# THE DYNAMIC MATHEMATICAL MODEL AND DIGITAL SIMULATION OF THE ENVIRONMENTAL CONTROL SYSTEM

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Abstract

A dynamic mathematical model has been developed for a typical air-cycle environmental control system (ECS). The model has taken into account for the effects of fluid compressibility, fluid momentum, energy storage and rotor inertia on the transient performance of ECS. The overall performance maps of turbine, compressor and fan are needed to formulate their models. A intervariable is introduced to transform the compressor's map. Their dynamic models, including ducts, are ordinary differential equations. The annular heat exchanger, due to the complexity of its structure and heat transfer in the core, more detailed analyses are presented. The dynamic models of heat exchangers are partial differential equations. All the models can be used to predict the dynamic performance of components under any aircraft missions or operating conditions. The methods of state-space and finite-difference are used to solve respective models, which are efficient under small and large perturbations. It is also possible to include such problems as change of state, variable thermodynamic properties, non-linear e ffects and pressure surges or flow oscillations. As an example, the paper analyzes the ECS and its main components. It has shown that there is a good agreement between the theoretical results and the flight test data.

## I. Introduction

With the development of the technology of aviation, modern aircraft requires more reliable, safe and comfortable environment for both passengers, aircrew and avionics, which urges the improvement and better designs of Environmental Control System (ECS) regardless of at the cost of more complex structure and the control sub-system. To improve or design a new system, the dynamic analysis is essential. As we know, When aircraft flies at high speed in the sky, its flight parameters such as flight height, Mach number as well as the operating conditions of the engine, are on continuously changing in wide ranges. These cases are even severe in fighter or attacker, for example, during maximum ascent and decent. The changes of some parameters are approximate to step changes. These parameters, being the boundary conditions of ECS, have a great influence upon ECS, and consequently ECS, is also on dynamic changing processes.

The great changes in the system dynamics over the complete working range of ECS lower the reliability of ECS. It is necessary that the system performances not only the steady but also the dynamic are thoroughly analyzed early in the system development in order to identify potential problems and correct them before expensive engineering changes. It proves very difficult to perform these analyses of the complex system by the conventional methods. With the advanced computer, digital simulation becomes possible. Using high-fidelity dynamic mathematical models, digital simulation provides a means of analyzing the behavior and interactions of these increasingly complex system prior to full-scale testing of the hardware, furthermore, it serves as an aid in solving problems that arise during the ground and flight test, and after the development phase.

The early research on the dynamics of ECS can date back to 1950's. At that time, the majority work was concentrated on the control of cabin temperature and pressure, and design of the controller. [1] Afterwards, many papers appeared on the dynamic performance of ECS components such as heat exchangers, [2], [3] but the papers relating to the research on the dynamics of overall system are rare in the literature. Until not very long ago, American Government has developed the Environmental Control Analysis System Program (EASY) with several versions. [4]

The dynamic mathematical model of a typical air cycle ECS is presented in this paper, which has taken into account the transient variations of temperature, pressure and air flow. In the end of the paper, the simulation results of the simple cycle system are given, however, the models can be applicable to other more complex system such as the high pressure water separation system (HPWS). The reason to choose simple cycle system is that some experiment data is available.

## II. Mathematical Model

Dynamic mathematical modeling has a very important role in system simulation and design, which requires the representation of system phenomena as a functional dependence between interacting input and output variables. The system to be modeled is a typical air cycle ECS which usually incorporates heat exchangers, connecting ducts, an air cycle machine (ACM) and other components. The model formulation is based on the basic equations of continuity, momentum, energy, heat transfer and state relationships. The complexity and expression of the models depend on not only the purpose but also the physical phenomena and difficulties of model formulation.

### Duct

The overall ECS models must include representations of the air flow in duct, i.e., the dynamic model of duct must account for the effects of fluid compressibility, fluid momentum, energy storage. A complete analytical model would be distributed, complex, and then difficult to solve. Of course, it is not practicable and effective. In practice, one of the extremes for modeling the duct flow process is that the duct is lumped to only one element. The results are simple and insensitive to the details of the duct. Also, they could be used as the dynamic model of volumes. The state variables of the duct are represented by the average parameters $P_D$, $T_D$ and $W_D$.

The following assumptions were used in the equation development of ducts:

1. Due to the smaller Mach number of the air in the duct, the dynamic behavior of the duct is based on the steady state behavior.
2. The duct is assumed to be a lumped system.
3. During dynamic process, the heat transfer between the air and the wall is $hA(T_w-T_D)$.
4. All pressure losses are represented by total pressure losses.
5. The outside of the duct is thermal isolated.

Mass balance:

$$\frac{dM}{dt} = W_1 - W_2 \qquad (1)$$

Energy balance:

$$(MCp)_w \frac{dT_w}{dt} = (hA)\ (T_D - T_w) \qquad (2)$$

Energy balance of the air:

$$\frac{dE}{dt} = (WCpT)_1 - (WCpT)_2 - (hA)\ (T_D - T_w) \qquad (3)$$

where E=Mu

Average pressure:

$$P_D = (P_1 + P_2 * Pk) / (1 + Pk) \qquad (4)$$

Pressure loss:

$$P_2 = P_1 - KpW_D^2T_D/P_1 \qquad (5)$$

where Kp is pressure coefficient determined from the experimental data.

Average temperature:

$$T_D = (T_1 + T_2 * Tk) / (1 + Tk) \qquad (6)$$

Average flow rate:

$$W_D = (W_1 + W_2 * Wk) / (1 + Wk) \qquad (7)$$

where,
coefficient Pk, Tk, and Wk are introduced to reduce the effect caused by the replacement of distributed -parameter system by lumped-parameter system.

### Heat Exchangers

Heat exchangers are most important components in ECS, therefore, the proper modeling of the heat transfer process is of prime importance to accurately describe the steady state and dynamic response of the system. There are many types of heat exchangers used in ECS, such as parallel, counter, cross-flow heat exchangers and annular heat exchangers. A huge amount of work can be found relating the former three types of heat exchangers. However for the last type, due to the complexity of its structure and heat transfer in the core, the emphasis is given in the model formulation. [5] The empirical models are effective in use, but lacking of experimental data, the physical approach is applied to heat exchanger modeling, which can reflect the dynamic behavior of exchangers in wide operating conditions at the cost of more execution time.

#### Parallel, Counter and Cross-flow Heat Exchangers.

Parallel, counter and cross-flow heat exchangers to be analyzed are described by the following hypotheses:

1. The physical properties and the fluid capacity rates are uniform at any instant.
2. No heat is conducted in the axial direction in the exchanger core or in the two fluids. The core wall offers no thermal resistance to flow of heat from one fluid to the other.
3. The exchanger shell or shroud is adiabatic and influences neither the steady-state nor the transient behavior of the fluid temperature.
4. Neither flow is mixed.
5. The thermal capacities of the masses of the two fluids contained (at any instant) in the exchanger are negligibly small relative to the thermal capacity MC of the exchanger core.

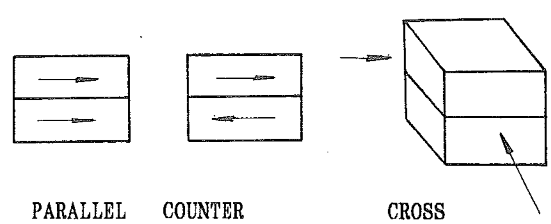


Figure 1. Parallel, Counter and Cross Flow HEX

Three partial differential equations are required to define the behavior of the heat exchangers. An energy balance on an element length of the exchanger core gives the first equation:

$$(MC)_w \frac{\partial T_w}{\partial t} = (hA)_a (T_a - T_w) + (hA)_b (T_b - T_w) \qquad (8)$$

The remainning two governing equations are readily obtained from energy conservation for each fluid.

$$(LWC)_a \frac{\partial Ta}{\partial X} = (hA)_a (T_w - T_a) \qquad (9)$$

$$\pm\ (LWC)_b \frac{\partial Ta}{\partial X} = (hA)_b (T_w - T_b) \qquad (10)$$

$$(LWC)_b \frac{\partial T_b}{\partial Y} = (hA)_b (T_w - T_b) \qquad (11)$$

In equation (10), the upper part of the double sign is for the case of parallel flow and lower part for the case of counterflow, while equation (11) is for the case of crossflow.

Annular Heat Exchanger. As illustrated diagrammatically in Figure 2, the annular heat exchanger

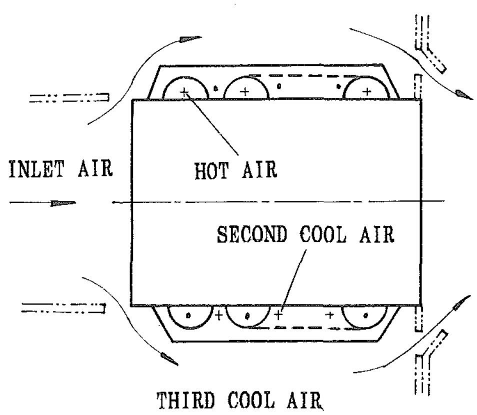


Figure 2. Annular Heat Exchanger Schematic

is installed in the inlet section of aircraft engine and its cool side is part of the inlet. Because of the special type of the structure and the place of installation, it greatly reduces the aircraft penalty and efficiently utilize the aircraft space. With distributing uniformly in each of the wavy channels, the hot bleed air flows in circular direction, which is cooled by three cool air flows. The inlet air of the engine plays a most important part in heat transfer, the second cool air flows in counter direction along the hot air, while the third cool air flows in cross direction to the second cool airflow.

The following idealizations are basic to the analysis besides mentioned above:

1. Heat is conducted in exchanger core material only in two directions, one is mutually perpendicular to the two fluid flow directions and the other is along the axial direction of the annular heat exchanger.
2. According to the experimental data, the influence of the third cool air in heat transfer is ignored in steady state. This idealization is also applicable in dynamic state.
3. Because the hot air flow rate is negligibly small as compared with that of inlet air, the temperature of the inlet air is not changed due to the heat transfer with hot air.
4. Each wavy channel has same temperature field.

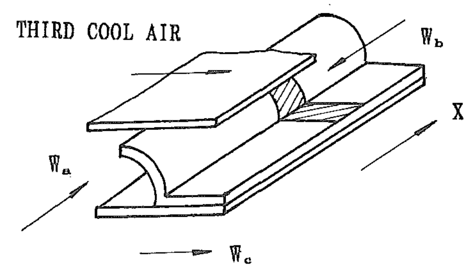


Figure 3. Stereo Stretch of One Wavy Channel

The stereo stretch of one wavy channel is shown in Figure 3. X direction in the Figure 3. is the hot air flow direction along the wavy channel. Based on above idealization, Considering an element of Figure 3 (shadowed), to obtain the temperature distribution by the analytical method is not easy work. In this paper, another method is used, i.e. lumping the element by a thermal resistance and thermal capacitance net work. Although this method is approximate, as long as the number and the location of nodes are properly chosen, the models can meet any accuracy requirements. An electrical analogy of the thermal network is shown in Figure 4 (The convective thermal resistances are not shown in Figure 4 for clearance) In Figure 4, twelve nodes are used in this paper to describe the dynamic behavior of the element, however because of the symmetry, only 7 nodes are used. In spite of the discretion in axial direction, the distributed characteristics of temperature in circular direction is still remained.

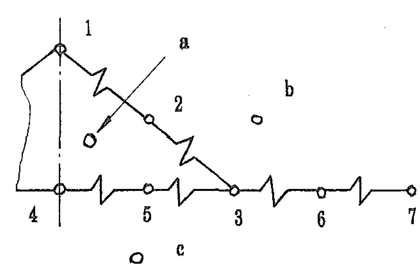


Figure 4. Thermal Network Electrical Analogy

Energy balance on the air nodes:

$$(LWC)_a \frac{\partial T_a}{\partial X} = \sum_j (T_j - T_a) / R_{ja} \qquad (12)$$

$$-(LWC)_b \frac{\partial T_b}{\partial X} = \sum_j (T_j - T_b)/R_{jb} \tag{13}$$

where:
$R_{ja}$, $R_{jb}$ are convective thermal resistance between air node a or b and adjacent core node j.

$$(MC)_{iw} \frac{\partial T_i}{\partial t} = \sum_j (T_j - T_i)/R_{ji} \tag{14}$$

where:
$R_{ji}$ is thermal resistance between core node i and adjacent node j, and i=1, 2, · · ·, 7.

Air Cycling Machine (ACM)

An air cycling machine composes of turbine, compressor or (and) fan. The different structure ACM is used in different air-cycle ECS. Taking a typical ACM used in ECS, the length of air passage is short and the volume is small, so the flow dynamics is of very high frequency. Since the rotor of the ACM rotates in extremely high speed. The kinetic energy is considerable. Therefore the most significant factor in determining the transient behavior of ACM is rotor inertia rather than fluid momentum.

The following hypotheses are needed during analysis:

1. The air velocity in the ACM is so high that the air flow is adiabatic.
2. The thermal capacities of the metal parts of the ACM and the air contained in the passage are negligible.
3. The contribution of flow dynamics in the ACM is assumed to be primarily high frequency in nature and is consequently ignored.

Turbine. The direct approach to modeling turbine is to apply overall steady state performance maps. Usually these maps relate the turbine speed factor, the flow factor, the turbine isentropic efficiency, and the turbine pressure ratio. To facilitate the digital simulation by the computer, it had better rearrange the maps to digital tables according to following formulae:

$$W_T = W_T(n_T, \pi_T) \tag{15}$$

$$\eta_T = \eta_T(n_T, \pi_T) \tag{16}$$

where,

$$W_T = W\sqrt{T_{T1}}/P_{T1} \tag{17}$$

$$n_T = n/\sqrt{T_{T1}}; \quad \pi_T = P_{T1}/P_{T2} \tag{18}$$

Angular momentum:

$$2\pi J_T \frac{dn}{dt} = Q_T - Q_R \tag{19}$$

where $Q_R$ is resistant torque acting on the shaft.

Power released by the air expansion:

$$N_T = WCpT_{T1}(1-\pi^{-\frac{K}{T}})\eta_T \tag{20}$$

where $K = (\gamma-1)/\gamma$

Torque equation;

$$Q_T = N_T/2\pi n_T \tag{21}$$

The discharge temperature of turbine:

$$T_{T2} = T_{T1}(1-(1-\pi^{-K})\eta_T) \tag{22}$$

Compressor. As in the case of turbine, the overall performance map is also used to describe the compressor. Sample map is shown in Figure 5.

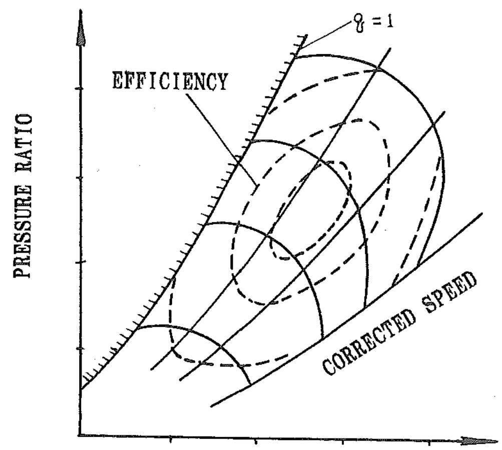


Figure 5. Sample Map

It is very difficult to accurately determine compressor's characteristics by such map, for there exists a surge line. Some transformation must be made about the map. A intervariable q is introduced. In other words, add q lines in the map, the line q=1 is just surge line, then gradually moves the line right downwards until meets the maximum operating boundary of compressor as shown in Figure 5. The transformed maps are shown in Figure 6. Based on Figure 6, rearrange the maps into digital tables according to following formulae:

$$W_C = W_C(n_C, q) \tag{23}$$

$$\pi_C = \pi_C(n_C, q) \tag{24}$$

$$\eta_C = \eta_C(n_C, q) \tag{25}$$

where,

$$W_C = W\sqrt{T_{C1}}/P_{C1} \tag{26}$$

$$n_C = n/\sqrt{T_{C1}}; \quad \pi_C = P_{C1}/P_{C2} \tag{27}$$

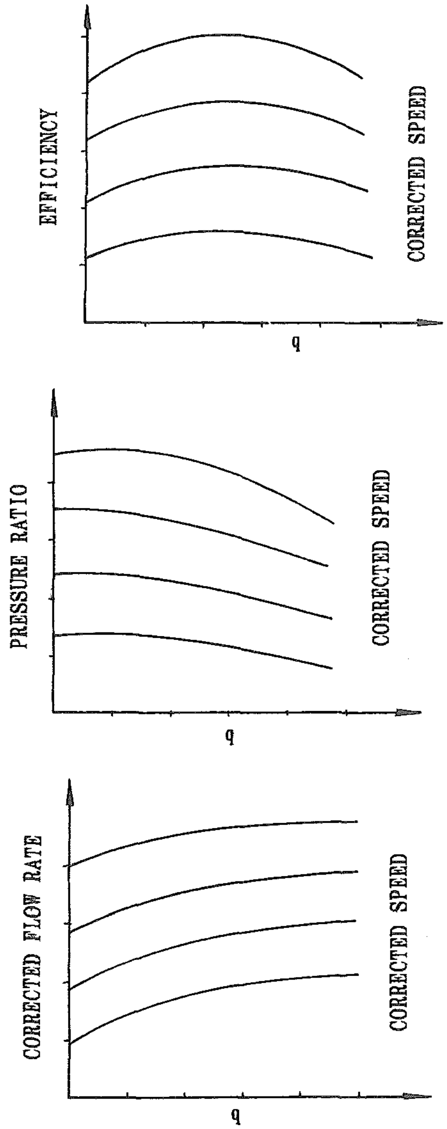


Figure 6. Transformed Maps

Angular momentum:

$$2\pi J_C \frac{dn}{dt} = Q_{cd} - Q_C \tag{28}$$

where $Q_{cd}$ is the driving torque from the turbine.

Power:

$$N_C = WCpT_{C1}(\pi^{K}-1)/\eta_C \tag{29}$$

Torque equation:

$$Q_C = N_C/2\pi n \tag{30}$$

Discharge temperature:

$$T_{C2} = T_{C1}(1-(\pi^{K}-1)/\eta C) \tag{31}$$

<u>Fan.</u> Fans discussed in this paper, can be classified into two types (according to their usage) - blowing cooling air or merely absorbing the turbine power. The best way to predict the steady-state performance is by their performance maps, but because the maps are not available, the approximate method- the principle of similitude, is used, which is approved efficiently in practical use.

Angular momentum:

$$2\pi J_F \frac{dn}{dt} = Q_{fd} - Q_F \tag{32}$$

where $Q_{fd}$ is the driving torque from the turbine.

when the conditions under the design case are known, the off-design conditions can be predicted by the principle of similitude.

$$\frac{N_{ds}}{N_F} = \frac{n_{ds}^3 \rho_{ds}}{n^3 \rho} \tag{33}$$

where $\rho$ is ambient air density of fan

Torque:

$$Q_F = N_F/2\pi n \tag{34}$$

<u>Link of Rotors.</u> Turbine, compressor and (or) fan are fixed in the same shaft, so their rotational speeds are the same. The work released by the expansion of high pressure and temperature is extracted by compressor and (or) fan. To determine the rotational speed of ACM, rotors must be linked together. Corresponding to different structure ACM, combine respective angular momentum equations and ignore frictional torque. Take an example of the ACM used in simple cycle, then the resistant torque equals the driving torque of the fan, therefore:

$$2\pi (J_T+J_F)\frac{dn}{dt} = Q_T - Q_F \tag{35}$$

<u>Valves</u>

Valves are primarily used in ECS to regulate airflow, temperature and pressure. Sometimes, the flow compressibility must be taken into consideration, for the pressure ratio across valves is larger than critical pressure ratio.

When the flow is sub-critical, then:

$$W = cFP/\sqrt{T}\left((\pi^{2/\gamma} - \pi^{(\gamma+1)/\gamma})\right)^{1/2} \tag{36}$$

where c is flow rate coefficient depending on the experimental data.

When the flow is super-critical, then:

$$W = cFP/\sqrt{T} \tag{37}$$

Orifice is used in ECS to limit maximum flow rate. The form of its formular is the same as that of valve, the difference is that the area of orifice can not be adjusted.

Pressure State

Because flow dynamics are typically high frequency and significantly increase the execution time of the digital simulation. It is not necessary to analyze all the pressure states of each components. Instead, the flow compressibility effects are lumped at isolated pressure nodes, and flows are generated at orifice-type devices such as turbine and valve. [6]

$$\frac{dP}{dt} = \frac{RT}{V}\sum W \tag{38}$$

If some components need special consideration of flow compressibility, for example, large volume, we can add more pressure nodes.

## Ⅲ. Solution of Models

As mentioned above, there are two types models-partial differential equations for heat exchangers and ordinary differential equations for duct, ACM and pressure nodes. The former equations are solved by finite-difference method, while the latter equations, after linearized, are solved by state-space technique. The reason is that the pressure states have higher frequencies than thermal states, That is to say, the method of solution to pressure states must in accordance with thermal states in computing time step so as to minimize the CPU time.

The FORTRAN program has been developed to analyze either steady or dynamic behavior of ECS. The dynamic mathematical models are implemented in FORTRAN subroutines for different system components. Other components could be included in subroutines. The equations solved by state-space method are linearized based on non-steady state points rather than steady state points, and the matrices of state-space equations are changing with each computing step. Therefore, the results are the same as obtained by original non-linear equations solved by Gear or other stiff methods.

The schematic of the simple system to be simulated is shown in Figure 7. The high temperature and pressure bleed air from aircraft engine compressor stage is precooled in the annular heat exchanger, the cool side of which is part of the engine inlet. The precooled air enters the cross-counter flow two-

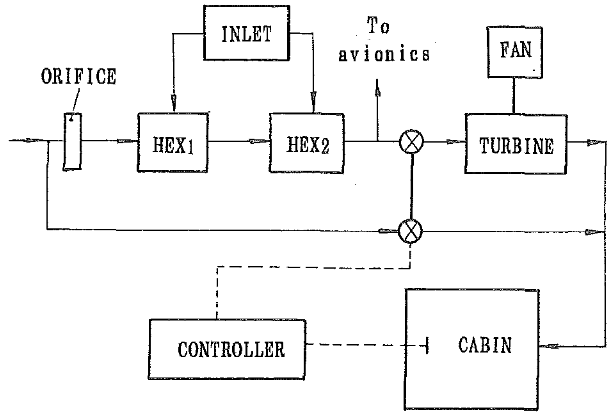


Figure 7. Simple-Cycle ECS Schematic

pass heat exchanger to be further cooled, then the air is expanded through an air cycling machine. After the power absorbed by the fan, a significant lower turbine discharge temperature results. By proper mixing with the hot air directly from bleed port regulated by a valve, the air is provided to modulate the cabin temperature.

The dynamic mathematical models must reflect not only the dynamic but also the steady performances. It is because ECS works in rather wide operating ranges and the system dynamic performances directly depend on the operating points. In this way, a good dynamic model should track the steady performance well and its simulation must reflect the problems of change of state, variable thermodynamics properties, non-linear effects.

## Ⅳ. Results of Digital Simulation

To verify the dynamic models, the dynamic behavior of a simple cycle ECS at two cases is simulated. Two cases are described as follows:

1. aircraft ascent in 300 seconds from take-off.
2. the sudden step change of bleed pressure.

Case 1 shows the effect of flight conditions on the system. During this period of 300 seconds, the aircraft takes off and ascends to 8000 m. The ambient temperature decreases from 37.2 ℃ ( summer ) to -20.7 ℃ (high altitude). The maximum Mach number is 0.83. Case 2 shows the effect of sudden change of bleed pressure on the system, including temperature, pressure, flow rate, and rotational speed of ACM.

Case 1

Because the cabin temperature is very high in summer, the regulating valve is fully closed in add-heat line, while fully open in cool line. The discharge temperature of turbine, after a little growing up in the beginning 40 seconds, then gradually runs down. The agreement of simulation results with flight test data is not very good in 40 seconds at first, but the results track well in remainning times, especially the pressure and flow rate. (Figure 8)

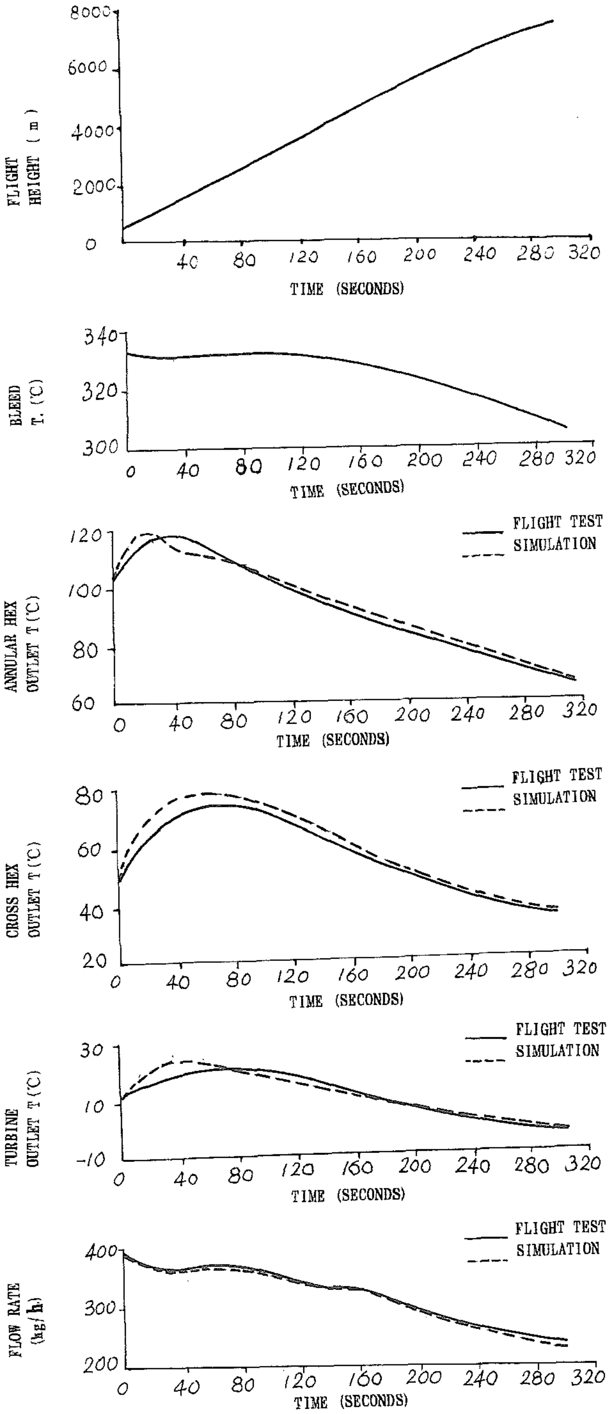


Figure 8. Aircraft Ascent

Case 2

The bleed pressure suddenly changes from 7.57 bar to 10.0 bar. (remainning parameters are the same as those at t=60 sec. in case 1 ) The position of the valve is the same as in case 1. The discharge temperature of the turbine decreases rapidly in 2 sec. at first (the effect of pressure dynamics on the turbine), then gradually increases to a constant (the effect of temperature). The time constant of ACM inertia is approximately equal to that of pressure dynamics. (Figure 9)

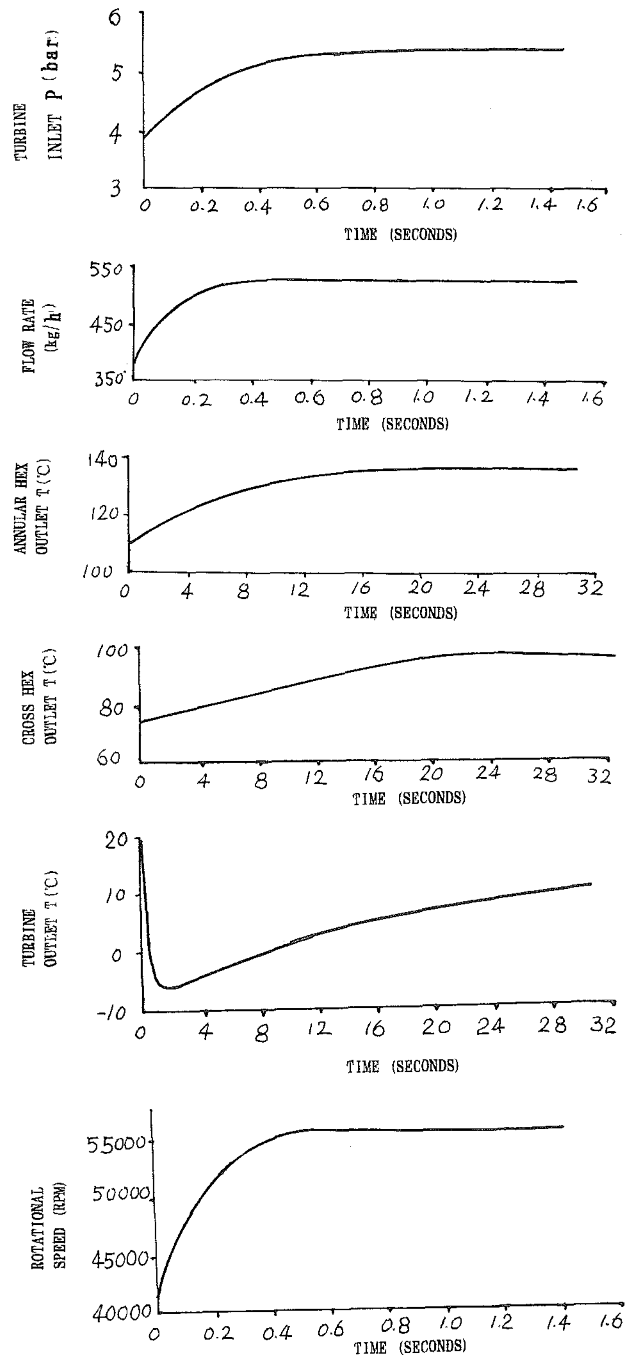


Figure 9. Sudden Pressure Change

## V. Concluding Remarks

The dynamic mathematical model for a typical air-cycle system has been presented. The annular heat exchanger is modeled by thermal resistance and capacitance. The models of heat exchangers are formulated by physical approach, which assures the proper simulation over the full operating range of systems. Only the effect of ACM rotor inertia is considered.

The simulation methods also have been proposed and proved to be efficiently in practical use. The digital simulation helps to better understand the dynamic behavior of air-cycle systems and to design or improve the ECS controller, furthermore, it can provide a tool to find potential problems in ECS design and hard ware test.

## Nomenclature

A = heat transfer, $m^2$
Cp = unit heat capacity of core material, J/kg · K
F = cross-sectional area, $m^2$
h = thermal conductance per unit heat transfer area, W/℃ · $m^2$
J = inertia, kg · $m^2$
L = fluid flow length of exchanger, m
M = mass, kg
N = power, W
n = rotational speed, RPM ( r/s )
P = pressure, bar
Q = torque, N · m
R = thermal resistor, K/W and gas constant (287.4 J/kg · K)
t = time, second
T = temperature, ℃
V = volume, $m^3$
W = mass flow rate of fluid, kg/s
X = distance from air a entrance, m
Y = distance from air b entrance, m
$\gamma$ = ratio of specific heats, dimensionless
$\rho$ = air density, kg/$m^3$
$\eta$ = efficiency, dimensionless
$\pi$ = pressure ratio, dimensionless

## Subscripts

1 = inlet
2 = outlet
a = hot fluid
b = cool fluid
c = inlet air of the engine
C1 = compressor inlet
C2 = compressor oitlet
D = duct
ds = design point
F = fan
T1 = turbine inlet
T2 = turbine outlet
w = wall.

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