AN ANALYSIS OF REDUCED ORDER SYSTEM FOR AIRPLANE GUST ALLEVIATION

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Abstract

The equations of motion describing a gust alleviation system are usually of high order. An optimal LQG control law would be of the same order as the system. This control law is sensitive to modeling errors, has poor stability margins, and is too complex to implement in a flight computer.

In this paper, a method is presented using optimization techniques for designing a reduced-order controller to minimize a performance index defined by a weighted sum of mean-square responses and control inputs. A truncated system is composed of key states of the original full-order system. The optimal feedback matrix and Kalman estimator gain matrix of the truncated system are chosen as the initial values of the controller design variables.

This method was applied to the synthesis of a gust alleviation controller for a model of a transport airplane. The responses of the airplane using the reduced-order controller are compared with the responses using the full-order controller.

Nomenclature

B_c	controller input matrix
B_g	input matrix of gust model
B_m	input matrix in equation (1)
B_{s1}	input matrix in equation (3)
B_{s2}	input matrix in equation (3)
C_c	controller measurement matrix
C_g	measurement matrix of gust model
C_m	measurement matrix in equation (2)
C_s	measurement matrix in equation (4)
F_c	controller dynamics matrix
F_g	dynamics matrix of gust model
F_m	dynamics matrix in equation (1)
F_s	dynamics matrix in equation (3)
J	performance index
Q	design output weighting matrix
R	design input weighting matrix
R_v	measurement noise intensity matrix
R_w	system noise intensity matrix
u	control input vector
u_m	input vector in equation (1)
v	measurement noise vector
w	system noise vector
w_g	input vector of gust model
x_c	controller state vector
x_g	state vector of gust model
x_m	state vector in equations (1),(2)
x_s	state vector in equations (3),(4)
y	measurement output vector
y_g	output vector of gust model

Introduction

The equations of motion describing a gust alleviation system of an airplane are usually of high order. It is particularly true for a system which represents the rigid and elastic motions, unsteady aerodynamics, and actuator dynamics. An optimal LQG control law would be of the same high order as the system. System using observer-based control law always exhibit poor stability margins, are sensitive to modeling errors, and made the implementation impractical in a flight computer.

In this paper, a reduced-order control law is used to reduce the cost of implementation. The design variables of the reduced-order controller can be searched to minimize a performance index defined by a weighted sum of mean-square responses and control inputs. The basis of this approach is described in rererences (1) and (2). The design variables of the reduced-order controller are chosen that they are the optimal feedback and Kalman estimator gain matrices, that are similar to reference (3). But these matrices are obtained from the truncated system of the full-order system in this paper. The truncation is based on the control objectives of gust alleviation. To increase the stability margins of the system, a fictitious noise is introduced at the inputs.[4]

This method is applied to the synthesis of a fourth-order gust alleviation control law for a model of a transport airplane. The responses of the airplane using the fourth-order control law are compared with the responses using the full-order (25th-order) control law.

Equations of Motion

The equations of motion are presented using the conventional modal approach. The motion of the airplane is described by a linear combination of its rigid and elastic modes. The forcing functions to the system are both control and gust inputs. Linear equations of longitudinal motion of an airplane that approximate the rigid modes are used in this study.[5] The equations for the flexible motion are written in linear combination of normal modes of vibration. The equations of motion and an accelerometer sensor output can be expressed as

$$\dot{x}_m = F_m x_m + B_m u_m \qquad (1)$$

$$y = C_m x_m + D_m u_m \qquad (2)$$

where F_m, B_m, C_m, and D_m are dynamics, input, and measurement matrices respectively, and x_m, u_m, and y are state, input, and output vectors respectively.

The main difficulty in the design of a control system is related to the presence of the unsteay aerodynamic forces. The unsteady aerodynamic matrix can be regarded as a transfer function, which contains a number of hidden states.

When a model of actuator dynamics and a model of gust turbulence are introduced, the state-spsce equations can be written as

$$\dot{x}_s = F_s x_s + B_{s1} u + B_{s2} w \qquad (3)$$

$$y = C_s x_s + v \qquad (4)$$

where x_s, u,and y are state vector of order N, input vector of order N_c, and measurement vector of order N_o respectively, w and v are system noise and measurement noise vectors respectively. Where w and v are modeled as zero-mean white noise processes with intensity matrices R_w and R_v respectively.

The reduced-order controller model is assumed to be [1],[2]

$$\dot{x}_c = F_c x_c + B_c y \qquad (5)$$

$$u = C_c x_c \qquad (6)$$

where x_c represents the controller state of order M, and $M \leq N$. F_c, B_c, and C_c are matrices of size $M \times M$, $M \times N_o$, and $N_c \times M$, respectively. Out of the $M(M+N_o+N_c)$ elements of the F_c, B_c, and C_c matrices, only $M(N_o+N_c)$ elements are independent, [6] and can be chosen as design variables.

Control law Synthesis

By augmenting the state vector, the gust alleviation system is represented as

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} F_s & B_s C_c \\ B_c C_s & F_c \end{bmatrix} \begin{bmatrix} x_s \\ x_c \end{bmatrix} + \begin{bmatrix} B_{s2} & 0 \\ 0 & B_c \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix} \quad (7)$$

The control law is synthesized using a nonlinear programing algorithm to search for the design variables which minimize a performance index J defined by a weighted sum of mean-square responses and control inputs.

$$J = \lim_{t \to \infty} E \left[y^T Q y + u^T R u \right] \quad (8)$$

$$= \mathrm{tr} \left[(C_s^T Q C_s) X_s \right] + \mathrm{tr} \left[RU \right]$$

where Q and R are symmetric weighting matrices and X_s and U are steady-state covariance matrices of the state x_s and input u, respectively. The gradients of J with respect to the elements of matrices F_c, B_c, and C_c are described in reference (3).

In this paper, the $M(N_o+N_c)$ elements of B_c and C_c are chosen as the design variables, that is the same as in reference (3).

The objective of model reduction is to simplify the control laws while preserving the significant dynamic properties relative to the control tasks. In a natural gust field, the low frequency gusts are dominant. Gusts of higher frequency only appear with small amplitude, as can be seen by the well-known Von Karman gust spectra. In analysis of gust response, fast dynamics are not significant with respect to the control task. The high-frequency modes are generally not modeled accurately and need not be fed back to meet the control objectives. The modes are ordered so so that the low frequency modes are retained in a truncated system.

The order of the truncated system is assumed as the same order of the reduced-order controller. The optimal feedback and Kalman estimator gain matrices of the truncated system are chosen as the initial values of controller design variables in this study. If a full-order controller is assumed, let $F_c=F_s-B_cC_s+B_{s1}C_c$, then it can be shown that the optimized values of B_c and C_c are identical to the Karman estimator and optimal full-state feedback gain matrices, respectively. When M<N, the reduced-order controller can be regarded as the Kalman estimator and optimal full-order feedback gain matrices of the truncated system which contains M key states for gust response. Thus we set

$$F_c = F_s - B_c C_s + B_{s1} C_c \quad (9)$$

where B_c and C_c are the Kalman estimator and optimal feedback gain matrices of the truncated system respectively.

Since F_c is a function of B_c and C_c, the total gradients of performance index with respect to B_c and C_c are given by

$$\frac{dJ}{dB_c} = \frac{\partial J}{\partial B_c} - \frac{\partial J}{\partial F_c} C_s^T \quad (10)$$

$$\frac{dJ}{dC_c} = \frac{\partial J}{\partial C_c} + B_{s1}^T \frac{\partial J}{\partial F_c} \quad (11)$$

a conjugate gradient optimization procedure can be used to minimize the performance index J.

In order to increase the stability margins of the system, it has been shown [4] that by introducing a fictitious input-noise, the stability margins of the full-order control laws can be improved. This procedure is also used in reduced-order controller design process. Thus the input-noise intensity matrix R_u becomes a design parameter in this design process.

Application

This synthesis method is applied to gust alleviation of a transport airplane model. Both rigid and elastic motion are included.

The power spectrum of turbulence is not flat over the range of mode frequencies. Thus it is necessary to include a model of the turbulence disturbance. This model has white noise as inputs and time-correlated gust velocities as outputs. The gust disturbance state model which approximate Von Karman's spectrum as a rational function is

$$\dot{x}_g = F_g x_g + B_g w_g \tag{12}$$

$$y_g = C_g x_g \tag{13}$$

where x_g is the state vector of gust model, y_g is the vector of time-correlated gust velocities, and w_g is a vector of zero-mean Gaussian white-noise sources with the spectral density matrix W_g.

The equations of motion of full-order system are derived using two rigid modes and three elastic modes. The resulting airplane model is a system of order 25. The input u is the actuator command signals and the output y is the accelerometer signals.

The weighting matrices are selected to be

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \quad R = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

and

$$R_v = \begin{bmatrix} 0.0005 & 0 \\ 0 & 0.0005 \end{bmatrix} \quad R_w = 2.0$$

$$R_u = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$$

The truncated system is composed of four states representing two low modes. The fourth-order controller is based on estimating the fourth-order truncated system. The initial values of the gain matrices B_c and C_c are obtained from the truncated system results.

The gust responses of a transport airplane model at the center of gravity and the wing tip are given in Figs. 1 and 2. The control input of control surface on the wing are given in Fig. 3. The results are shown that the responses of the fourth-order controller very close to the responses of the full-order optimal feedback control law.

Conclusions

A method of synthesizing a reduced-order optimal feedback control law is presented for gust alleviation of an airplane. The controller design variables are selected by employing a nonlinear programming algorithm to minimize a performance index defined by a weighted sum of mean-square responses and control inputs. The initial values of the design variables are chosen on the base of a truncated system, that are the Kalman estimator and optimal feedback gain matrices of the truncated system.

The gust alleviation system of a model of a transport airplane represented by a 25th-order system. A reduced fourth-order control law is synthesized, and the responses at the center of gravity and wing tip are compared with the full-order optimal feedback control law. The responses of fourth-order system are very close to the responses of 25th-order optimal control law.

References

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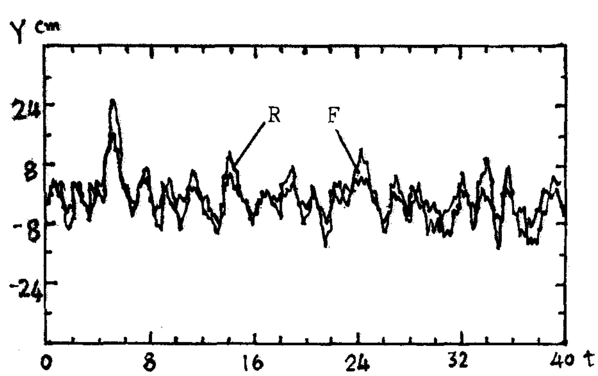

Fig. 1 The Displacements of 4th-Order Controller (R) and 25th-Order Controller (F) at Wing Tip.

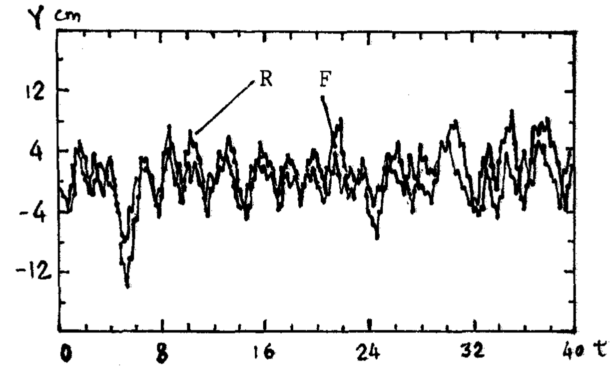

Fig. 2 The Displacements of 4th-Order Controller (R) and 25th-Order Controller (F) at The Center of Gravity.

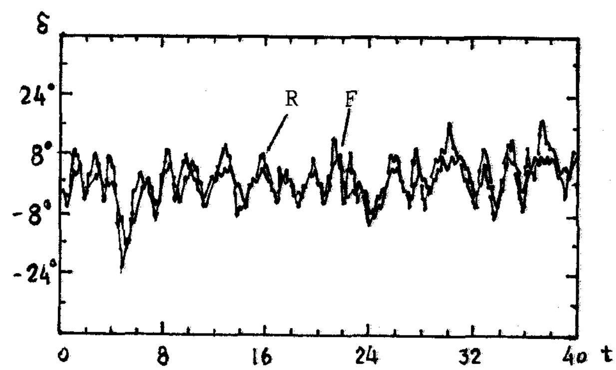

Fig. 3 The Inputs of 4th-Order Controller (R) and 25th-Order Controller (F) at Wing Tip.