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Investigation of Precise Approach and Landing of Civil Aircraft Using Integrated System Based on GPS

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Abstract

The possibility of using integrated system based on GPS to guide the civil aircraft precision approach and landing is investigated. Using eigenstructure assignment method, the longitudinal and lateral-directional autopilot of a feeder aircraft are designed. Using the state output of the aircraft-autopilot system as filtering object and the output of the integrated system based on GPS as the filtering measurement, a Kalman filter is designed. To guess the guidance accuracy of the whole system, covariance analysis and Monte Carlo simulation method are used. The reaults showed that the integrated system of GPS with the aid of altimeter is possible to meet the guidance accuracy requirement of ILS, if the accuracy of the altimeter is high enough.

1. Introduction

The GPS (Global Positioning System) technique plays an increasing important role in many respects in the last ten years. It has been used not just for vehicle navigation, but also for geodesy, gravimetry and so on [1,2,3].

One of its interesting application for China is how to use the GPS technique to guide the civil aircraft approach and landing. China really is a large country, we can not buy so many ILS instruments to equip our so many small airports and feeder aircraft due to cost and maintenance reason.

When the GPS technique is used for civil aircraft approach and landing, the guidance accuracy requirement is still worth discussing, but the IFR regulations of ICAO should be referenced here at first.

Some of the precision approach and landing regulation of ICAO is shown in Fig.1, and the guidance accuracy requirement is listed in Table 1.

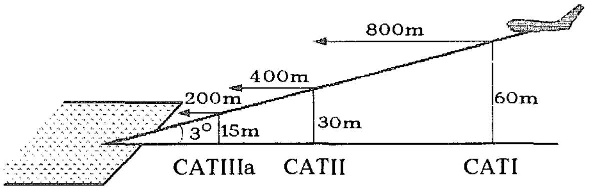

Fig.1. Categories and Visual Conditions of IFR

Table 1. Guidance Accuracy Requirement of ILS

Category	Decision Altitude(m)	Lateral Accuracy(2σ)	Vertical Accuracy(2σ)
I	60	17.1 m	4.1 m
II	30	5.7 m	1.7 m
IIIa	15	4.02 m	0.5 m

From Table 1, it could be seen that the guidance accuracy requirement is just several meters, and the vertical accuracy requirement is still one order smaller than the lateral accuracy requirement. It means, though a better GPS technique, for example DGPS, is used, how to meet the accuracy requirement in two directions at the same time perhaps still is a problem.

Trying to find a simple and cheaper way to solve this problem, an integrated system of GPS aided by gyroscope and /or altimeter is adopted.

In this paper, the state equations of aircraft-autopilot system are introduced firstly. Then using the state output of the aircraft-autopilot system as the filtering object and the output of the integrated system based on GPS as the measurement, a Kalman filter is designed. At last, covariance analysis and Monte Carlo method are used to quess the guidance accuracy of the whole system.

From the simulated results it could be seen that the guidance accuracy could not be meet if only the GPS signal is used. But the accuracy perhaps could be meet, even for CATIII, if an integrated system of GPS aided by altimeter is used, as long as the accuracy of the altimeter is high enough (for example, 2 sigma = 0.3 Meter).

2. State Equations of Aircraft-Autopilot System

Assume the approach and landing of the aircraft is performed automatically. Then based on the flying quality specification MIL-F-8785C, the longitudinal and lateral- directional autopilot are designed[4].

(1) Longitudinal state equation of aircraft—autopilot system.

The linearized longitudinal state equation of aircraft- autopilot system could be written briefly as

$$\left.\begin{aligned} \dot{X}_1 &= A_1 X_1 + B_1 U_1 \\ Y_1 &= C_1 X_1 \\ U_1 &= F_1 Y_1 \end{aligned}\right\} \quad (1)$$

Here the state vector

$$X_1 = [u \ \ \alpha \ \ q \ \ \vartheta \ \ \delta_e \ \ \delta_t \ \ x_e \ \ z_e]^T$$

the control vector

$$U_1 = [\delta_e \ \ \delta_t]^T$$

the output vector

$$Y_1 = [u \ \ q \ \ \vartheta \ \ \alpha \ \ z\ ]^T$$

The system matrix A_1, control matrix B_1, output matrix C_1 have to be omitted here, because of the space of this paper is limited.

$$F_1 = \begin{bmatrix} k_{eu} & k_{eq} & k_{e\vartheta} & k_{e\alpha} & k_{ez} \\ k_{tu} & 0 & k_{t\vartheta} & 0 & 0 \end{bmatrix}$$

(2) Lateral—directional state equation of airraft— autopilot system.

The linearized lateral-directional state equation could be written briefly as

$$\left.\begin{aligned} \dot{X}_2 &= A_2 X_2 + B_2 U_2 \\ Y_2 &= C_2 X_2 \\ U_2 &= F_2 Y_2 \end{aligned}\right\} \quad (2)$$

Here the state vector

$$X_2 = [\beta \ \ p \ \ r \ \ \varphi \ \ \psi \ \ \gamma \ \ \delta_a \ \ \delta_r]^T$$

the control vector

$$U_2 = [\delta_a \ \ \delta_r]^T$$

the output vector

$$Y_2 = [\,p \ \ r \ \ \varphi \ \ \psi \ \ \gamma\ ]^T$$

The system matrix A_2, control matrix B_2, output matrix C_2 are also omitted due to the limited space of of the paper. The feedback matrix

$$F_2 = \begin{bmatrix} k_{ap} & 0 & k_{a\varphi} & k_{a\psi} & k_{ay} \\ 0 & k_{rr} & k_{r\varphi} & 0 & 0 \end{bmatrix}$$

(3) Feedback matrix determination of longitudinal autopilot.

Assume the feeder aircraft Y7 is approaching with altitude 1000 meter, Mach number 0.25, and the glide path angle -3.0 degree. Then according to the flying quality requirement and using the eigenstructure assignment method the constrained output feedback matrix is calculated as[5].

$$F_1 = \begin{bmatrix} -0.007922 & -0.2825 & 0.005957 & -0.4591 & -0.00062 \\ -0.000026 & 0.0 & 0.00119 & 0.0 & 0.0 \end{bmatrix}$$

and the assigned eigenvalues are listed in Table 2.

Table 2 Assigned Eigenvalues of Aircraft(Longitudinal)

Short Period Mode	Phugoid Mode	InnerLoop of Control System Mode		X Mode	Z Mode
-0.54±1.32 j	-0.1 ± 0.1 j	-10.0	-10.0	0.0	0.0

(4) Feedback matrix determination of lateral—directional autopilot.

Using the same reference flight condition and eigenstructure assignment method, the constrained output feedback matrix is calculated as

$$F_1 = \begin{bmatrix} 0.9632 & 0.0 & 3.007 & -0.1253 & -0.000349 \\ 0.0 & 0.1946 & 6.141 & 0.0 & 0.0 \end{bmatrix}$$

and the assigned eigenvalues are listed in Table 3.

Table 3. Assigned Eigenvalues of Aircraft(Late.-Direc.)

Deutch Roll	Spiral Mode	Coupled Mode	InnerLoop of Control	ψ Mode	γ Mode
-0.6±3.5j	-0.005	-6.39±9.64j	-13.351	-2.142	-0.004

3. Kalman Filter Design

In order to improve the guidance accuracy, a Kalman filter is designed. In which, the state output of the aircraft autopilot system is used as the filtering object, and the output of the integrated system based on GPS is used as the filtering measurement[6].

The advantages of this Kalman filter are obvious. That means, the aircraft dynamic characteristics has been considered, the low data rate of the GPS could be compensated by the high data rate of the aid instrument, and the redundancy of the flight guidance system is also improved.

The longitudinal and lateral-directional state equation of the aircraft-autopilot system could be combined together and be written as

$$\dot{X} = AX + BU + W$$

Here the state vector

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$A=\begin{bmatrix} A_1+B_1F_1C_1 & 0 \\ 0 & A_2+B_2F_2C_2 \end{bmatrix}$$

$$B=\begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$$

$$U=\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

W is the system noise, and its variance matrix $Q=E\{WW^T\}$. After discreting, the state equation of aircraft-autopilot system could be written as

$$X_K=\Phi_{K,K-1}\ X_{K-1}+H_{K-1}U_{K-1}+\Gamma_{K-1}W_{K-1} \quad (3)$$

Here

$$\Phi_{k,k-1}=1+A\Delta T+\frac{1}{2}A^2\Delta T^2$$

$$H_{k-1}=[\frac{\Delta T}{2}(1+A\Delta T+\frac{1}{2}A^2\Delta T^2)+\frac{\Delta T}{2}]B$$

$$\Gamma_{k-1}=Q\Delta T+[AQ+(AQ)^T]\frac{T^2}{2!}+\{AQ+(AQ)^T+[AQ+(AQ)^T]^T\}\frac{T^3}{3!}$$

The measuement equation could be written in the form of $Y_k=CX_k+V_k$

Here V_k is the measurement noise, and the variance matrix of V_k is $R=E\{V_kV_k^T\}$.

If P_k is *the variance matrix of system estimatio* error, and $P_{k,k-1}$ is the square mean error matrix of one step prediction, then the Kalman filter equation will be

$$\hat{X}_k=\hat{X}_{k,k-1}+K_k(Y_k-C\hat{X}_{k,k-1})$$

$$\hat{X}_{k,k-1}=\Phi_{k,k-1}\hat{X}_{k-1}+H_{k-1}U_{k-1}$$

$$K_k=P_{k,k-1}C^T(CP_{k,k-1}C^T+R_k)^{-1}$$

$$P_{k,k-1}=\Phi_{k,k-1}P_{k-1}\Phi^T_{k,k-1}+\Gamma_{k-1}Q_{k-1}\Gamma^T_{k-1}$$

$$P_k=(1+K_kC)P_{k,k-1}$$

Based on the equations mentioned above, the guidance accuracy could be estimated with covariance analysis method. To verify the results of covariance analysis, Monte Carlo simulation method is also used.

4.Guidance Accuracy Analysis

The state equation of aircraft-autopilot system in discret form is mentioned before, formula (3). The simplified measurement equation could be written as

$$Y=C_gX_k+V_g \quad (4)$$

$$C_g=\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&U&0&0&0&0&0&0&0 \\ 0&U&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix}$$

Here V_g is the measurement noise, and if $R=E\{V_gV_g^T\}$, then

$$R=\mathrm{diag}[\sigma_{vx}\ \sigma_{vy}\ \sigma_{vz}\ \sigma_x\ \sigma_y\ \sigma_z]$$

σ_x, σ_y and σ_z are the components of σ in the three axis of the coordinate, and σ is the mean error of velocity which is measured by GPS receiver.

From equations (3) and (4), *and using the* covariance analysis and Monte Carlo method, the guidance accuracy of the whole system has been estimated.

The accuracy analysis are done for several cases.

(1) **Case 1.** Only GPS signal is used and no system noise is added.

In this case, the results of covariance analysis are shown in Fig.2 with dotted lines, and the results of Monte Carlo simulation are shown in the same figure with solid lines.

The results showed that the work of the Kalman filter is stable, the state estimation error approach to zero when time T is infinitive.

(2)**Case 2.** Only GPS signal is used but the system noise is added.

For this case, the covariance analysis and Monte Carlo simulation results are shown in Fig.3, respectively.

From these results, it could be seen that the work of the Kalman filter is good, the steady state could be obtained in less than twenty seconds. The guidance accuarcy in lateral direction could meet the requirement of CAT IIIa, but can not meet the requirement of vertical direction at all.

(3)**Case 3.** Integrated system of GPS aided by rate gyroscope

When the pitch rate gyroscope is used as an aid instrument to GPS, the measurement equation of the integrated system should be written as

$$Y=\begin{bmatrix} C_g \\ C_{wq} \end{bmatrix}X_k+\begin{bmatrix} V_g \\ V_{wq} \end{bmatrix}$$

$$C_{wq}=[0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

Similarly, if the roll and yaw rate gyroscopes are used to aid the GPS, the measurement equations should be written as

$$Y=\begin{bmatrix}C_g\\C_{wp}\end{bmatrix}X_k+\begin{bmatrix}V_g\\V_{wp}\end{bmatrix}$$

$$Y=\begin{bmatrix}C_g\\C_{wr}\end{bmatrix}X_k+\begin{bmatrix}V_g\\V_{wr}\end{bmatrix}$$

$$C_{wp}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\end{bmatrix}$$

$$C_{wr}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\end{bmatrix}$$

The results of covariance analysis and Monte Carlo method are shown in Fig.4, 5, 6, respectively. The results showed that if the signal of anyone of the rate gyroscope is added to the GPS, the influence on the position accuracy improvment is very small, only the estimation accuracy of the relevant rate is improved and the rate error is suppressed.

(4)**Case 4.** Integrated system of GPS with the aid of altimeter.

When the altimeter is used as an aid instrument to GPS, the measurement equation of the integrated system should be written as

$$Y=\begin{bmatrix}C_g\\C_{wh}\end{bmatrix}X_k+\begin{bmatrix}V_g\\V_{wh}\end{bmatrix}$$

$$C_{wh}=\begin{bmatrix}0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\end{bmatrix}$$

The covariance analysis and Monte Carlo simulation results are shown in Fig. 7, repectively.

The results showed that if the altimeter signal is added to GPS, the improvement of guidance accuracy in lateral direction is small, but the guidance accuracy improvement in vertical direction is very clear. If the accuracy of the altimeter could be higher and higher, then the guidance accuracy requirment in vertical direction would be better and better, even the requirement of CATIIIa also could be meet.

In order to investigate the relationship between altimeter accuracy (1 sigma) and the guidance accuracy requirement (2 sigma), for different given altimeter accuracy the guidance accuracy analysis with the same procedure mentioned before is done, and the results are shown in Fig. 8.

From the curves, we could see that when the accuracy of the altimeter is higher and higher, the guidance accuracy in vertical direction is smaller and smaller, but there is no influence in the lateral direction.

If σ_h is used as the independant variable, and the $2\sigma_z$ as the dependent variable, the curve could be matched by the formula

$$2\sigma_z=0.06118+1.58\sigma_h-0.268(\sigma_h)^2+$$
$$+0.02331(\sigma_h)^3-0.0008106(\sigma_h)^4$$

Or it could be expressed as

$$\sigma_h=0.187\,e^{0.9154(2\sigma_z)}$$

Based upon these formula and the guidance accuracy requirement of ILS regulation, Table 4 could be created.

Table 4 Altimeter Accuracy Required by Precision Approach Requirement

Category	Error Limit of Vertical Direction ($2\sigma_z$) (m)	Required Altimeter Accuracy ($1\sigma_h$) (m)
CAT I	4.1	7.97
CAT II	1.7	0.89
CAT IIIa	0.5	0.295

From Table 4, if the guidance accuracy ($2\sigma_z$) of CATIIIa in vertical direction 0.5 meter is required, then it is easy to see that the required altimeter accuracy ($1\sigma_h$) should be 0.295 meter.

5.Conclusions

(1)The GPS technique is a very useful and powerful tool for vehicle navigation. Especialy for civil aircraft approach and landing, it would has a brilliant future.

The guidance accuracy requirements of ILS in lateral and vertical direction are different---one order difference. If only the GPS is used for civil aircraft approach and landing, perhaps it would be not so easy to meet the requirement in both directions at the same time.

(2)Integrated system of GPS with the aid of altimeter may be, is a simple and cheaper way to solve this problem. According to the investigation of this paper, the guidance accuracy requirement of ILS, even for CATIIIa, is possible to meet, as long as the accuracy of the altimeter is high enough.

(3)As an example, a typical feeder aircraft Y7 is used and calculated in this paper. Based upon the covariance analysis and Monte Carlo simulation results which we have gotten in the paper, a relationship between altimeter accuracy and the guidance accuracy requirement is obtained.

Using this relationship, we could guess what kind of altimeter we should have, radio or laser, if the guidance accuracy requirement of ILS we would like to meet.

6. Reference

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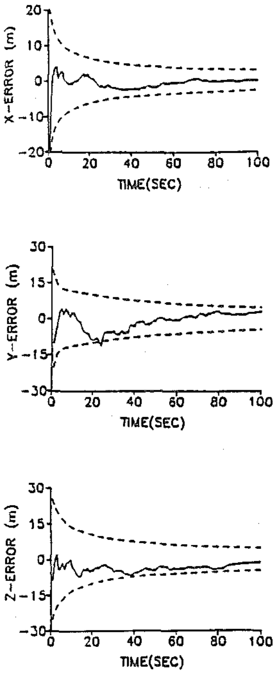

Fig.2 Position Error Estimation
(Only GPS, No System Noise)
----- Covariance Analysis(2 Sigma)
—— Monte Carle Method

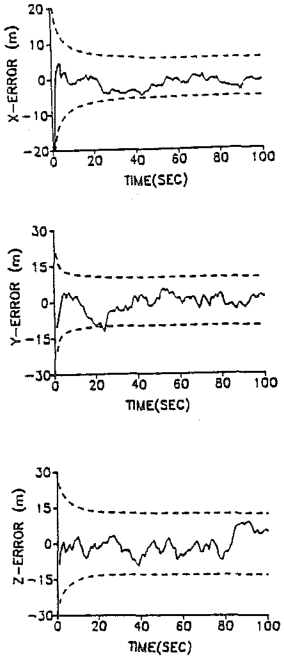

Fig.3 Position Error Estimation
(Only GPS, With System Noise)
----- Covariance Analysis(2 Sigma)
—— Monte Carle Method

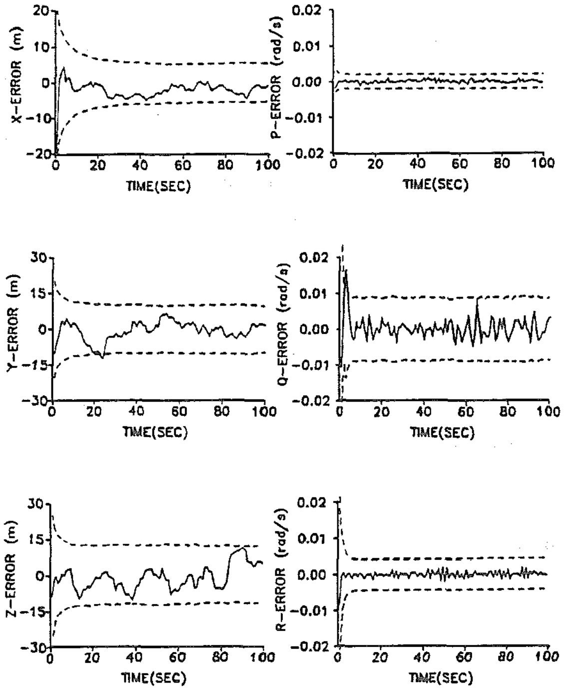

Fig.4 Position Error and Rate Error Estimation
(GPS aided by Pitch Rate Gyro)
----- Covariance Analysis(2 Sigma)
—— Monte Carle Method

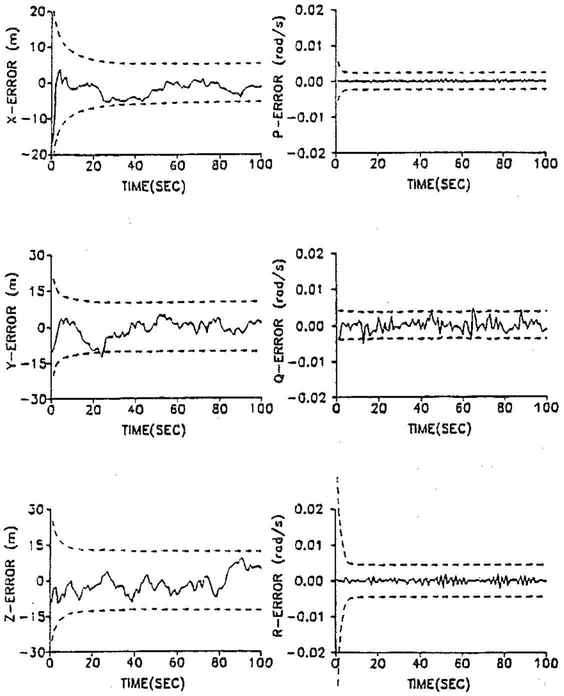

Fig.5 Position Error and Rate Error Estimation
(GPS aided by Roll Rate Gyro)
----- Covariance Analysis(2 Sigma)
——— Monte Carle Method

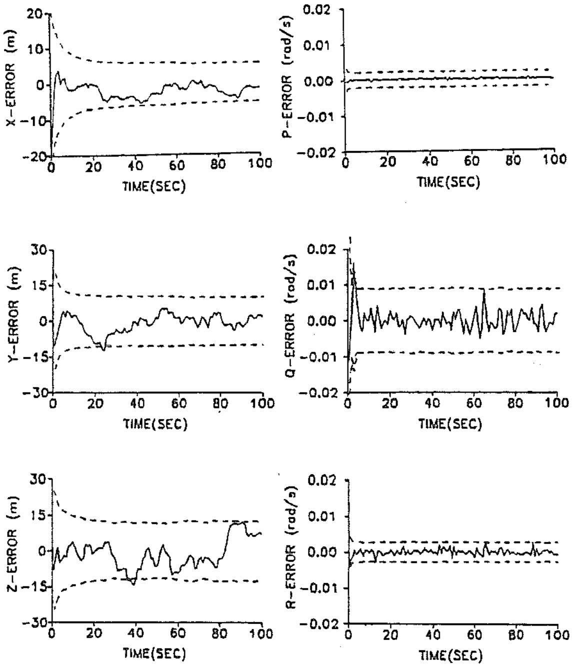

Fig.6 Position Error and Rate Error Estimation
(GPS aided by Yaw Rate Gyro)
----- Covariance Analysis(2 Sigma)
——— Monte Carle Method

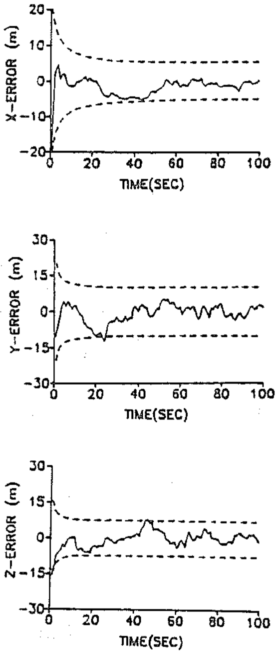

Fig.7 Position Error Estimation
(GPS aided by Altimeter)
----- Covariance Analysis(2 Sigma)
——— Monte Carle Method

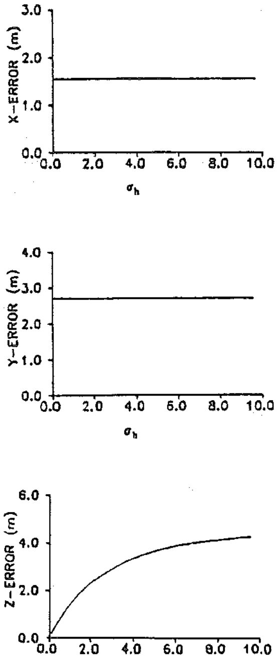

Fig.8 Position Error Estimation
for Diffrrent Altimeter Accuracy
(1 Sigma, m)