NON-LINEAR MATHEMATICAL, THERMAL MODELS OF GAS TURBINE ENGINES AND THEIR APPLICATION IN OPERATION

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Abstract

In this lecture, a brief survey is offered of the models of steady-state and transient duties elaborated so far, and also their scheme of algorhythms is given. Then the mode of adaptation of the models and jet-plants is dealt with, as well as the experiences gained in the course of adaptation. In addition, the results of the tests carried out with turbojet- and turbofan models under different conditions, as well as the consequences derived from them are imparted.

Nomenclature

j	serial number of compressor or turbine
N	number of rotors
$\dot{m}_c$	inlet mass-flow rate of the compressor
$\Delta\dot{m}_c$	air flow extracted from the compressor and used for cooling and in technological processes
$\dot{m}_c$	inlet mass-flow rate of the turbine
$\Delta\dot{m}_r$, $\Delta\dot{m}_s$	masses of air recirculated into the gas flow for cooling the rotor and stator blades
$\dot{m}_f$	fuel-flow rate delivered into the burner
$\dot{m}_n$	gas flow rate of the nozzle
n	number of revolutions
p_I, p_{II}	static pressures of streams at the entrance into mixing chamber
P	powers available at one rotor
η_m	mechanical efficiency
η_c	compressor efficiency
η_T	turbine efficiency
A_c	compressor inlet cross-sectional area
F_t	thrust
f	fuel-to-air ratio
$q(\lambda)_j$	non-dimensional mass-flow rates
p^*	stagnation pressure
T^*	stagnation temperature
s	specific fuel consumption
$\frac{F_t}{\dot{m}}$	specific thrust.

Set of equations for simulation

Mathematical models represent by themselves the description of jet-engines motion and the processes taking place within them by using equations. In the equations, the conditions for the co--operations of the co-acting components, on the one hand, and their properties and characteristics are included, on the other hand.

In final analysis, from thermal point of view, the jet-engine can be characterized by conservation equations (conservation of energy and mass-rate) set up for the components, as well as by establishing the equation describing the control system.

In the case of known gas characteristics prior to the entrance and given component characteristics, the number of the required equations is identical with the number of components bringing about modifications in the energy-flow. In case of steady-state plant duties, this number is equal to the sum of numbers of compressors turbines, burners and rotors, while in case of transient processes, in addition to those enumerated above, other components as e.g. storages subject to time--dependent modifications is their content can be added.

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Steady-state duties

Equations for conservation of mass-rate ranging from the inlet cross-section to the outlet from the nozzle (two-spool turbojet according to the arrangement shown in Fig.1) are the following:

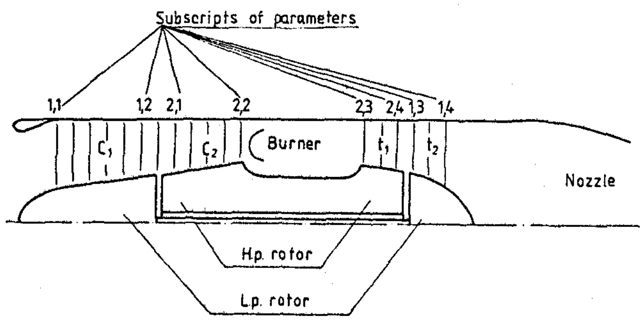

Fig. 1 Arrangement scheme of two-spool turbojets

for compressors:

$$\dot{m}_{c\,j+1}-\dot{m}_{cj}-\Delta\dot{m}_{cj}=0 \qquad (j=1\ldots N-1) \qquad (1)$$

for burner:

$$m_{cN}+\dot{m}_f-\dot{m}_{tN}=0 \qquad (2)$$

for turbines:

$$\dot{m}_{tj+1}+\Delta\dot{m}_{rj+1}-\Delta\dot{m}_{tj}+\Delta\dot{m}_{sj}=0 \qquad (j=1\ldots N-1) \qquad (3)$$

for nozzle:

$$\dot{m}_{t1}+\Delta\dot{m}_{r1}-\dot{m}_n=0 \qquad (4)$$

Equations for the conservation of energy for each rotor:

$$P_{ti}\,\eta_{mi}-P_{ci}-\Delta P_i=0 \qquad (i=1\ldots N) \qquad (5)$$

The above equations are completed with those describing the rules of controlling (e.g. n = comstant, or T_3 = constant etc.) to form the set of equations to be solved.

$$n_1 = \text{const} \qquad (6)$$

With a two-spool turbofan having a mixing-chamber and a low pressure compressor equipped with primary fan-stages (Fig.2), in equations (1) and (3): j=2.. ..N, while equations (2) and (4) transformed into the following form:

$$\dot{m}_{cN+1}+\dot{m}_f-m_{tN+1}=0 \qquad (7)$$

$$\dot{m}_{t2}+\Delta\dot{m}_{r2}-\dot{m}_n=0 \qquad (8)$$

and as an additional equation, the mixing condition of flows will be added:

$$p_I - p_{II} = 0 \qquad (9)$$

In this case, the rule of controlling will be expressed by:

$$n_3=\text{const} \qquad (10)$$

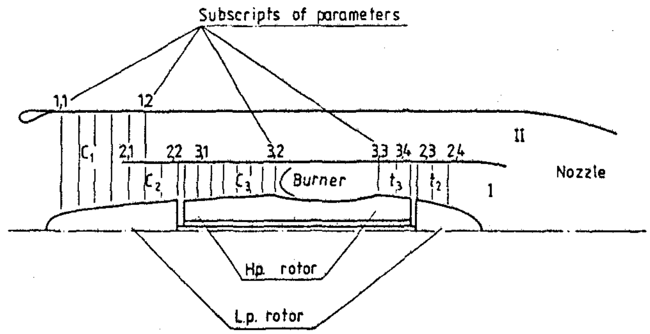

Fig. 2 Arrangement scheme of two-spool turbofans

The temperature- and composition-dependences of gas characteristics should be taken into consideration as a function of the requirements raised on the model.

The characteristics of the compressor and turbine were determined in advance for a shorter running-time to be ensured, and they were built into the set of modelling equations by means of section-wise quadratic polynomials having coefficients determined by cross-regression.

The ambient temperature and pressure of the jet-plant are either given, or else they can be determined in a simple way with the knowledge of the flight speed and flight height[2].

The set of non-linear equations (1) - (6), or (1), (3), (5), (7) - (10) with real contents substituted into them can be solved by numerical method[3] selected properly according to non-dimensional mass flow-rates $q(\lambda)_j$, pressure ratios of the turbines (π^j_t), number of revolutions (n), and fuel-to-air ratio (f) as unknown quantities.

From viewpoint of computer technique, it seems appropriate to take the relative values (related to the take-off duty) of the independent variables instead of using their absolute values.

To solve the set of equations of this kind, the Newton-Raphson method proved to be effective because the initial values of the solutions can be estimated with an accuracy required by this method, while the convergence can become doubtful only in case of an extremely wrong assumption of initial values. The required partial derivatives were replaced by the difference quotients formed by increasing the independent variable to its 1,001-fold value.

The scheme of algorhythms for the mathematical model of the steady-state duties is illustrated in Fig.3.

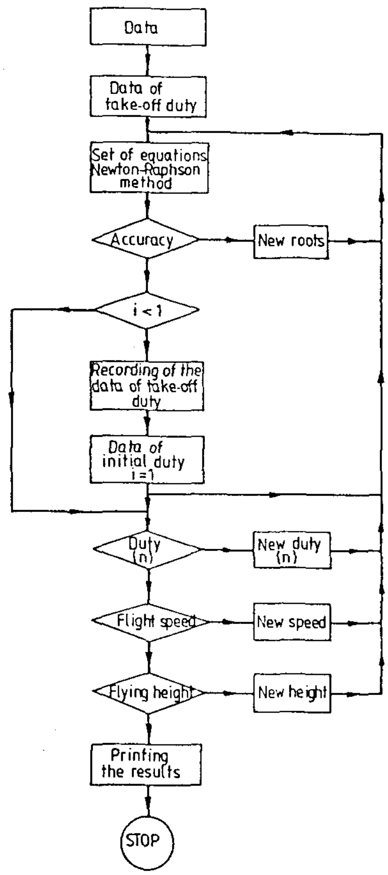

Fig. 3 Scheme of algorhythms for models of steady-state duties

Transient duties

The set of modelling equations will be altered depending on the number and character of the *material(mass)- and* energy storages taken into consideration during the transient processes of the jet-engine.

The material- and energy storages can be characterized by differential equations describing the modification of the mass-flow-rate in the form:

$$\dot{m}_1 - \dot{m}_2 = \frac{dm}{dt}, \text{ where } m = \frac{p^* V}{R\,T^*} \qquad (11)$$

or by those describing the modification of the energy content in the form:

$$(\dot{m}_1 T_1^* - \dot{m}_2 T_2^*)\, c_p\, dt = c_v (T^* dm + m\, dT^*) \qquad (12)$$

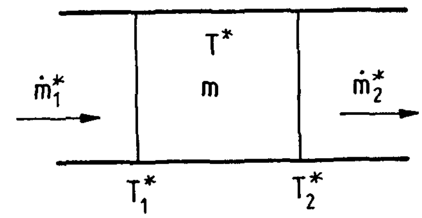

Fig. 4 Parameters of mass-storages

The designations following those used in Fig. 4 are:
m - mass of gas contained in the storage: p^* - mean stagnation pressure of the storage: V - volume of the storage: T^* -mean stagnation mass temperature: m_1, m2 - mass-flow rates entering or leaving the storage, resp.

The storing of kinetic energy can take place within the units performing motion in the energy-flow. The most considerable effect is exerted by the kinetic energy storage-capacity of the rotors.

The motion equation of kinetic-energy storages can be interpreted in the form:

$$\frac{dn_i}{dt} = \frac{P_{ti}\,\eta_{mi} - P_{ci} - \Delta P_i}{4\pi^2 \Theta_i\, n_i} \qquad (13)$$

Here Θ_i - inertial moment of the i^{th} rotor.

Inasmuch, the quasi-stationary character of the processes is assumed, and from among the storages only the energy accumulation yielded from the inertia of rotors is taken into consideration, while the energy accumulation of other storages as an insignificant one in comparison to the formers is neglected, then the set of equations describing the transient processes coincides with the set of equations modelling the steady-state duties, with the exception of (6) or (10) and the equations describing the rules of controlling.

The unsteady-state heat-transfer between the working fluid and the structural elements of the jet-engine in case of free run out process is taken into consideration by relationship:

$$T_3^* = T_2^* + \frac{T_3^* - T_2^*}{e^{ct}}$$

Where T_2^*, T_3^* - stagnation temperatures leaving the compressor and entering the turbine respectively, c - constant, t - time.

The neglections can be compensated in the course of model adaptation.

Equations (13) should be substituted instead of (5), or (8), respectively. The rules of controlling are expressed by:

$$n = f(t) \qquad (14)$$

or in case of free run out process

$$f = 0. \qquad (15)$$

For solving the set of non-linear, differential equations composed of equations (1) - (4), (13) and (14) or (15), or else (1), (3), (5), (7) - (9), (13) and (14) or (15) simulating the transient duties, according to our experiences, the method both of Runge-Kutta and Euler can be applied properly however the latter can insue a satisfactory accuracy only in the case of a step-interval shorter than the 1/3-1/5 fraction of the time-constant occuring in the set of equations ($\Delta\tau < 0{,}001$ sec.). The Runge-Kutta method of the fourth order serving for automatic selection of the step-interval and applied here for the calculation of the free running process (ranging from the idling to the full shut down) of the jet-engine required the smallest step-interval of 0,05 sec.

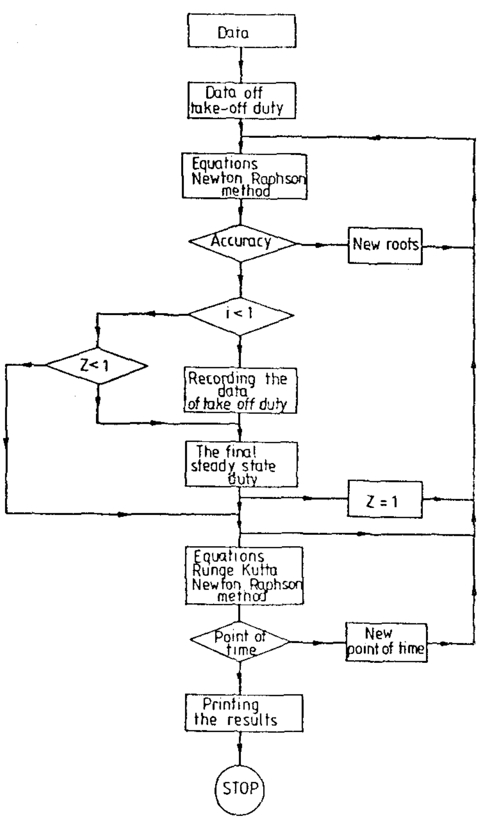

Fig. 5 Scheme of algorhythms for models of transient duties

As a consequence of construction of the set of equations, besides the Runge--Kutta method of the fourth order and within it, the Newton- Raphson method was applied for solving the set of non--linear algebraic equqtions.

The scheme of algorhythms forthe mathematical model of the process is illustrated in Fig.5.

Adaptation of the mathematical model to the simulated jet-engine

In the overwhelming majority of cases the correction of the model, and the adaptation of the jet-engine constituting the object of the model and the model itself are required. In this case, the adaptation was carried out on the basis of measurement results.

In the course of this adaptation, several internal elements of the mathematical model were altered in a way that its complexity and build-up (its set of equations) remain unchanged however the solutions yielded by the model approximate most the measurement results.

In a jet-engine, the elements to be altered can be represented by different loss-factors, efficiences, constants of semi-empirical relationships, as well as by other quantities chosenproperly. For selecting them, the following fundamental points should be taken into consideration:

- they should have the least reliability,
- they should exert a considerable influence on the value ot the measured characteristics,
- they should have differing effects on any of the measured characteristics,
- no interdependence can exist between them
- the number of the measured characteristics and that of the constants to be varied should be in accordance with each other,
- the foreknown inaccuracies occuring in the model should be compensated by the selection, as far as possible.

Prior to the selection of the quantities to be varied, it is advisable to carry out a previous examination for establishing the sensibility of parameters, which points out what an influence the factors to be altered exert on the value of the mentioned characteristics.

As far as possible, a lot of parameters should be measured for the jet-engine in different duties, and the model should be calculated in the same duties.

The measured parameters have different physical nature, and they were determined by different methods and with different accuracies. Therefore, the measurement results should be converted to the same accuracy[5], which with the exclusion of correlation between the measurement errors can be carried out on the basis of relationship:

$$P_{ij}^* = P_{ij}\sqrt{g_i} \qquad (16)$$

or by applying the weighting according to duties, relationship:

$$P_{ij}^{*} = P_{ij}\sqrt{g_i\ g_j} \qquad (17)$$

can be used,
where: P_{ij} - is the i^{th} parameter in j^{th} duty, $g_i = \sigma_o^2/\sigma_i^2$ - is the weight of measurement for the i^{th} duty. Here σ_i is the quadratic mean error of the i^{th} parameter, σ_o is the quadratic mean error per unit weight: g_j - is the weight of the duty.

The difference between the converted measured (p^{*}) and the calculated parameters (P) should be taken the smallest in all duties. This condition is satisfied by applying the method of least squares.

Relative value $\bar{A}_n = A_n/A_{n\ max}$ of quantity A_n to be varied can always be calculated as follows:

$$\bar{A}_n = \hat{A}_{on} + \Delta\bar{A}_n \qquad (18)$$

Inasmuch differencies $\Delta\bar{A}_n$ are suitably small, then the functions of the jet-engine parameters calculated in several duties by using the mathematical model can be expanded into Taylor's series according to those differencies like this:

$$\bar{P}_{ijc} = (\bar{P}_{ij})_o + \left(\frac{\partial \bar{P}_{ij}}{\partial \bar{A}_1}\right)_o \Delta\bar{A}_1 + \left(\frac{\partial \bar{P}_{ij}}{\partial \bar{A}_2}\right)_o \Delta\bar{A}_2 + \dots + \left(\frac{\partial \bar{P}_{ij}}{\partial \bar{A}_n}\right)_o \Delta\bar{A}_n \qquad (19)$$

where: $\bar{P} = \dfrac{P}{P_{nom}}$ - is the relative parameter value as compared to the nominal duty: subscript "c" indicates the calculated value.

The deviation between the parameters measured in the different duties and those calculated on the basis of the model is:

$$P_{ijc} - P_{ijm} = \delta_{ij} \qquad (20)$$

Instead of the weighting of the measured parameter values, the weighting of the deviations can be carried out, and as a result of this, the weighted quadratic sum of the deviations is:

$$S = \sum_i \sum_j \left(\delta_{ij}\sqrt{g_i\ g_j}\right)^2 \qquad (21)$$

$$S = \sum_i \sum_j \Big[\Big(\left(\frac{\partial \bar{P}_{ij}^{*}}{\partial \bar{A}_1}\right)_o \Delta\bar{A}_1 + \left(\frac{\partial \bar{P}_{ij}^{*}}{\partial \bar{A}_2}\right)_o \Delta\bar{A}_2 + \dots + \left(\frac{\partial \bar{P}_{ij}^{*}}{\partial \bar{A}_n}\right)_o \Delta A_n + (P_{ijo} - P_{ijm})\sqrt{g_i\ g_j}\Big]^2 \qquad (22)$$

The condition of fulfilling the minimum is:

$$\frac{\partial S}{\partial \bar{A}_1} = 0;\ \frac{\partial S}{\partial \bar{A}_2} = 0 \dots;\ \frac{\partial S}{\partial \bar{A}_n} = 0 \qquad (23)$$

and from this set of linear equations, unknowns $\Delta\bar{A}_1 \dots \Delta\bar{A}_n$ can be calculated, and as a result of this, the new values of the quantities to be varied can be derived in the form:

$$(\bar{A}_n)_{k+1} = (\bar{A}_n)_k + (\Delta\bar{A}_n)_k \qquad (24)$$

where: k - is the serial number of approximation.

Similarly, the adaptation of the model for transient processes can also be performed with the exception that in this case subscript "j" belongs to the point of time of the measurement calculated from the initial steady-state duty as a different one.

As experiences show, the adaption can be performed successfully if the number of the factors to be varied as selected according to the contitions is smaller than the product of the number of the measurement duties and that of the measured parameters.

The scheme of algorhythms for the adaptation of the models is shown in Fig.6.

In Fig. 7, the convergence of the adaptation process is illustrated in the case of the combinations of the numbers for different measured parameters and duties.

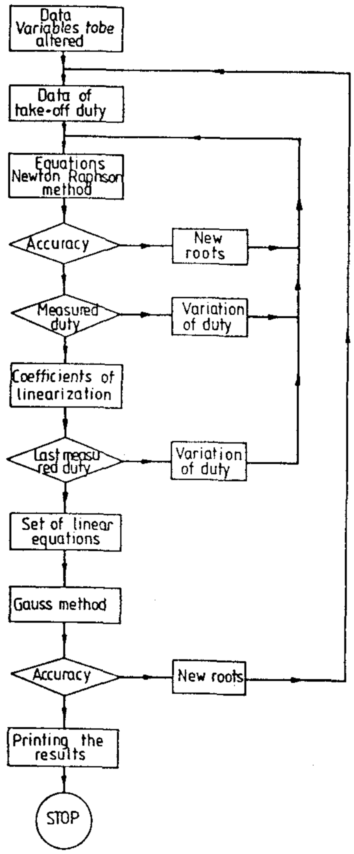

Fig. 6 Scheme of algorhythms for adaptation processes

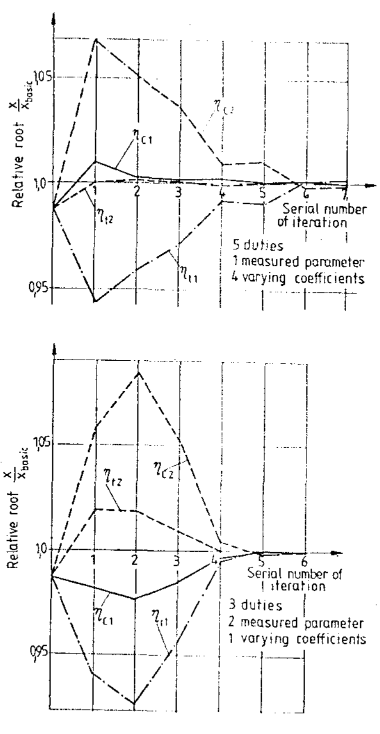

Fig. 7 Convergence of adaptation processes in case of combination of numbers for different measured parameters and duties

Applicability of models in operation

The adapted model is suitable for the examination of the processes taking place within the jet-plant, and for the determination of the values of parameters not to be measured. The analysis of the jet-engine behaviour can be implemented under different simulated conditions to detect the parameter- sensibility in steady-state and transient duties. These examinations show that the sensibility of the measured or potentially measurable parameters of the jet-engine varies with the single characteristics (efficiency, cross-sections, losses stc.) (Fig.8).

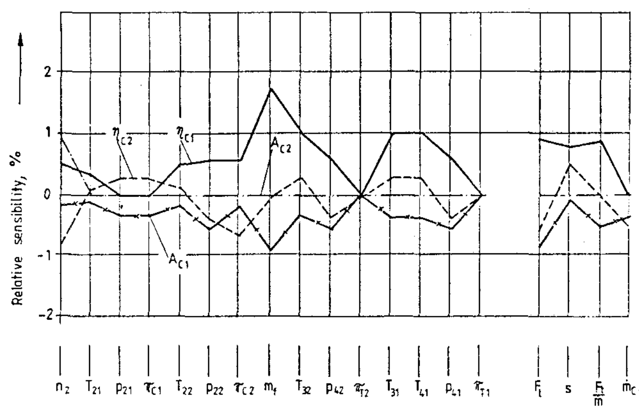

Fig. 8 Effect of 1 % reduction of compressor characteristics exerted on the parameters of two-spool turbojet.

In the case of identical parameters and characteristics, the sensibility varies in different steady-state duties (Fig.9), however it will be different with the transient processes, too (Fig. 10).

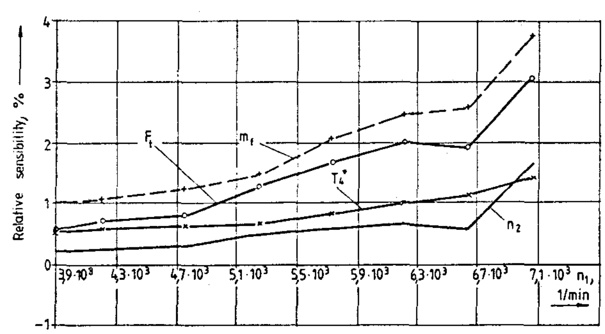

Fig. 9 Effect of 1 % reduction of high -pressure turbine efficiency exerted on the parameters of two-spool turbofan in different duties

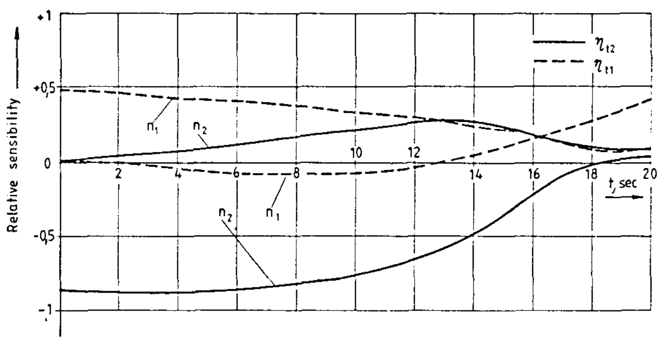

Fig.10 Effect of 1 % reduction of turbine efficiencies exerted on the parameters of two-spool turbojet during free run-out processes

These latter circumstances show that if the determined conditions are fulfilled in diagnostics there is a possibility of measuring less parameters in more duties instead of measuring more parameters in a single duty, or obtaining more pieces of information from less measurement points.

Inasmuch, the undamaged model adapted before is adapted again to the new measurement results of the damaged jet-engine, then the cause of the damage can be localized more accurately. In this case, the factor characteristic of the damage should be found beyond any doubt among the factors to be varied as selected on the basis of the conditions detailed above.

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