Calculation of Sound Field Radiated by Oscillating Cascade

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Abstract

The unsteady aerodynamics of oscillating cascade has been extensivly studied, both experimentally and theoretically, by various invertigators. However, Much of the earlier work was conarned with the calculation of the flow field of cascade and the relevant aeroelastic problems,while there seem to be few results in current literature about the aeroacoustic characteristics of oscillating cascade. An aeroacoustic model for oscillating cascade is described in this paper. The characteristics of frequency and propagation of the sound generated by oscillating cascade is analysed by combining the model with the calculation of unsteady field of the cascade . The numeried calculation shows some interesting results.

I. Introduction

When a fan / compressor operates in non–design state, some unsteady phenomena will appear. For example, many experiments show that once the blades in compressors stall or flutter, the sound will be quite different from that in the normal condition of the compressors. Then, what reasons are responsible for the changes of the sound? and what information does it transmit to us about the unsteady flow? So far our knowledge about this is still insufficient. Therefore it appears that some theoretical investigation is needed, which may be useful both for further understanding the mechanisms of the self–exciting vibration of blades and for the fault diagonoses and monitorings of turbomachines.

There has been a great deal of research work[1–4] about aeroacoustics and aeroelasticity of turbomachines. By using similar method, an aeroacoustic model about the oscillating cascade of a fan / compressor is described in this paper.

First, suppose a single stage compressor in an infinite circular duct containing an uniform flow with velocity U. Hence the sound pressure produced by the compressor can be formulated by applying generalized Lighthill equation[5]. Second, unsteady force caused by blade vibration can be transformed into Fourier series. So one can obtain an expression which is related to the geometry of a fan / compressor and unsteady force amplitude and frequency of blade vibration. Third, the unsteady force of blades can be obtained by numerically solving unsteady linerized potential equation with the technique of computational fluid dynamics.

The model indicated as above can be applied to investigate the frequency and propagation characteristics of sound generated by oscillating cascade. Preliminary numerical results given in this paper have shown some interesting conclusions.

II. Aeroacoustic model of oscillating cascade

Suppose a isolated rotor in an infinite circular duct (see Fig 1) containing a uniform flow with velocity U. For subsonic flow, the dipole sources produced by the forces exerted on the blade will be dominant. Hence, the sound pressure generated by the rotor can be expressed as[5].

$$P(\vec{X},t)=\int_{-\infty}^{+\infty}\int_{S_m} n_i\frac{\partial G}{\partial y_1} f_i(\vec{Y},\tau)dS(\vec{Y})d\tau \tag{1}$$

where $\vec{X},\vec{Y}$ are the coordinates of sound sources and observer position respectively. $f(\vec{X},\tau)$ are the forces acted on the surface of blades, G is Green function and

$$G=-\frac{1}{4\pi i}\sum_m\sum_n\frac{\psi_m(k_{m,n}r)\psi_m(k_{m,n}r')}{\Gamma_{m,n}}exp[im(\varphi-\overline{\varphi}]$$

$$\cdot\int_{-\infty}^{+\infty}\frac{1}{K_{n,m}}exp[-i\omega(t-\tau)+\gamma^{\pm}_{n,m}(y_1-x_1)]d\omega \tag{2}$$

$$K_{n,m}=\sqrt{k_0^3-\beta^2k_{m,n}^2}$$

$$k_0=\frac{\omega}{C_0}$$

$$\gamma^{\pm}_{n,m}=\frac{Mk_0}{\beta^2}\pm\frac{K_{n,m}}{\beta^2},\qquad \beta^2=1-M^2$$

$$\psi_m(k_{m,n}r)=AJ_m(k_{m,n}r)+BY_m(k_{m,n}r)$$

$$\Gamma_{m,n}=\int_{R_h}^{R_d}|\psi_{m,n}|^2rdr$$

Where $J_m(k_{m,n}r)$ and $Y_m(k_{m,n}r)$ are the first kind Bessel function and the second kind Bessel function respectivly, and $k_{m,n}$ is the characteristic root. M is the Mach number of axial flow velocity . In addition, the 'plus' sign represents the acoustic wave which propagates upstream, while 'minus' sign represents the wave which propagates downstream.

It is usual to express the forces acted on the surface of blades in terms of an axial thrust component f_T and a circumferencial drag component f_D , i.e.,

$$f=\{f_T, -f_D sin\overline{\varphi},\ f_D cos\overline{\varphi}\}$$

and

$$f_i\frac{\partial}{\partial y_i}=\frac{f_D}{r'}\frac{\partial}{\partial\varphi}+f_T\frac{\partial}{\partial y_i} \tag{3}$$

For convenience sake, the source integrals are expressed in terms of a coordinate system that rotates with blades. The cylindrical coordinates corresponding to this frame are r', y_1 and

$$\varphi'=\overline{\varphi}-\Omega\tau \tag{4}$$

Inserting eq(4) and the Green function (2) into eq(1) and carrying out the differentiations yields

$$P(\overline{X},t)=\frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{\psi_m(k_{m,n}r)}{\Gamma_{m,n}}e^{im\varphi}\int_{-\infty}^{+\infty}\frac{e^{-i(\gamma_{n,m}x_1+\omega i)}}{K_{m,n}}\int_{S_m}\int_{-\infty}^{+\infty}\psi_n(k_{m,n}r')\times(\frac{m}{r'}f_D-\gamma^{\pm}_{n,m}f_T)e^{-i(m\varphi'-\gamma^{\pm}_{n,m}y_1)}e^{i(\omega-m\Omega)\tau}d\tau d\omega dS(\overline{Y}) \tag{5}$$

As seen from eq(5), the sound pressure p($\overline{X}$, t) depends on the forces f_T,f_D excrted on the surface of blades and its frequency characteristics.

Suppose the blade vibrates harmonically, so the blade force can be expressed as

$$\overline{f}_\alpha(\tau)=\overline{A}_\alpha e^{-i\Omega'\tau}\qquad(\alpha=T,D) \tag{6}$$

where

$$\overline{A}_\alpha=\frac{\Omega'}{2\pi}\int_0^{\frac{2\pi}{\Omega}}\overline{f}_\alpha(\tau)e^{i\Omega'\tau}d\tau$$

and Ω' represents the angular frequency of blade vibration.

Naturally, substituting (6) Into (3) yields

$$\rho'=\sum_{m=-\infty}^{\infty}\rho(\overline{X})e^{-i(\Omega'+m\Omega)t} \tag{7}$$

where

$$\rho(\overline{X})=\frac{1}{2C_0^2}\sum_{n=1}^{\infty}\frac{\psi_m(k_{m,n}r)}{\Gamma_{m,n}K_{n,m}}e^{i(m\varphi-\gamma^{\pm}_{n,m}x_1)}\times(mD^{\pm}_{n,m}-\gamma^{\pm}_{n,m}T^{\pm}_{n,m})$$

$$D^{\pm}_{n,m}=\int_m\psi_m(k_{m,n}y')e^{i(\gamma^{\pm}_{n,m}y_1-m\varphi')}\overline{A}_D dr'd\varphi'$$

$$T^{\pm}_{n,m}=\int_m\psi_m(k_{m,n}r')e^{i(\gamma^{\pm}_{n,m}y_1-m\varphi')}\overline{A}_T r'dr'd\psi' \tag{8}$$

$$K_{n,m}=\sqrt{k_0^2-\beta^2k^2_{m,n}}=\sqrt{(\frac{\Omega'+m\Omega}{C_0})^2-\beta^2k^2_{m,n}} \tag{9}$$

$$\gamma^{\pm}_{n,m}=\frac{Mk_0}{\beta^2}\pm\frac{K_{n,m}(k_0)}{\beta^2}=\frac{M(\Omega'+m\Omega)}{\beta^2C_0}\pm\frac{K_{n,m}}{\beta^2} \tag{10}$$

To reduce the region of integration to one blade, arbitrarily select one blade as a reference blade, Assign this blade the number 1, and the remaining blades, in the direction of decreasing , the number 2 through B. Let the unsteady force of the first blade be $A^k_\alpha(r',\varphi',\tau)$, then the corresponding point blades should be

$$A^k_\alpha=A^k_\alpha(r',\varphi-\frac{2\pi}{\beta}(k-1))$$

So, the total blade forces are

$$A_\alpha=\sum_{k=1}^{B}A^k_\alpha(r',\varphi'-\frac{2\pi}{B}(k-1)) \tag{11}$$

Because the conditions of geometry and flow of blades are considered as same, the magnitude of the force exerted on each blade will also be same Therefore substituting eq(11) into eq(7) yields

$$\rho'=\frac{1}{4\pi C_0^2}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{\psi_m(k_{m,n}r)}{\Gamma_{m,n}}e^{im\varphi}\int_{-\infty}^{\infty}\frac{e^{-i(\gamma^{\pm}_{n,m}x_1+\omega t)}}{K_{n,m}}\int_A\psi_m(k_{m,n}r')e^{i[\gamma^{\pm}_{n,m}y_1-m\varphi']}\times\int_{-T}^{T}\sum_{k=1}^{B}e^{i\frac{2\pi(k-1)m}{B}}(A^1_D\frac{m}{r'}-\gamma^{\pm}_{n,m}A^1_T)e^{-i\Omega'\tau}e^{1(\omega-(\Omega'+m\Omega))\tau}r'dr'd\varphi'd\omega \tag{12}$$

$$\sum_{k=1}^{B}e^{i\frac{2\pi(k-1)m}{B}}=\begin{cases}B & (m=SB)\\ 0 & (m\neq SB)\end{cases} \tag{13}$$

So, eq(12) can be written as

$$\rho'=\sum_{S=-\infty}^{\infty}\rho_B(x)e^{-i(\Omega'+SB\Omega)t} \tag{14}$$

where

$$\rho_B(x)=\frac{B}{2C_0^2}\sum_{n=1}^{\infty}\frac{\psi_{SB}(K_{SB,n}\ r)}{\Gamma_{SB,n}K_{n,SB}}e^{i(SB\varphi-\gamma^{\pm}_{n,SB}x_1)}\times(SBD^{\pm}_{n,SB}-\gamma^{\pm}_{n,SB}\ T^{\pm}_{n,SB}) \tag{15}$$

$$T^{\pm}_{n,SB}=\int_A\psi_{SB}(k_{SB,n}r')e^{i(\gamma^{\pm}_{n,SB}y^c_1-SB\varphi')}\overline{A}^1_T r'dr'd\varphi'$$

$$D^{\pm}_{n,SB}=\int_A\psi_{SB}(k_{SB,n}r')e^{i(\gamma^{\pm}_{n,SB}y^c_1-SB\varphi')}\overline{A}^1_D dr'd\varphi'$$

$$K_{n,SB}=\sqrt{(\frac{\Omega'+SB\Omega}{C_0})^2-\beta^2k^2_{SB,n}} \tag{16}$$

$$\gamma^{\pm}_{n,SB}=\frac{M(\Omega'+SB\Omega)}{\beta^2C_0}\pm\frac{K_{n,SB}}{\beta^2} \tag{17}$$

According to eq(16), the sound frequency due to oscillating cascade can be written as

$$f=\frac{1}{2\pi}(\Omega'+SB\Omega)\qquad(s=0,\pm1,\pm2,\cdots)$$

when $\Omega'=0$, $f=\frac{1}{2\pi}SB\Omega$, which is the blade passing frequency under the normal working condition. Besides it is obvious that the sound frequency of oscillation cascade is different from that in static state by adding a term $SB\Omega$.

Further, from eq(16), one knows when

$$K^2_{n,SB}=\left(\frac{\Omega'+SB\Omega}{C_0}\right)^2-\beta^2k^2_{n,SB}<0 \tag{18}$$

the sound wave will attenuate with distanec increase.

For subsonic relative Mach number, it can be shown that the sound generated by steady Blade loading will decay exponentially fast at larg values of $|x_1|$. But even in the same case, whether the sound wave produced by oscillating cascade decay or propagate will completly determined by eq(18).

III. Unsteady Aerodynamic Model

First of all, consider the model of inviscid flow. Two–dimensional potential flow is assumed to exist. Expand full velocity potential Φ in Fourier series with respect to time t and leave only first order term, i.e. $\Phi=q_{-\infty}X+\Phi_0$. After some simplification and reduction from full velocity potential equation, we have

$$\begin{cases}(1-M_0^2)\Phi_{0xx}+\Phi_{0yy}=0 & (19)\\ a_0^2=a_\infty^2+\frac{r-1}{2}(q^2_{-\infty}-U_0^2) & (20)\end{cases}$$

and

$$\begin{cases}(1-M_0^2)\Phi_{1xx}+\Phi_{1yy}-\frac{u_0}{a_0^2}[(\gamma+1)u_{0x} \\ \quad +2i\omega]\Phi_{1x}+\frac{1}{a_0^2}[\omega^2-i\omega(\gamma-1)u_{0x}]\Phi_1=0 & (21)\\ a^2=a_0^2-(\gamma-1)(u_0\Phi_{1x}+i\omega\Phi_1)e^{i\omega t} & (22)\end{cases}$$

Eq.(19) is non–linear mixed–type differential equation for steady perturbation velocity potential Φ_0. Its type depends on local steady Mach number M_0. After eq. (1) being solved, eq. (21) of unsteady pertubation potential amplitude Φ_1 becomes a linear mixed–type differential equation with variable coefficients which depends on the steady solution. A mixed difference approach has been developed first to solve eq. (1), and then to solve eq.(2) from the solution of eq.(1).

After doing that, we can have unsteady pressure distributions along profile.

IV. Numerical Results and Analysis

Using the aeroacoustic model and unsteady aerodynamic model presented as above, some preliminary numerical results have been carried out based on the geometriccl parameters of a transonic rotor facility in BUAA[6]. Fig.2–7 are the computational results obtained. In these figures, x coordinate represents the d:stance from the sound sources, while y coordinate does the sound pressure level generated by the isolated rotor. The numerical results show that whatever blade vibration appears, such as bending and tortion vibration or bending–tortion coupling vibration, whether the sound wave generated by oscillationg cascade decays or propagates with the distance from the sound souces is mainly dependent on the relative Mach number of inlet flow and the frequency of blade vibration beside the geometrical parameters of a fan / compressor. This means that for decaying wave, one can not know whether the blade vibrate or not by measuring far acoustic field, for the propagating wave, an observer, in principle, can know the acoustic pressure radiated by oscillationg cascade in any position. But in later case, the measurement will be affected by the acoustic field produced by steady blade forces. The calculating results also show that the steady blade forces will produce higher sound pressure level than the unsteady blade forces in near acoustic field. So it may be feasible to obtain the acoustic signals of oscillating cascade in far acoustics field because the acoustic wave radiated by the steady blade forces will decay more rapidly with the distance from the sound souces for subsonic relative flow.

V. Summary

An aeroacoustic model for an oscillating cascade is described in this paper, and the relevant numerical results have been carried out. Besed on the analysis of the frequency and propagation characteristics of the sound wave generated by oscillationg cascade. Some experimental phenomena that we have observed about oscillating cascade may be explained preliminarily. On the other hand, this model is only related to an isolated compressor rotor. For multi–stage compressor, the sound will become more complicated. So how to idetify the sound field generated by blade vibration is still a problem that requires a great deal of further research effort.

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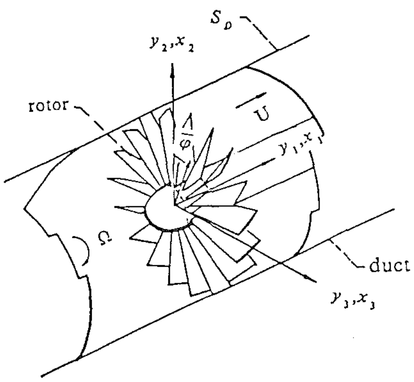

Fig1 A compressor in an infinite duct

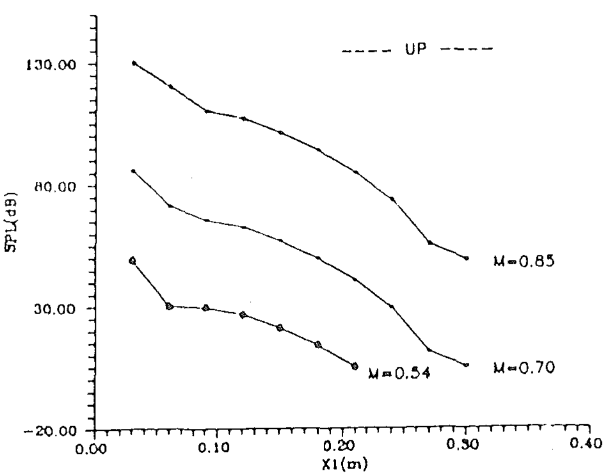

Fig.2 decaying acoustic wave radiated by blade tortion vibration (M–Mach number, Up–upstream)

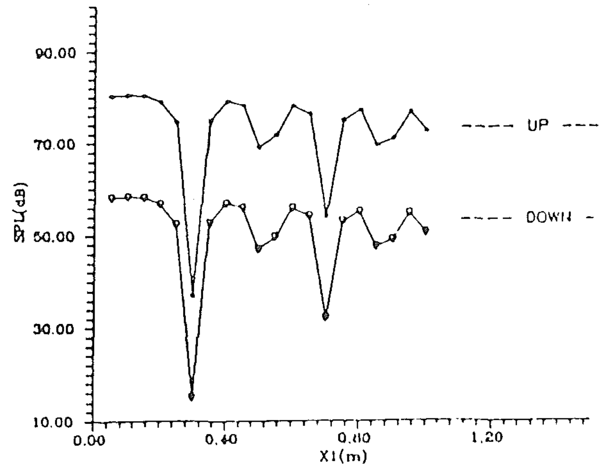

Fig.3 propagating acoustic wave radiated by blade bending vibration (Up–upstream, down–downstream)

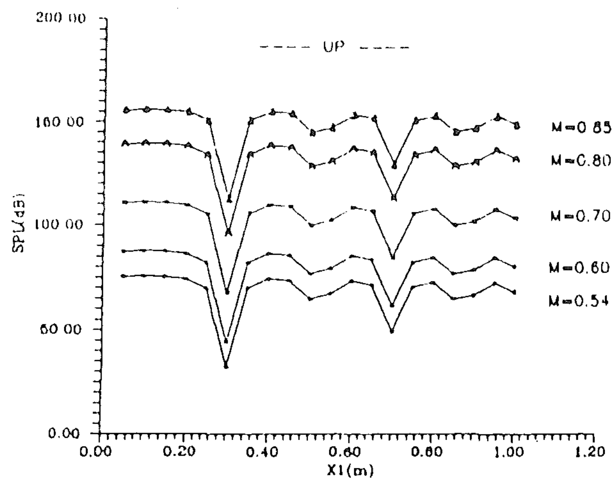

Fig.4 variation of sound pressare with Mach number (tortion vibration, upstream)

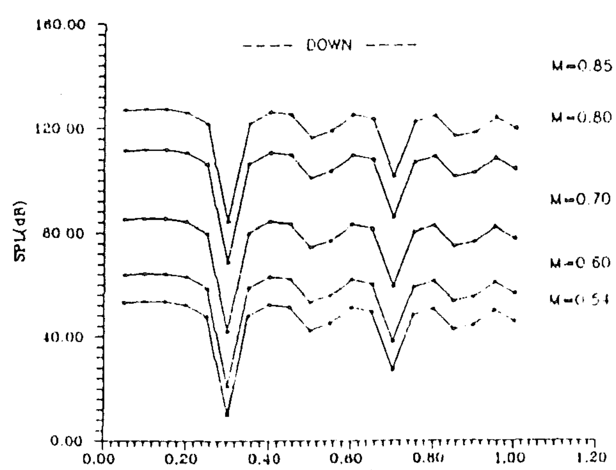

Fig.5 Variation of Sound pressure with Mach number (tortion Vibration, downstream)

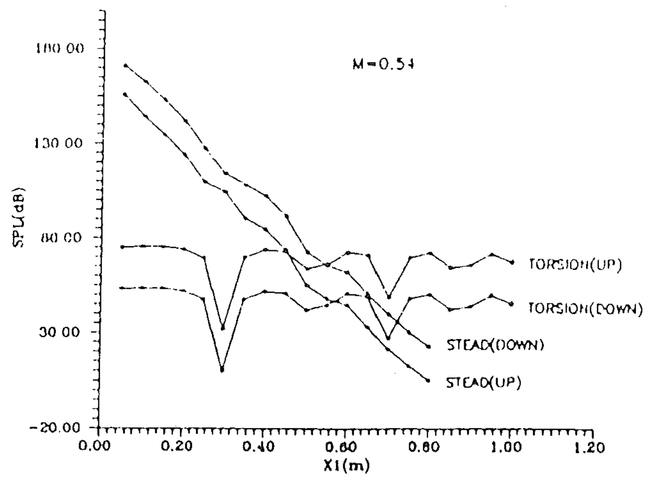

Fig.6 Comparison in prpagating acoustic wave due to blade bending vibration with the decaying wave due to the steady blade forces.

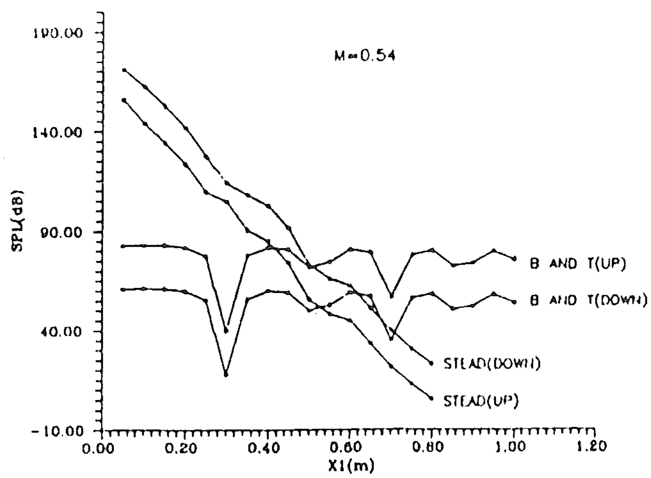

Fig.7 Comparison in propagating acoustic wave due to bending–tortion coupling vibration with the decaying wave due to the steady blade forces.