MATHEMATICAL MODELING OF OPTIMAL PASSIVE CONTROL OF ROTOR HEAD VIBRATIONS

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Abstract

The procedure of synthesis of complete dynamic model of helicopter structure including rotor head and corresponding passive controler in a first part of the paper is presented. At the end of this part of the paper the nonreduced form of the dynamics of mentioned system in a decoupled form on a two subvector of system generalised coordinates is presented. The first one is a subvector of generalised coordinates ccresponding to a non-conected nodes of elastic displacements to a rotor head as a superelement of the helicopter structure. The second subvector consists of generalised coordinates of elastic structure displacements ccresponding to a nodes which are conected whith rotor head and passive controler together. The coresponding structural viscous damping of elastic conections between rotor head and helicopter structure and between rotor head and its passive controler are included in the model also. A procedure of synthesis representing dynamic model of the system which consists of eqvivalent transformed generalised coordinates of the mentioned second system subvector is presented in the second part of the paper.

The numerical results of synthesis optimal absorber of rotor head vibrations depending on its viscous structural damping is presented in a last part of the paper. Equivalent absorber efectivnees is defined as a amplitude ratio between amplitudes with and without absorber. Absorber efectivnees depending on generalised factor of viscous structural damping of absorber beam and viscous structural damping of rotor head conections to the helicopter structure is presented on the diagram.

Introduction

Helicopter structural vibrations are caused by the rotor oscilating resultant force which acts in vertical direction. This vibrations can be compensated on a several ways. One of them is to bilt a resonator mass on a rotor head. In this paper is not discused which of the possible methods of compensation structural vibrations is the most efective. The procedure and coresponding numerical method for absorber synthesis is presented in the paper only.

Absorber is one of the possible passive controlers for compensation helicopter structural vibrations. General performances of passive control systems are not of high qualities as they are in a cases of active control systems of vibrations. In this paper a original powerfull technics for numerical synthesis of system absorber is presented. This procedure is based on a given results of several calculated examples of numerical absorber synthesis.

Complete investigations presented in this paper are based on a linear theory for the case of small elastic displacements of whole helicopter structure including absorber. Any nonlinear efects in this paper are not threated.

Nondimensional factors and parameters

Total mass of helicopter fuselage, mass of rotor with gear-box and frequency of rotors blade rotation are the input constants for numerical simulation of the absorber synthesis problem.

Total absorber mass is given in a form

$$M_a = K_a \frac{\mu^2}{\Omega^2} \tag{1}$$

where K_a is a eqvivalent vrtical translation stifnees, Ω is the frequency of the resultant rotor lift force and μ is a nondimensional factor of absorber synhronisation.

Factor of viscous damping is defined in a following form

$$\delta_a = \theta_a \sqrt{M_a K_a} \tag{2}$$

where θ_a is a nondimensional parameter of absorber damping.

Factor of viscous damping of structure of rotor and transmision arm to the helicopter structure is given in a form

$$\delta_o = \theta_o \sqrt{M_r K_o} \tag{3}$$

where θ_o is a nondimensional parameter of damping, M_r is a total mass of rotor and transmision together and K_o is eqvivalent stifnees of transmision arm.

The measure of viscous damping of absorber beam is equal

$$\lambda = \frac{\theta_a}{\mu} \Omega \tag{4}$$

The measure of reduced frequency of absorber can be defined in a form

$$\omega_{red} = \frac{\sqrt{1 - \theta_a^2}}{\mu} \tag{5}$$

The next parameter of interest can be presented as a ratio between equivalent stifnees in a form

$$\beta = \frac{K_a}{K_o} \tag{6}$$

This parameter defines the efectivnees of absorber just because of its influence on a total mass of a absorber.

Mathematical modeling of system dynamics with absorber

In a paper |1| mathematical model of absorber is simplified as model of translation two rigid bodies coresponding to the helicopter fuselage and rotor with gear-box, conecteed to a third one as a absorber, including viscous damping between them. In a practise this aproximation is not so usable because the real system is too complicated.

In this paper is presented the methcd of synthesis of complete finite element model of the system dynamics. Evaluated mathematical model is used for numerical calculations of system amplitudes. On the given diagrams numerical results of amplitudes calculations for the various hipothetical examples are presented. It must be noted that the given model of system dynamics is cleaned of any parameter which is not so efective on a system dynamics.

The system is described as a system of three bodies multipally conected with elastic and viscous damped conections under the action of frequent external forces and moments. Complete dynamic model of helicopter structure together with absorber can be described by using finite element method. It is a system of five superelements coresponding to the fuselage, gear-box arm, reduction gear-box and rotor together, elastic beam of absorber and absorber only. Each of the vectors of generalised displacement coordinates for coresponding superelements consists of two subvectors. First one is a subvector of generalised coordinates of elastic displacements corespondi-ng to the internal nodes. The second one is a subvector of generalised cocrdinates of elastic displacements of coupling nodes between conected super-elements. Mentioned subvector can be written in a folowing vector form:

- $X_f=\{X_f^i,X_f^c\}^T$ - subvector of generalised coordinates of elastic displacements of fuselage;
- $X_{agb}=\{X_f^c,X_{agb}^i,X_{rf}^c\}^T$ - subvector og generalised coordinates of elastic displacements of gear-box arm
- $X_r=\{X_{rf}^c,X_r^i,X_{rb}^c\}^T$ - subvector of generalised coordinates of elastic displacements of reductor gear-box together withrotor;
- $X_{aab}=\{X_{rb}^c,X_{aab}^i,X_{ab}^c\}^T$ - subvector of generalised coordinates of elastic displacements of absorber elastic beam;
- $X_{ab}=\{X_{ab}^c,X_{ab}^i\}^T$ - subvector of generalised coordinates of elastic displacements of absorber.

Complete vector of system elastic displacements can be written in a subvector form as

$$X=\{X_f^i,X_f^c,X_{agb}^i,X_{rf}^c,X_r^i,X_{rb}^c,X_{aab}^i,X_{ab}^c,X_{ab}^i\}^T \tag{7}$$

X_f^i - subvector of internal fuselage generalised coordinates;

X_f^c - subvector of coupling generalised coordinates between fuselage and arm of reduction gear-box;

X_{agb}^i - subvector of internal arm generalised coordinates;

X_{rf}^i - subvector of coupling generalised coordinates between reduction gear-box and its arm;

X_r^i - subvector of internal generalised coordinates of reduction gear-box and rotor together;

X_{rb}^c - subvector of coupling generalised coordinates between reduction gear-box and absorber beam;

X_{aab}^i - subvector of internal generalised coordinates of absorber beam;

X_{ab}^c - subvector of coupling generalised coordinates between absorber and its beam;

X_{ab}^i - subvector of internal generalised coordinates of absorber only.

Mathematical model of system dynamics can be presented in a next matrix form

$$\bar{M}\ddot{X} + 2\bar{D}\dot{X} + \bar{K}X = \bar{F} \tag{8}$$

where are $\bar{M}$ - matrix of generalised mases, $\bar{D}$ - matrix of generalised dampings and $\bar{K}$ - matrix of generalised stifnees. $\bar{F}$ is a generalised vector of external forces. The map of these system matrices of zero (0) and nonzero (X) coeficients can be presented in the following submatrix form

$$\bar{M},\bar{D},\bar{K} \rightarrow \begin{bmatrix} x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & x & x & x & x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x & x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & x & x & x & x & x & 0 \\ 0 & 0 & 0 & 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & 0 & 0 & x & x & x & x \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x \end{bmatrix}$$

First six subvectors (including subvector X_{rb}^c as a last one) in a given serie written by relation (7), defines the state of the system without absorber. Given form of the system matrices $\bar{M}$,$\bar{D}$ and $\bar{K}$ is a complete one including all generalised coordinates of interest. The next questions may be of interest to us as:

- what are the possibilities of dynamic

reducing the complete dynamic model of the system by using method of superelements?

- what are the usable aproximations of the complete system which makes simplification in procedure of absorber synthesis?

The answer on a first question can be positive if exists next asumptions:

- the eigenvalue domain of the system dynamics without absorber must be well separated of the coresponding eigenvalue domain of the aditional absorber system. This assumption guaranties that there are no resonance efects caused by the external periodic rotor lift forces;

- the coresponding eigen values for the each substructure component as a decoupled dynamic system must be also well separated in the seam meaning as it is given before. In that case the procedure of system reduction will be so efective. Complete procedure of reducing for this case will be presented in the following part of the paper.

The second question needs a numerical analysis of the full problem. As we can see on a given diagrams, the following conclusions can be derived:

- influence of generalised structural viscous damping of the system without absorber on a system motion is too less than the influence of generalised damping of absrober (the main roll acts the generalised structural damping of the absorber elastic beam). It means that this system damping without absorber can be neglected in a procedure of absorber synthesis;

- generalised mases of absorber elastic beam can be also neglected. The total absorber mass is equal aproximatelly ten percents of total mass of reduction gearbox and rotor together. The efects which can be given are about ten times less amplitude of fuselage elastic displacements.

Procedure of system reduction

Complete dynamic model of the system without absorber consists of six subvectors of generalised coordinates. It can be reduced on a three subvectors only. Second subvector of the fuselage dynamic and coupling subvector between reductor gear-box and its arm and between reductor gear box and elastic absorber beam makes the vector of reduced system state without absorber. As it is shown in a paper |2| we can asume the folowing linear combinations in a matrix form between system coordinates

$$X_{h2} = -Q \cdot X_{h1} \tag{9}$$

where are

$$X_{h1} = \{X_f^c, X_{rf}^c, X_{rb}^c\}^T \text{ and } X_{h2} = \{X_f^i, X_{agb}^i, X_r^i\}^T \tag{10}$$

Transformation matrix Q of system generalised coordinates can be derived from the coresponding matrix algebraic Ricatti-equation |2| . It means that reduced system is partly rigid body. Its state is defined by the eigen-vector of the reduced system which is determined by using transformation matrix of generalised coordinates Q.

Consider hypotetical dynamic system in a diferential matrix form

$$M\ddot{X}_h + 2D\dot{X}_h + KX_h = F_h \tag{11}$$

If we introduce the change of coordinates in a form

$$\dot{X}_h = Y_h \tag{12}$$

we gives the system in a first order form

$$\begin{vmatrix} \dot{Y}_h \\ \dot{X}_h \end{vmatrix} = \begin{bmatrix} -2M^{-1}D & -M^{-1}K \\ I & 0 \end{bmatrix} \begin{vmatrix} Y_h \\ X_h \end{vmatrix} + \begin{vmatrix} M^{-1}F_h \\ 0 \end{vmatrix} \tag{13}$$

This system can be reduced at first step on a next form

$$\dot{Y}_h = (-2M^{-1}D + M^{-1}KL)Y_h + M^{-1}F_h \tag{14}$$

where is

$$X_h = -L \cdot Y_h \tag{15}$$

Matrix L is a solution of the next matrix algebraic Ricatti equation

$$KL - 2D + ML^{-1} = 0 \tag{16}$$

The next step is to used the same procedure for reducing the system given in form (14). We can transform coordinates of subvector Y_h in a form

$$Y_{h2} = -M \cdot Y_{h1} \tag{17}$$

where M is a solution of coresponding matrix algebraic Ricatti equation of the system (14). Linear combinations between initial generalised coordinates of the system (11) is given in a matrix form presented by relation (9). Transformation matrix Q is given in a form

$$Q = -(L_{22}M - L_{21}) \cdot (L_{12}M - L_{11})^{-1} \tag{18}$$

where matrices $L_{11}, \ldots, L_{22}$ are the submatrices of the matrix L.

Numerical results of system dynamics simulation

On the folowing diagrams in this paper the numerical results of system amplitudes ratio for the amplitude input with constant frequency are shown.

Presented results are given for the cases of variation of synhronisation parameter and parameters of viscous structural damping of reductor arm and absorber beam. Mentioned parameters where defined before.

Presented numerical results are given for hypotethical example with the following values of system parameters:

- total mass of reductor gear-box with rotor : M_r = 700 kg
- total mass of helicopter fuselage: M_f = 5700 kg
- equivalent vertical stifnees of reductor arm : $K_c = 3 \cdot 10^7$ N/m
- equivalent vertical stifnees of absorber beam : $K_a = 1\ 10^6$ N/m
- frequency of input force: $\Omega = 100\ s^{-1}$

Equivalent vertical stifnees is calculated for the assumed hypothetical example.

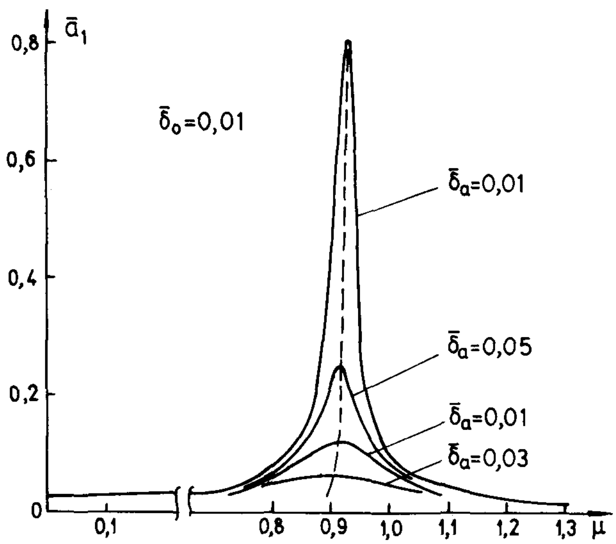

Fig. 1

On a diagrams (Fig. 1 and 2) the curves of absorber amplitude ratio output in a vertical direction depending on a value of synhronisation parameter for various values of viscous damping parameter of reductor arm and absorber beam.are presented, The sensitivity of the amplitude ratio gain is too higher for the variations of viscous damping parameter of absorber beam then reductor arm. This results shows that we can neglect the influence of structural viscous damping in a problem of absorber synthesis.

We can see also that the maximum amplitude ratio not corespconds the theoretical value of synhronisation parameter (equal 1). The curve which conects the points of maximum amplitude ratio is a little bit on the left side. For the higher values of the viscous structural damping parameter of absorber beam this curve decreased to a smaller values of synhronisation parameter (in example the value of synhronisation parameter tends to 0,9). This conclusion gives the possibilities to reduced the amplitudes ratio of absorber displacements asuming the values of synhronisation parameter greater than the coresponding one for its maximum ratio without any losses in absorber efects.

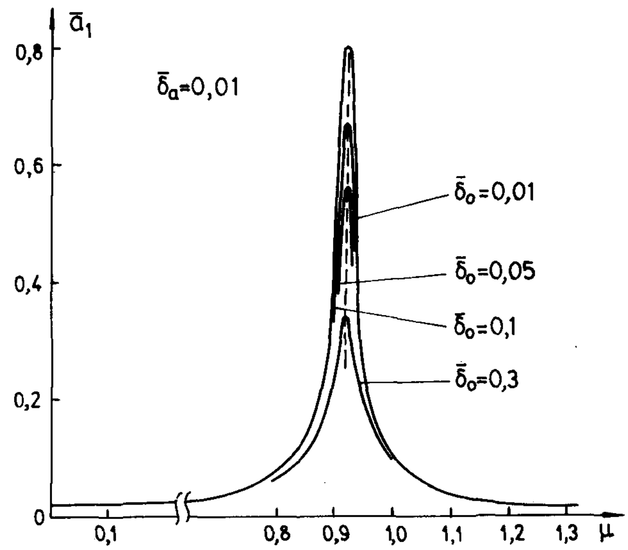

Fig. 2

On a diagrams (Fig. 3 and 4) the curves of rotor head amplitude output in vertical direction are presented.

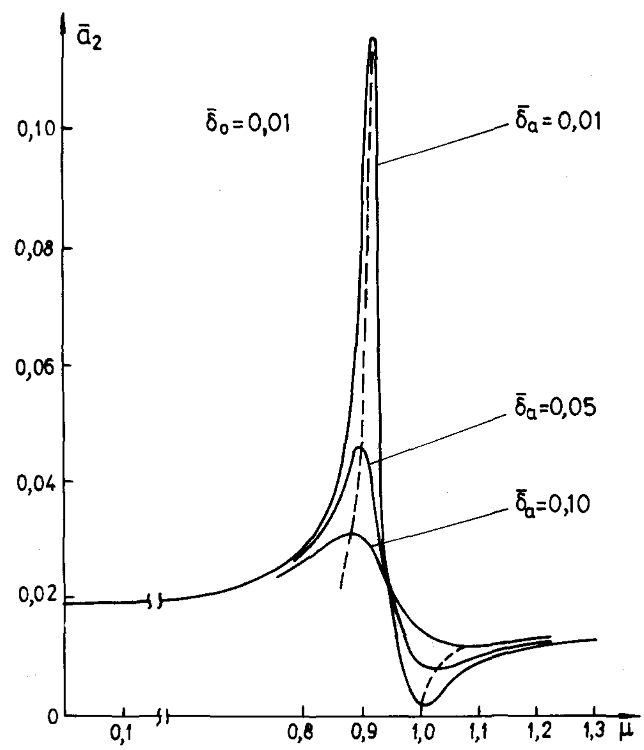

Fig. 3

On a diagrams (Fig. 3) are presented curves depending on viscous damping parameter of absorber beam. Sensitivity of absorber apsorption efectivnees depending on viscous damping parameter is very high. On a diagrams (Fig. 4) it is shown that the sensitivity of apsorption efectivnees depending on viscous damping parameter of reductor arm is very low, specially in a domain around value of synhronisation parameter equal 1. This fact shows that the influence of viscous damping of reductor arm is practicaly negligable The most efective apsorption coresponds to the value of synhronisation parameter equal 1.

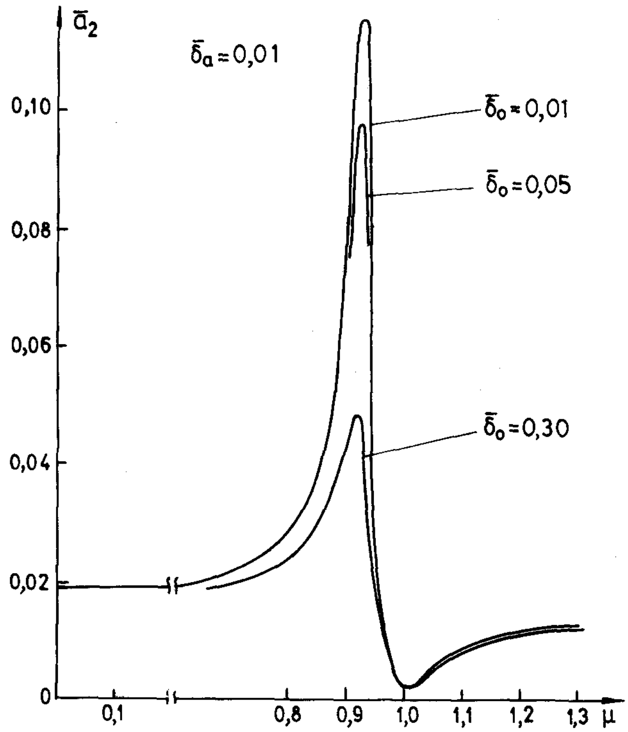

Fig. 4

On a diagrams (Fig. 5 and 6) the curves of fuselage amplitude output ratio in a vertical direction are presented. Efects of system vibrations apsorption are too much better for a lower values of viscous damping parameter of absorber beam. The influence of viscous structural damping of reductor arm can be neglected in a practical calculations. Maximum efect of vibrations apsortion ccresponds to a value of synhronisation parameter equal 1. Total efects are not too bad for the values of synhronisation parameter between 1,0 and 1,05. In a presented example the amplitude ratio is about ten times less then the ccreposnding value for the system without absorber.

The very bad efect is a high degree of system sensitivity on viscous damping parameter of absorber.

On a diagrams (Fig. 7 and 8) the numerical results of simulation of some other hypotetical examples of absorber synthesis are presented. The coresponding curves of amplitude system output ratio can be very diferent than the usual ones, derived in the first example on a diagrams (Fig. 1 up to Fig. 6). Curves number 1 and 3 are the usual ones, but the curves number 2, 4 and 4`are so diferent than the first ones (curves 4 and 4`are for the same example).

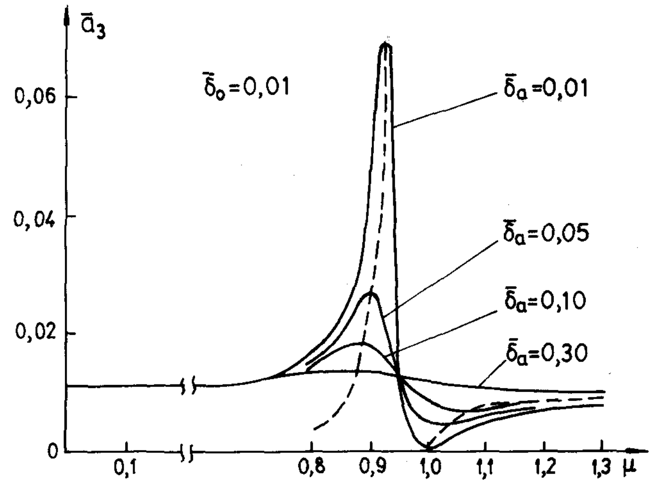

Fig. 5

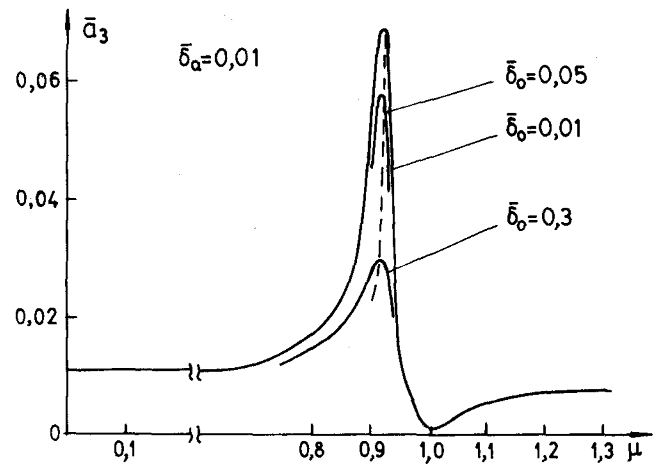

Fig. 6

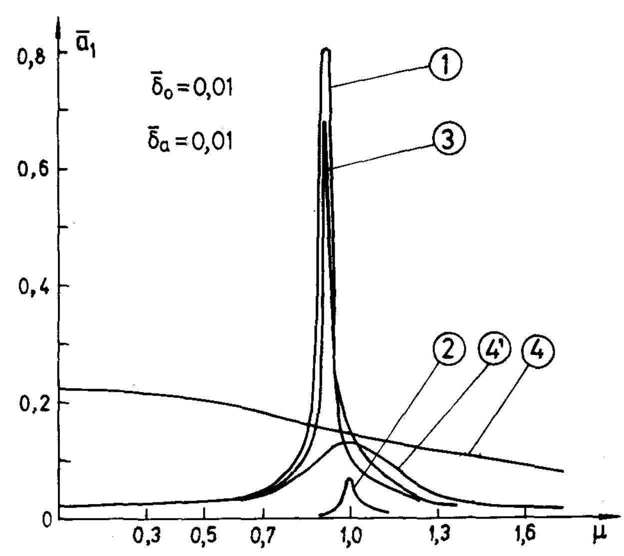

Fig. 7.

The main reason of these efects is a high degree of viscous damping in these cases. Total value of viscous damping coeficients can be derived by multiplication viscous damping parameter of absorber beam with equivalent helicopter structure stifnees. In this cases equivalent structure stifnees is greater than for the first two cases. The same efects we gives for the high values of viscous damping parameter of absorber beam. It is shown on a diagrams (Fig. 5) (curve for $\bar{\delta}_a = 0{,}3$). These efects acts when the curves of the maximum and minimum amplitude ratio are too separated.

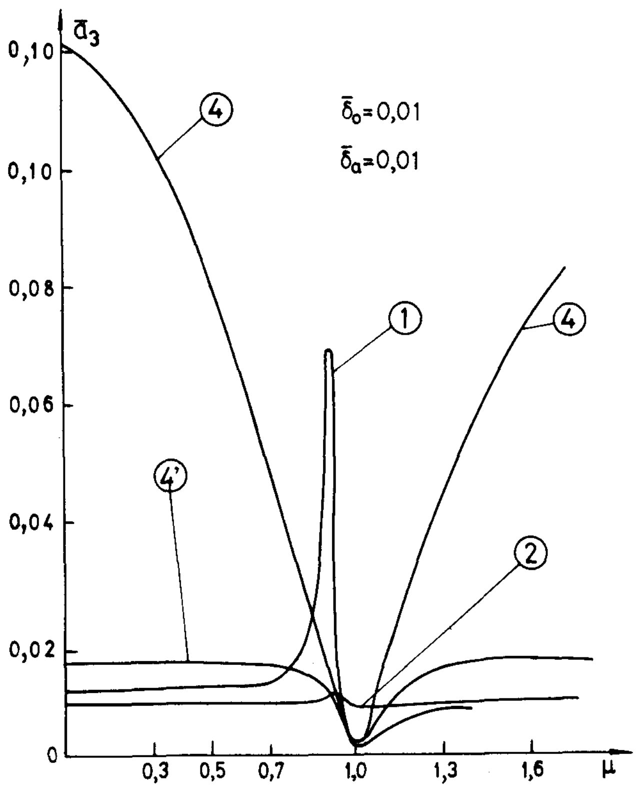

Fig. 8

<u>Reduced mathematical model of the system dynamics with absorber</u>

Dynamic model of the absorber together with its elastic beam can be reduced by using following aproximation:

$$X_{a2} = T \cdot X_{a1} \tag{19}$$

where are

$$X_{a1} = \{X^{c}_{rb}, X^{c}_{ab}\}^T \quad ; \quad X_{a2} = \{X^{i}_{aab}, X^{i}_{ab}\}^T \tag{20}$$

Procedure of its reducing and determination of transformation matrix T is presented in a part of this paper before (like for the matrix Q).

Before system reducing it is convenient to neglect submatrix coresponding to a viscous structural damping of the system components except absorber beam. In that case dynamic model of the system without absorber can be described aproximately in a form:

$$M_h \ddot{X}_h + K_h X_h = F_h ; \quad X_h = \{X_{h1}, X_{h2}\}^T \tag{21}$$

After transformation of generalised coordinates in a form (9) we gives the reduced system in a form

$$M'_h \ddot{X}_{h1} + K'_h X_{h1} = F'_h \tag{22}$$

where are

$$\begin{aligned} M'_h &= M_{h11} + M_{h12} \cdot Q \\ K'_h &= K_{h11} + K_{h12} \cdot Q \\ F'_h &= F_{h1} \end{aligned} \tag{23}$$

Dynamic model of absorber with elastic beam together can be written in a following matrix form

$$M_a \ddot{X}_a + 2D_a \dot{X}_a + K_a X_a = 0 \tag{24}$$

where are

$$X_a = \{X_{a1}, X_{a2}\}^T$$

After transformation (19) of generalised coordinates we gives the reduced form of the system (24) in a form

$$M'_a \ddot{X}_{a1} + 2D'_a \dot{X}_{a1} + K'_a X_{a1} = 0 \tag{25}$$

where are

$$\begin{aligned} M'_a &= M_{a11} + M_{a12} \cdot T \\ D'_a &= D_{a11} + D_{a12} \cdot T \\ K'_a &= K_{a11} + K_{a12} \cdot T \end{aligned} \tag{26}$$

Finaly, the complete reduced form of the system dynamic model can be written in a matrix form

$$\begin{aligned} M'_h \ddot{X}_{h1} + K'_h X_{h1} &= F'_h \\ M'_a \ddot{X}_{a1} + 2D'_a \dot{X}_{a1} + K'_a X_{a1} &= 0 \end{aligned} \tag{27}$$

or in integral form

$$M \ddot{X} + 2D \dot{X} + K X = F \tag{28}$$

where is

$$X = \{X^{c}_{f}, X^{c}_{rf}, X^{c}_{rb}, X^{c}_{ab}\}^T \tag{29}$$

Structure of the system matrix coeficients can be maped in a form

$$M,D,K \rightarrow \left[\begin{array}{ccc:c} x & x & x & 0 \\ x & x & x & 0 \\ x & x & x & x \\ 0 & 0 & x & x \end{array}\right]$$

<u>Conclusion</u>

Numerical results of calculation of system amplitudes ratio shows that the limit value of parameter β must be less than 0,05. In the case that parameter β is up to 0,1 the efectivnees of absorber is not to high. If the parameter β is less than 0,01 the absorber efectivnees will be very bad. Optimal results are given for the case in which parameter β is equal 0,03. This value of β gives the value of ratio between absorber and total rotor head mass equal 0,10 ÷ 0,14. In this case the resonant caracter of absorber dynamics is very clear and the procedure of its numerical calculation is very stable. In other cases calculations are very sensitive to a value of factor of absorber viscous damping. These efects are shown on a presented diagrams.

References

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