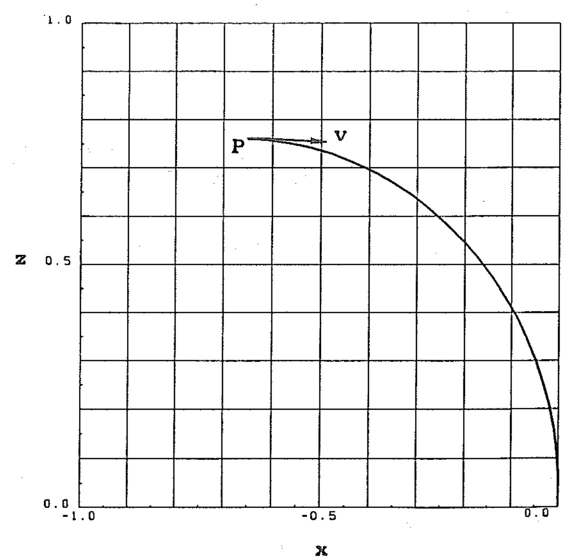

Fig. 2 : Reentry vehicle relative trajectory

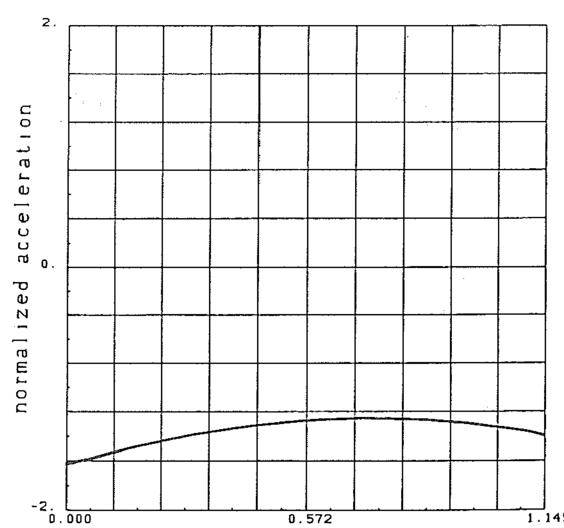

Fig. 3 : Normalized acceleration versus time