A NEW METHOD FOR CALCULATION OF HELICOPTER MANEUVERING FLIGHT*

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Abstract

In this paper a set of nonlinear equations governing the helicopter in maneuvering flight is developed and solved based on finite difference methods for evaluating the extreme value of function. The inverse solution technique is improved and from which a good result of the sample calculation of level turn maneuver is obtained. Beginning with a set of prescribed flight conditions, this method calculates the flight path, velocity and other kinematic parameters first, and then the control displacement, flight attitude and the load factor for a helicopter during the maneuvering are calculated.

Key words: helicopter, maneuvering, calculation.

Ⅰ. Introduction

With the continuous development of helicopter applications, the maneuverability targets have been placed on the design requirements for helicopters, especially, for armed helicopters. The main rotor is the key component of a helicopter. It decides the maximum lift capability which is the fundamental factor of maneuvering capability for a helicopter. Ref. 1 ~4 presented the theoretical or experimental results referred to the maneuverability of a helicopter.

Synthesizing the power available, the limits of control and attitude angle and so on, a set of equations governing the helicopter during maneuvering flight is established in this paper. Also a nonlinear iteration method is developed for the solution. Based on these theoretical formulae and calculation methods, almost any kind of maneuvering flights of helicopters can be calculated.

The conventional method is in this way to find the vehicle's response to the given control inputs and then the flight path can be found. On the contrary in this paper, an inverse solution is adopted. The solution process begins with a defined flight path, then the control deflections, the attitudes and the load factor required to fly the helicopter with this maneuvering will be obtained.

The first attempts of the inverse solution method are referred to fixed wing aircrafts[5,6,7]. Recently, T. Cerbe and G. Reichert of Braunschweig University in W. Germany used a similar method to study optimization of helicopter take off and landing performances[8], but they did not deal with the dynamics of the whole helicopter and cut out the rotor flap motion. At the same time, D. G. Thomson and R. Bradley investigated the inverse solution for a prescribed flight path based on a set of linearised equations of motions[9]. Their methods are valid only for the calculation of longitudinal maneuvers since it is confined to small-angle assumption in the linearized analysis.

Because of difficulties in mathematical representation and complexity of helicopter maneuvering flight, the inverse solution technique of Euler equations is still in the continuously developing stage. Flight dynamicists are making efforts to study the mathematical simulation and solution technique for helicopter maneuvering flight[10,11]. A new attempt is made in this paper. Firstly, the time derivatives of flap motion coefficients and induced velocity model derived from vortex theory are adopted, so that the governing equation of helicopter maneuvering flight is derived more properly; Second, based on the mathematical principle of finite-difference methods for evaluating the extreme value of function, the nonlinear inverse solution of general governing equations is obtained simultaneously; Finally, a sample calculation is given with level turn maneuver.

Ⅱ. Governing Equations

Dynamic equations

The conventional approach for flight dynamics calculation involves the solving process of Euler equation (1) for the six unknown state variables, V_X, V_Y, V_Z the translational velocities, and $\omega_X, \omega_Y, \omega_Z$ the angular velocities, in a body fixed frame (see Fig. 1),

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$$\left.\begin{aligned}
&F_x - mgsin\Theta_s - m(\dot{V}_x - \omega_z V_y + \omega_y V_z) = 0 \\
&F_y - mgcos\Theta_s cos\gamma_s - m(\dot{V}_y - \omega_x V_z + \omega_z V_x) = 0 \\
&F_z + mgcos\Theta_s sin\gamma_s - m(\dot{V}_z - \omega_y V_x + \omega_x V_y) = 0 \\
&M_x - I_x \frac{d\omega_x}{dt} - (I_z - I_y)\omega_y\omega_z - I_{yz}(\omega_z^2 - \omega_y^2) \\
&\quad - I_{xy}\omega_x\omega_z + I_{zx}\omega_x\omega_y + I_{xy}\frac{d\omega_y}{dt} + I_{zx}\frac{d\omega_z}{dt} = 0 \\
&M_y - I_y \frac{d\omega_y}{dt} - (I_x - I_z)\omega_z\omega_x - I_{zx}(\omega_x^2 - \omega_z^2) \\
&\quad - I_{yz}\omega_x\omega_y + I_{xy}\omega_y\omega_z + I_{yz}\frac{d\omega_z}{dt} + I_{xy}\frac{d\omega_x}{dt} = 0 \\
&M_z - I_z \frac{d\omega_z}{dt} - (I_y - I_x)\omega_x\omega_y - I_{xy}(\omega_y^2 - \omega_x^2) \\
&\quad - I_{zx}\omega_y\omega_z + I_{zy}\omega_x\omega_z + I_{zx}\frac{d\omega_x}{dt} + I_{zy}\frac{d\omega_y}{dt} = 0
\end{aligned}\right\} \quad (1)$$

Where the helicopter attitude angles (Θ_s, γ_s, ψ_s) are the Euler angles relating the body fixed frame to the earth fixed frame of reference.

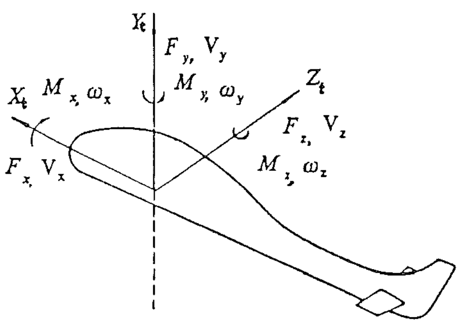

Fig.1 Body fixed axes system

For the flapping motion of blades, only the zero and first harmonics are concerned. The elastic deflection of the blade can be written as

$$Y_1 \approx \eta_1(r)q_1(\psi) = \eta_1(r)[a_0(t) - a_1(t)cos\psi - b_1(t)sin\psi]$$

Starting from the force on the blade section, taking the method of separation of variables and the property of orthogonality of the mode shape, the partial differential equation of the blade elastic axis can be changed into the following set of tip-path-plane equations:

$$\begin{bmatrix}\ddot{a}_0\\ \ddot{a}_1\\ \ddot{b}_1\end{bmatrix} + [A]_{3\times3}\begin{bmatrix}\dot{a}_0\\ \dot{a}_1\\ \dot{b}_1\end{bmatrix} + [B]_{3\times3}\begin{bmatrix}a_0\\ a_1\\ b_1\end{bmatrix} = [C]_{3\times1} \quad (2)$$

Kinematic equations

The relations of velocity and acceleration are presented in detail in Ref. 12. Seeing Fig. 2, the velocity components in the earth fixed frame are

$$\left.\begin{aligned}
\dot{X}_E &= Vcos\theta_p cos\psi_h \\
\dot{Y}_E &= Vsin\theta_p \\
\dot{Z}_E &= Vcos\theta_p sin(-\psi_h)
\end{aligned}\right\} \quad (3)$$

The accelerations can be found by differentiating equations (3).

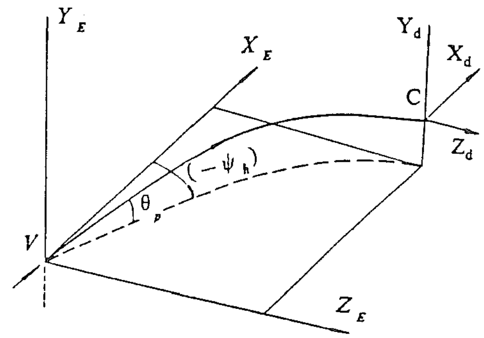

Fig.2 Description of the maneuvering flight path

According to the projection relations in the body fixed frame, the following relations of angular velocities can be derived

$$\left.\begin{aligned}
\dot{\gamma}_s &= \omega_x - tg\Theta_s(\omega_y cos\gamma_s - \omega_z sin\gamma_s) \\
\dot{\Theta}_s &= \omega_y sin\gamma_s + \omega_z cos\gamma_s \\
\dot{\psi}_s &= (1/cos\Theta_s)(\omega_y cos\gamma_s - \omega_z sin\gamma_s)
\end{aligned}\right\} \quad (4)$$

Angles equations

The preceding equations include eight angles: Θ_s, γ_s, ψ_s, α, β, θ_p, ψ_h, γ_h, but only five of them are independent variables. They are related with each other by the following relations which can be derived through mutual transformation among the earth fixed frame, body fixed frame and wind reference axes

$$\left.\begin{aligned}
sin\beta &= [sin\gamma_s sin\Theta_s cos(\psi_s - \psi_h) + cos\gamma_s \\
&\qquad sin(\psi_s - \psi_h)]cos\theta_p - sin\gamma_s cos\Theta_s sin\theta_p \\
sin\alpha &= \{[cos\gamma_s sin\Theta_s cos(\psi_s - \psi_h) - sin\gamma_s \\
&\qquad sin(\psi_s - \psi_h)]cos\theta_p - cos\gamma_s cos\Theta_s sin\theta_p\}/cos\beta \\
sin\gamma_h &= (sin\Theta_s cos\alpha sin\beta - cos\gamma_s cos\Theta_s sin\alpha sin\beta \\
&\qquad + sin\gamma_s cos\Theta_s cos\beta)/cos\theta_p
\end{aligned}\right\} \quad (5)$$

Complementary equations

Using the transformation relations between body axes and wind axes, the following complementary equations for the angular velocities can be derived.

$$\begin{bmatrix}
\omega_x cos\alpha cos\beta - \omega_y sin\alpha sin\beta + (\omega_z - \dot{\alpha})sin\beta \\
\omega_x sin\alpha + \omega_y cos\alpha - \dot{\beta} \\
-\omega_x cos\alpha sin\beta + \omega_y sin\alpha sin\beta + (\omega_z - \dot{\alpha})cos\beta
\end{bmatrix}
= \begin{bmatrix}
\dot{\psi}_h sin\theta_p + \dot{\gamma}_h \\
\dot{\psi}_h cos\theta_p cos\gamma_h + \dot{\theta}_p sin\gamma_h \\
-\dot{\psi}_h cos\theta_p sin\gamma_h + \dot{\theta}_p cos\gamma_h
\end{bmatrix} \quad (6)$$

The equations (1) through (6) are called the equations governing the helicopter during any maneuvering flight.

The Euler equations are the dominative ones. The governing equations can be simplified according to the defined conditions for a specific maneuver. For example, the calculation of level turn maneuver in this paper is carried out by setting$\theta_p = \beta = 0$

Ⅲ. The Solving Process

Taking the level coordinated turn as a sample calculation, the inverse solution technique can be described with the following two stages.

The first stage: calculation of kinematic parameters

The calculation of kinematic parameters is to make provision for solving the history of the control deflections and flight attitudes. According to the predefined parameters, such as equivalent radius Re, transient proportional factor K_m, velocity V and total turning angle ψ_{he}(see Fig. 3) and using the methods presented in Ref. 12, the variation of flight path, turn rate and load factor with time during this maneuver can be calculated.

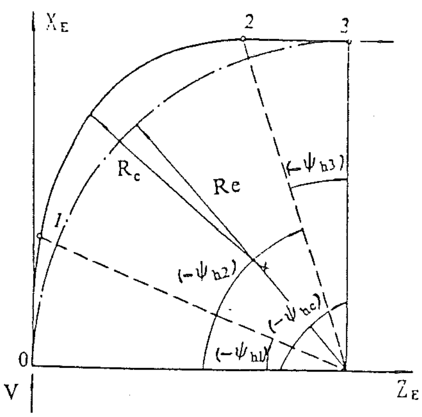

Fig. 3 The flight path of level turn

The second stage: the inverse solution of the governing equations

Euler equations(1) may decide only six unknown variables. In this paper four control variables $\theta_0, \theta_c, \theta_s, \theta_{tr}$ and two attitude angles Θ_s, γ_s are chosen as a set of key variables and denoted by $x = (\theta_0, \theta_c, \theta_s, \theta_{tr}, \Theta_s, \gamma_s)^T$. Then all kinematic variables, componental forces and moments are expressed as the function of x. In this step the equations (1) are not simplified by any linearization and so the governing equations are suitable for any maneuvering flight. However, equations (1) will be complicated nonlinear equations for the complexity of helicopters themselves. Setting $\beta = \theta_p = 0$ in the governing euqations, the following basic kinematic relations can be obtained from equations (3) through (6).

$$\left.\begin{aligned} \psi_s &= \psi_h + tg^{-1}(-tg\gamma_s sin\Theta_s) \\ \alpha &= sin^{-1}[cos\gamma_s sin\Theta_s cos(\psi_s - \psi_h) \\ &\quad - sin\gamma_s(sin\psi_s - \psi_h)] \\ \gamma_h &= sin^{-1}(sin\gamma_s cos\Theta_s) \end{aligned}\right\} \quad (7)$$

$$\dot{\psi}_s = \dot{\psi}_h + \frac{(-1)}{1 + (-tg\gamma_s sin\Theta_s)^2}(sec^2\gamma_s\dot{\gamma}_s sin\Theta_s + tg\gamma_s cos\Theta_s\dot{\Theta}_s) \quad (8)$$

where $\dot{\Theta}_s$, and $\dot{\gamma}_s$ at time t_j can be calculated by using the backward difference method, i. e.

$$\dot{\Theta}_{s,j} = \frac{\Theta_{s,j} - \Theta_{s,j-1}}{t_j - t_{j-1}}, \dot{\gamma}_{s,j} = \frac{\gamma_{s,j} - \gamma_{s,j-1}}{t_j - t_{j-1}} \quad (9)$$

Also, $\ddot{\alpha}_j$ and $\ddot{\gamma}_{h,j}$ can be treated similarly.

Then the following parameters should be calculated progressively

$$\left.\begin{aligned} \omega_x &= \dot{\gamma}_s + sin\Theta_s\dot{\psi}_s \\ \omega_y &= cos\Theta_s cos\gamma_s\dot{\psi}_s + sin\gamma_s\dot{\Theta}_s \\ \omega_z &= -cos\Theta_s sin\gamma_s\dot{\psi}_s + cos\gamma_s\dot{\Theta}_s \end{aligned}\right\} \quad (10)$$

$$\dot{\alpha} = \omega_z + \dot{\psi}_h sin\gamma_h \quad (11)$$

$$\dot{\gamma}_h = (\omega_x - \dot{\psi}_h cos\gamma_h sin\alpha)/cos\alpha \quad (12)$$

$$\left.\begin{aligned} \dot{\omega}_x &= (\ddot{\gamma}_h cos\alpha + \dot{\gamma}_h(-sin\alpha)\dot{\alpha}) + \ddot{\psi}_h cos\gamma_h sin\alpha \\ &\quad + \dot{\psi}(-sin\gamma_h\dot{\gamma}_h sin\alpha + cos\gamma_h cos\alpha\dot{\alpha}) \\ \dot{\omega}_y &= (-\ddot{\gamma}_h sin\alpha - \dot{\gamma}_h cos\alpha\dot{\alpha}) + \ddot{\psi}_h cos\gamma_h cos\alpha \\ &\quad + \dot{\psi}_h(-sin\gamma_h\dot{\gamma}_h cos\alpha + cos\gamma_h sin\alpha\dot{\alpha}) \\ \dot{\omega}_z &= \ddot{\alpha} - \ddot{\psi}_h sin\gamma_h - \dot{\psi}_h cos\gamma_h\dot{\gamma}_h \end{aligned}\right\} \quad (13)$$

Finally, the calculation of rotor flap motion and componental forces and moments of the rotor and other parts could be done. The solution of equation (1) begins with substitution of all componental forces and moments as well as all kinematic variables into (1). Generally speaking, the numerical solution methods for such a set of nonlinear euqations have not been ripe. The Levenberg-Marguardt finite-difference method for solving problem of the least squares is adopted in this paper. The left side of Euler euqations is defined as function $f_i(i=1, \cdots\cdots, 6)$. The summation of square of function f_i is expressed as $SSQ = f_1^2(x) + \cdots\cdots + f_6^2(x)$. The simulation of valid solution of Euler equations requires the minimization of SSQ.

The calculation proceeds according to recurrence steps. The calculation begins with level flight ($t_0 = 0$). Then the every next transient state is calculated at each step of time Δt. For the transient state at t_j, the initial value takes the final value of t_{j-1} transient state and the iteration calculation proceeds repeatedly until the result satisfies convergence criteria of the governing equations.

Ⅳ. Analysis of a Sample Calculation

The prescribed conditions

Helicopter Bo-105 performs a 90° left turn maneuver at constant velocity $V = 157km/h(\mu = 0.2)$, with the effective radius Re=250m and the transient proportional factor $K_m=0.1$

The kinematic parameters

From the defined conditions, $\psi_{he}=\frac{\pi}{2}$, the track angle in the entry part $\psi_{h1}=K_m\psi_{he}$, and the track angle in the withdrawing part $\psi_{h3}=K_m\psi_{he}$ (see Fig. 3), then the following results can be obtained as

$t_1=1.5s$, $t_2=7.7s$, $t_3=9.2s$

i.e. the transient turn in either entry part or withdrawing part takes 1.5 seconds and the steady turn part takes 6.2seconds.

The turn radius in steady turn section is $R_c=214.67m$ and the turn rate(r/s) in three sections are respectively

section 0—1, $\dot{\psi}_{h1}=-0.109588t^3+0.254366t^2$

$0\leqslant t<1.5$

section 1—2, $\dot{\psi}_{h2}=V/Rc=0.203023$ $1.5\leqslant t<7.7$

section 2—3, $\dot{\psi}_{h3}=0.109588t^3-2.798027t^2$

$+23.61649t-65.779358$ $7.7\leqslant t<9.2$

The load factor during this maneuver is

$$n_T(t) = \sqrt{1+(\dot{\psi}_h V/g)^2}$$

The variation of the load factor with time is shown in Fig. 4.

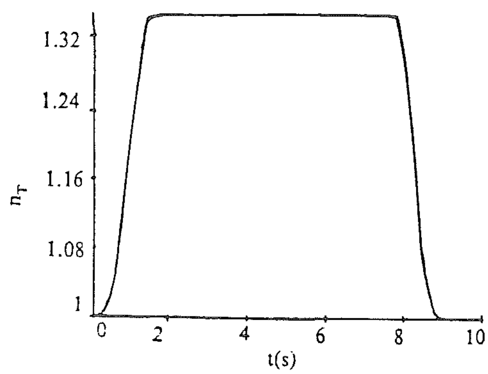

Fig. 4 The variation of load factor

The maximum value is $n_T=1.348$, occuring in the steady turn section

The control deflections and attitude angles

The time history of control deflections is calculated, among which the lateral cyclic pitch and tail rotor collective pitch are shown in Fig. 5 and Fig. 6. The variation of pitch angle, bank angle and yaw angle of the helicopter with time are shown in Fig. 7 through Fig. 9. The thrust vector of the main rotor has to tilt for making the helicopter into turn maneuver and so the main rotor collective must increase to keep the helicopter flying at a constant alititude in the entry phase. In steady turning the main rotor collective keeps constant. Then the collective decreases in withdrawing phase and recovers gradually to the value of level flight at last. The rolling motion during turning is controlled by the lateral cyclic control (see Fig. 5). In the entry phase, the pilot deflects the stick leftward to accelerate the helicopter rolling and then pulls the stick back somewhat until the proper bank angle of the helicopter is established. In the withdrawing phase, the contrary deflection in lateral control is performed. The corresponding bank angle of the helicopter is shown in Fig. 8. When the helicopter enters left turning, the gyroscopic moment of the main rotor due to helicopter rolling leftward causes the helicopter nose up(see Fig. 7). To keep the helicopter flying at a constant speed, the stick must be pushed forward to eliminate the nose-up motion. In the steady turning phase, no rolling exists and the longitudinal cyclic pitch keeps constant. In withdrawing phase, the conditions are inverse.

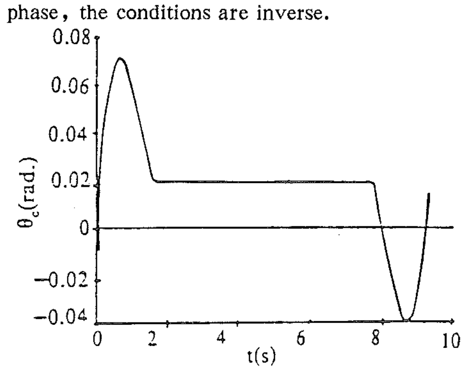

Fig.5 The time history of lateral cyclic pitch

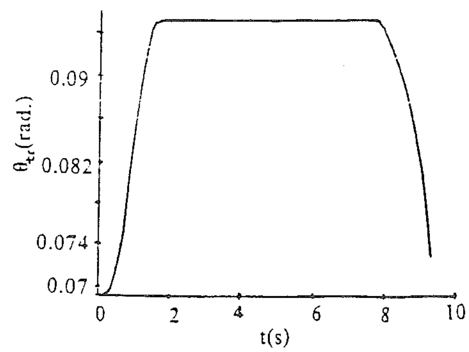

Fig.6 The time history of collective control of tail rotor

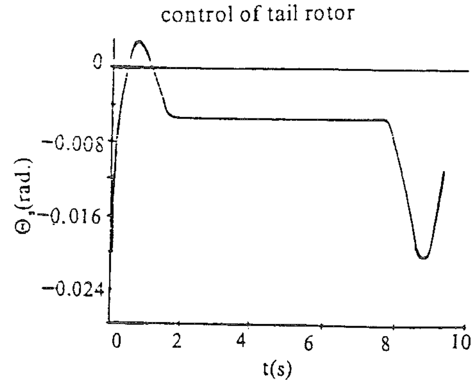

Fig. 7 The variation of pitch angle of the helicopter

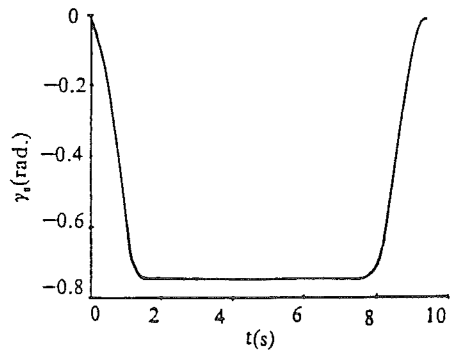

Fig.8 The variation of bank angle of the helicopter

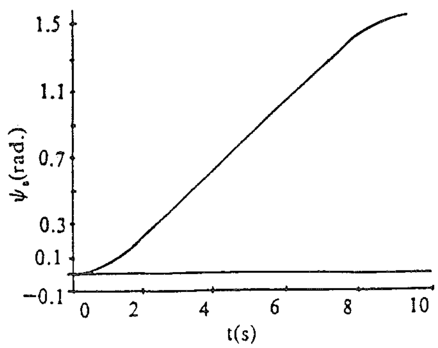

Fig. 9 The variation of yaw angle of the helicopter

The thrust of the tail rotor must be varied correspondently to keep the helicopter in coordinated turn (zero sideslip). The time history of the tail rotor collective is shown in Fig. 6. According to such a control process, the 90° left turn is completed as shown in Fig. 9.

The variation of the angle of attack is similar to the pitch angle of the helicopter since the helicoper turns keeping a constant altitude. The time histories of angular velocities and accelerations are also calculated. As an example, the yawing angular accelaration is shown in Fig. 11.

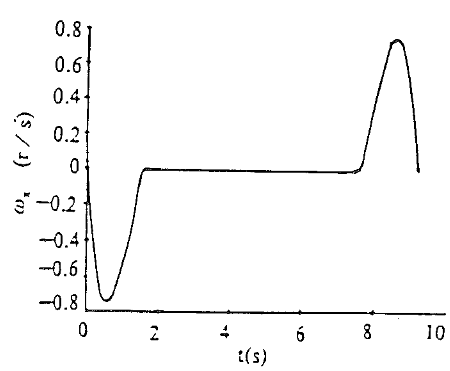

Fig.10 The time history of rolling rate

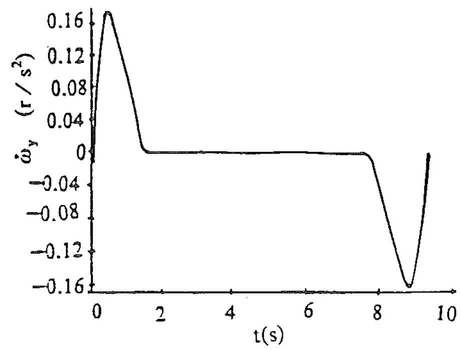

Fig.11 The time history of yawing angular acceleration

All the results are qualitatively reasonable, but until now no datum from flight tests can be used to verify the calculation. Subtracting the control values obtained here from the total travel of the control, much surplus still exists. Therefore, this turn maneuver is a rather slow maneuver. The accuracy of the calculating results themselves can be judged by the max $|fi|$ or SSQ value. In this example, the value of max $|fi|$ is within the order of 0.1×10^{-4} and the SSQ, 0.1×10^{-8}. More than four effective digits are the same for two successive values of X^n and X^{n+1} in iteration.

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