A MODEL FOR PREDICTING DAMAGE INDUCED FATIGUE LIFE OF LAMINATED COMPOSITE STRUCTURAL COMPONENTS

by

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Abstract

This paper presents a model for predicting the life of laminated composite structural components subjected to fatigue induced microstructural damage. The model uses the concept of continuum damage mechanics, wherein the effects of microcracks are incorporated into a damage dependent lamination theory instead of treating each crack as an internal boundary. Internal variables are formulated to account for the effects of both matrix cracks and internal delaminations. Evolution laws for determining the damage variables as functions of ply stresses are proposed, and comparisons of predicted damage evolution are made to experiment. In addition, predicted stiffness losses, as well as ply stresses are shown as functions of damage state for a variety of stacking sequences.

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Introduction

Structural applications using composite materials continue to increase in the aerospace engineering community. This is primarily due to the fact that aerospace applications are driven by high specific strength accompanied by low mass. Although composites are expensive when compared to monolithic materials, their improved properties can make them cost effective alternatives in such applications as space structures, wherein the cost of launching a pound of mass is exorbitant.

Although composites possess many inviting material characteristics, there are nevertheless some shortcomings. Possibly the most significant one is the fact that most composites develop microstructural damage when subjected to long term fatigue conditions. This is due to the fact that combining two materials with dissimilar mechanical properties results in a multitude of stress singularities when external loads are applied. These tremendous gradients contribute to stable microcraking which continues to evolve throughout the life of the structural component, and, in many circumstances, the damage leads to ultimate fracture of the part and subsequent failure of the structure.

Experimentally, this evolution of damage is illustrated by the x-ray micrographs shown in

Figs. 1-4 [2], wherein successive damage states are shown for a coupon of graphite/epoxy $(0/90_2)_s$ composite subjected to fatigue loading. In the photos the coupons have been subjected to uniaxial loading in the vertical direction, leading to transverse matrix cracks in the 90° plies (horizontal lines), longitudinal splits in the 0° plies (vertical lines) and interply delaminations between the 0° and 90° plies (shaded grey areas). Typically, the matrix cracks induce stress concentrations which precede the growth of delaminations, as shown in Fig. 5. Furthermore, this damage is accompanied by a small loss in component axial stiffness, as shown in Fig. 6. It has been conclusively determined that the damage is stress induced, so that the damage state and resulting stiffness are strongly affected by load history. In addition, the growth of damage is dependent on stacking sequence, as illustrated in Fig. 7. Note that the delaminations tend to propagate normal to the loading direction in a $(0/90_3)_s$ laminate, as opposed to vertically in a $(0/90_2)_s$ laminate. It is hypothesized that these variations are due to differences in ply level stresses caused by changing the ply stacking sequence. In fact, this process can be considered to be in some sense beneficial because the integration of two separate brittle materials results in damage, which is a ductile-like phenomenon, thus causing load transferral not unlike that which occurs in metals.

The current procedure for predicting life in composites seems to be largely phenomenological in nature; that is, the method of analysis is in most cases dependent on component geometry, load history, and even stacking sequence and environmental conditions. Probably the most direct procedure utilizes ad hoc failure laws not unlike Miner's rule [2,3]. As a result, they apply only to a predetermined geometry and stacking sequence. Perhaps the most ambitious approach uses linear elastic finite elements and treats each crack as an internal boundary subject to growth determined by fracture mechanics criteria [4,5]. While this is desirable and probably the most accurate approach, it would require supercomputing capability for a typically complicated damage state.

For the past several years the authors have been developing an alternative model for predicting damage development in laminated composites. The model can be used for any

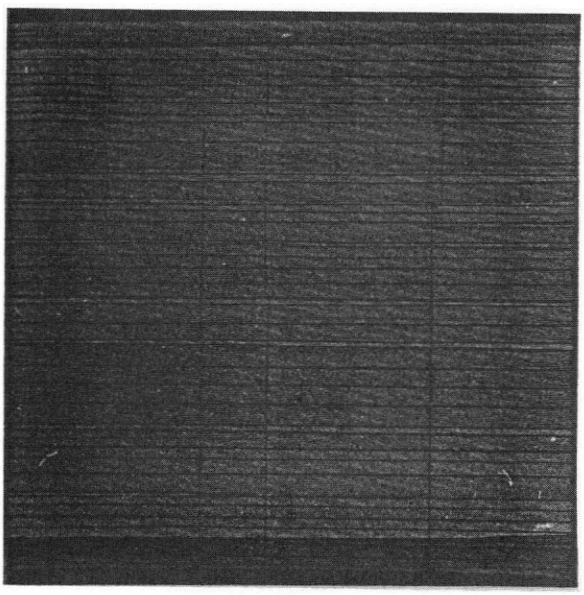

Fig. 1. Enlarge X-ray radiograph of #1 $(0/90_2)_s$ LVE, C=260,000, (f=2.0 Hz, R=0.1, $L_{max.}$=10.230 KN)

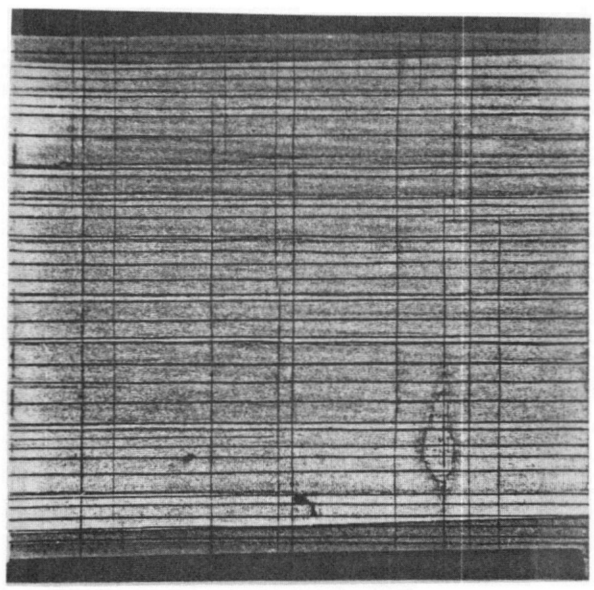

Fig. 2. Enlarged X-ray radiograph of #1 $(0/90_2)_s$ LVE, C=588,000, (f=2.0 Hz, R=0.1, $L_{max.}$=10.230 KN)

stacking sequence, using only initial linear elastic orthotropic ply properties. It has the capability to predict the effects of matrix cracks and delaminations on ply stresses. Therefore, it can be used to predict damage evolution as a function of load history such as that shown in Figs. 1-4. Furthermore, the model may be used to model the responses of beams and plates with stress gradients, so that it is not component specific. At the time of this writing the model has reached a fairly advanced state, although it would be premature to call the model mature at this point. This paper gives a brief review of the current state of development of the model. Although we do not pretend to suggest that the model is a usable design tool at this time, we are hopeful that the methodology suggested herein is moving in the right direction.

Model Development

The structure is modelled as a simply connected domain, with the effects of microcracking reflected by a set of internal state variables (ISV's) which enter the problem description via the constitutive equations. Thus, the necessity to model each crack with tens (or hundreds) of finite elements is obviated with little loss of accuracy and considerable computational savings. To this end, the approach is similar to self-consistent schemes [6] and global-local methods [7] utilized in other applications.

The necessary parts of the model are as follows: 1) a kinematic description of the damage state; 2) a damage dependent set of ply level stress-strain relations which account for matrix cracking; 3) a damage dependent lamination theory which models the effects of interply delaminations; 4) a set of damage evolution laws for predicting the load history dependence of the damage state at each material point; 5) a structural algorithm for modelling the response

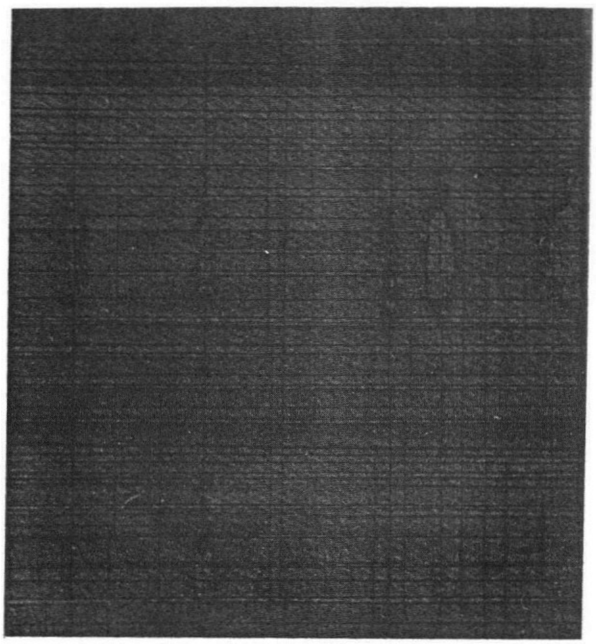

Fig. 3. Enlarged X-ray radiograph of #1 $(0/90_2)_s$ LVE, C=588,000, (f=2.0 Hz, R=0.1, $L_{max.}$=10.230 KN)

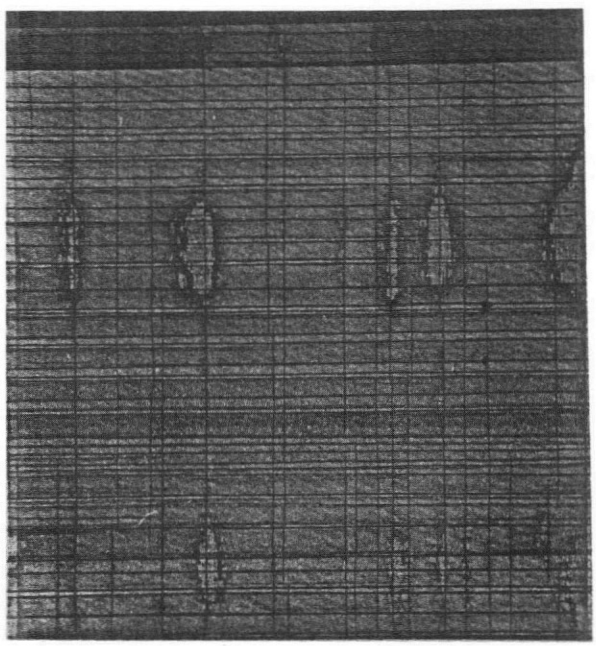

Fig. 4. The state of a delamination group in specimen #1 of $(0/90_2)_s$ laminate, C=1,030,000, (f=2.0 Hz, R=0.1, $L_{max.}$=10.230 KN)

of components with spatially variable and load history dependent stresses and damage; and 6) a failure function for predicting unstable crack growth leading to loss of component structural integrity. These parts are described briefly in the following subsections.

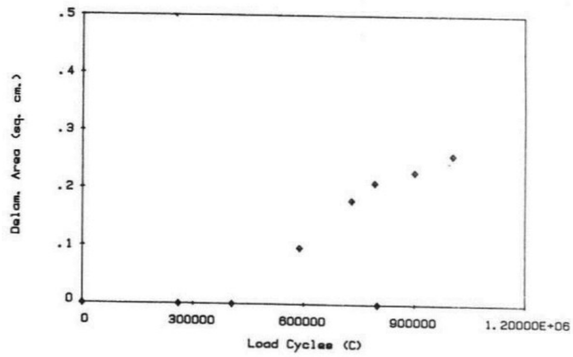

Fig. 5. Delamination Area vs Load Cycles Graph: $(0/90_2)_s$ Specimen

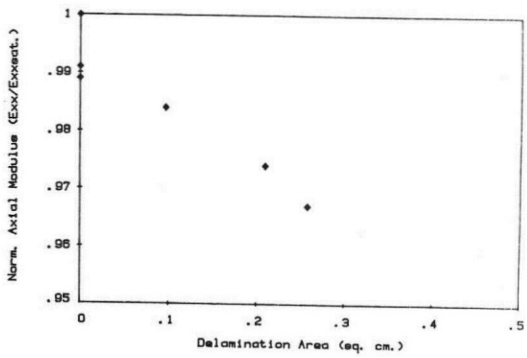

Fig. 6. Normalized axial Modulus vs Delamination Area Graph: $(0/90_2)_s$ Specimen

Kinematic Description of the Matrix Cracking

Since matrix cracks appear to be the first mode of damage to occur in all cases, it would seem that they should be modelled first. Typically, these are of a scale which is very small compared to the geometry of a structural component. Thus, it is hypothesized that a local volume element may be selected which is small compared to the boundary value problem of interest and, at least for the case of matrix cracking, the damage can be assumed to be statistically spatially homogeneous in this element. Modelling the effects of matrix cracks would then appear to be ideally suited to the approach taken in continuum damage mechanics. In this approach, first proposed by L.M. Kachanov in 1958 [8], it is hypothesized that the effect of microcracks may be locally averaged on a scale which is small compared to the scale of the structural component. Although the procedure has been extensively utilized in the literature, until recently it has not been applied to laminated orthotropic media [9-13].

A straightforward and direct approach to averaging the kinematic effects of cracks within the local volume element was taken by Vakulenko and M.L. Kachanov in 1971 [14]. This average is given by the following second order tensor:

$$\alpha^M_{ij} = \frac{1}{V_L}\int_{S_c} u^c_i n^c_j ds \qquad (1)$$

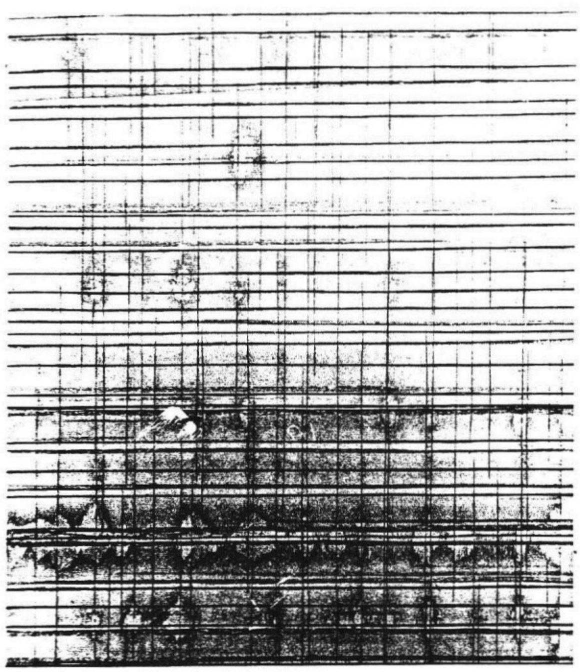

Fig. 7. State of grouping of delaminations, shown in Fig. 5. in #1 $(0/90_3)_s$ laminate, c=200,000 (f=2.0 Hz., R=0.1 $L_{max.}$=10.230 KN)

where α_{ij}^M is the ISV for matrix cracking in each ply, V_L is an arbitrarily chosen local volume element of ply thickness which is sufficiently large that α_{ij}^M does not depend on the size of V_L, u_i^C are crack opening displacements in V_L, u_i^C are the components of a unit normal to the crack faces, and S_C is the surface area of matrix cracks in V_L, as shown in Fig. 8.

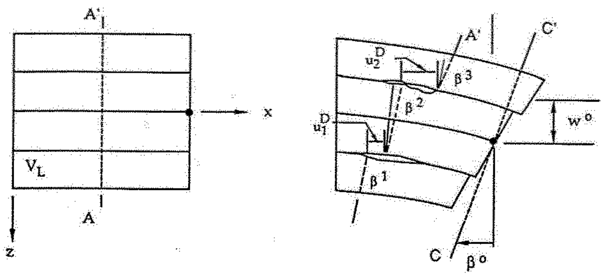

Fig. 8. Deformation Geometry for Region A_L.

Ply Level Stress-Strain Relations

For the case where all matrix cracks are normal to the laminate plane the local volume averaging process can be shown to result in the following ply level stress-strain relations [15,16]:

$$\begin{Bmatrix} \sigma_{L_x} \\ \sigma_{L_y} \\ \sigma_{L_z} \\ \sigma_{L_{yz}} \\ \sigma_{L_{xz}} \\ \sigma_{L_{xy}} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & Q_{15} & Q_{16} \\ Q_{12} & Q_{22} & Q_{23} & Q_{24} & Q_{25} & Q_{26} \\ Q_{13} & Q_{23} & Q_{33} & Q_{34} & Q_{35} & Q_{36} \\ Q_{14} & Q_{24} & Q_{34} & Q_{44} & Q_{45} & Q_{46} \\ Q_{15} & Q_{25} & Q_{35} & Q_{45} & Q_{55} & Q_{56} \\ Q_{16} & Q_{26} & Q_{36} & Q_{46} & Q_{56} & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{L_x} - \alpha_{xx}^M \\ \epsilon_{L_y} - \alpha_{yy}^M \\ \epsilon_{L_z} - \alpha_{zz}^M \\ \gamma_{L_{yz}} - 0 \\ \gamma_{L_{xz}} - 0 \\ \gamma_{L_{xy}} - \alpha_{xy}^M \end{Bmatrix} \tag{2}$$

where Q_{ij} are components of the elastic (undamaged) modulus tensor in ply coordinates, and the subscripts L imply thay the components of the stress and strain tensor are locally averaged [15].

Damage Dependent Lamination Theory

Unlike the ply level model for matrix cracking, statistical homogeneity cannot be assumed for delaminations. Although statistical homogeneity appears to hold in the plane of the laminate, the same cannot be said for the through-thickness direction. Thus, damage is accounted for via area averaging in the laminate plane, accompanied by a kinematic assumption through the thickness. Accordingly, the laminate equations are constructed by assuming that the Kirchhoff-Love hypothesis may be modified to include the effects of jump displacements u_i^D, v_i^D, and w_i^D, as well as jump rotations β_i^D and ψ_i^D for the ith delaminated ply interface, as shown in Fig. 9. Mathematically, then [17]

$$u(x,y,z) = u^o(x,y) - z\,[\beta^o + H(z-z_i)\,\beta_i^D] + H(z-z_i)\,u_i^D \tag{3}$$

$$v(x,y,z) = v^o(x,y) - z\,[\psi^o + H(z-z_i)\,\psi_i^D] + H(z-z_i)\,v_i^D \tag{4}$$

and

$$w(x,y,z) = w^o(x,y) + H(z-z_i)\,w_i^D \tag{5}$$

where the superscripts "o" imply undamaged midsurface quantities, and $H(z-z_i)$ is the Heavyside step function. Also, a repeated index i in a product is intended to imply summation, and the superscripts D imply displacement components across the delamination.

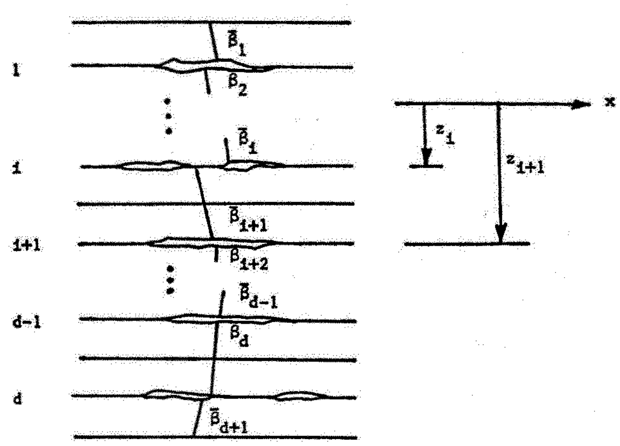

Fig. 9. Schematic of Delaminated Region in a Composite Layup.

The displacement equations are averaged over the local area in order to produce locally averaged displacements to be utilized in the laminate formulation. Thus,

$$u_L(x,y,z) = \frac{1}{A_L} \int_{A_L} [u^{\circ} - z(\beta^{\circ} + H(z-z_i)(\beta_i^D)) + H(z-z_i)\ u_i^D] dxdy \tag{6}$$

$$v_L(x,y,z) = \frac{1}{A_L} \int_{A_L} [v^{\circ} - z(\psi^{\circ} + H(z-z_i)(\psi_i^D)) + H(z-z_i)\ v_i^D] dxdy \tag{7}$$

and

$$w_L(x,y,z) = \frac{1}{A_L} \int_{A_L} [w^{\circ} + H(z-z_i)\ w_i^D] dxdy \tag{8}$$

By averaging the displacements, the delamination jump discontinuities are also averaged over A_L.

The resultant laminate equations may be obtained by substituting equations (6) through (8) into the strain-displacement equations, and this result into equations (2). This result is then integrated through the laminate thickness and the divergence theorem is employed to obtain the following laminate equations [17]:

$$\{N\} = \sum_{k=1}^{n} [Q]_k (z_k - z_{k-1}) \{\epsilon_L^o\} - \frac{1}{2} \sum_{k=1}^{n} [Q]_k (z_k^2 - z_{k-1}^2) \{\kappa_L\}$$

$$+ \sum_{i=1}^{d} [\bar{Q}_1]_i \ t_i \begin{Bmatrix} 0 \\ 0 \\ \alpha_{1i}^D \\ \alpha_{2i}^D \\ \alpha_{3i}^D \\ 0 \end{Bmatrix} + \sum_{i=1}^{d+1} (z_i - z_{i-1}) [\bar{Q}_2]_i \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \alpha_{4i}^D \\ \alpha_{5i}^D \\ 0 \end{Bmatrix}$$

$$\sum_{k=1}^{n} [Q]_k (z_k - z_{k-1}) \{\alpha^M\}_k \tag{9}$$

$$\{M\} = \frac{1}{2} \sum_{k=1}^{n} [Q]_k (z_k^2 - z_{k-1}^2) \{\epsilon_L^o\} - \frac{1}{3} \sum_{k=1}^{n} [Q]_k (z_k^3 - z_{k-1}^3) \{\kappa_L\}$$

$$+ \sum_{i=1}^{d} [\bar{Q}_3]_i \ t_i^2 \begin{Bmatrix} 0 \\ 0 \\ \alpha_{1i}^D \\ \alpha_{2i}^D \\ \alpha_{3i}^D \\ 0 \end{Bmatrix} + \sum_{i=1}^{d+1} [\bar{Q}_4]_i (z_i^2 - z_{i-1}^2) \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \alpha_{4i}^D \\ \alpha_{5i}^D \\ 0 \end{Bmatrix}$$

$$- \frac{1}{2} \sum_{k=1}^{n} [Q]_k (z_k^2 - z_{k-1}^2) \{\alpha^M\}_k \tag{10}$$

where {N} and {M} are the resultant forces and moments per unit length, respectively, and $\{\alpha^M\}_k$ and $\{\alpha_i^D\}$ represent the damage due to matrix cracking and interply delamination, respectively. Furthermore, n is the number of plies, and d is the number of delaminated ply interfaces.

The internal state variable for delamination, $\{\alpha_i^D\}$, is obtained by employing the divergence theorem on a local volume element of the laminate.
The resulting procedure gives [17]

$$\alpha_{1i}^D = \frac{2}{V_{Li}} \int_{S_{2i}} w_i^D n_z dS \tag{11a}$$

$$\alpha_{2i}^D = \frac{2}{V_{Li}} \int_{S_{2i}} v_i^D n_z dS \tag{11b}$$

$$\alpha_{3i}^D = \frac{2}{V_{Li}} \int_{S_{2i}} u_i^D n_z dS \tag{11c}$$

$$\alpha_{4i}^D = \frac{1}{A_L} \int_{S_{2i}^B} \psi_i^D n_z dS \tag{11d}$$

$$\alpha_{5i}^D = \frac{1}{A_L} \int_{S_{2i}^B} \beta_i^D n_z dS \tag{11e}$$

where the subscript i is associated with the ith delaminated ply interface. Furthermore, V_{Li} is equivalent to $t_i A_L$, where t_i is the thickness of the two plies above and below the delamination [17]. By definition, the z component of the unit normal, n_z, is equivalent to unity. The matrices [Q] with subscripts k are the standard elastic property matrices for the undamaged plies. The matrices $[\bar{Q}]$ with subscripts i apply to the ith delaminated ply interface. They represent average properties of the plies above and below the delamination [17].

While the damage-dependent laminate analysis model may be used to predict any of the effective engineering moduli for a laminate, experimental results are only available for the axial modulus and Poisson's ratio. Therefore, the general utility of the model is demonstrated by comparing model predictions to experimental results for E_x and ν_{xy} [18].

Model predictions have been made for a

typical graphite/epoxy system. The bar chart shown in Fig. 10 compares the model predictions to the experimental values for the engineering modulus, E_x, for combined matrix cracking and delamination. The delamination interface location and percent of delamination area are listed in the figure underneath the laminate stacking sequence. As can be seen, the comparison between model results and the experimental results is quite good. Some limited results for Poisson's ratio are given in Fig. 11 using the same bar chart format. With the exception of the $[0/90_2]_s$ laminate, these results are also quite good. Note that these results have been obtained for a single set of input data which do not depend on stacking sequence.

Evaluation of Ply Stresses

In order to evaluate the stress state in each ply, it is first necessary to substitute displacement equations (6) through (8) into the locally averaged strain-displacement equations. Utilizing the divergence theorem on this result will then give the following equations for the strains in each ply [19]:

$$\varepsilon_{L_x} = \varepsilon^o_{L_x} - z\,[\kappa_{L_x} + H(z-z_i)\,\alpha^D_{5i}] + H(z-z_i)\,\alpha^D_{3i} \quad (12)$$

$$\varepsilon_{L_y} = \varepsilon^o_{L_y} - z\,[\kappa_{L_y} + H(z-z_i)\,\alpha^D_{4i}] + H\,(z-z_i)\,\alpha^D_{2i} \quad (13)$$

$$\varepsilon_{L_z} = \varepsilon^o_{L_z} + H(z-z_i)\,\alpha^D_{1i} \quad (14)$$

$$\varepsilon_{L_{yz}} = \varepsilon^o_{L_{yz}} - [\kappa_{L_{yz}} + H(z-z_i)\,\alpha^D_{4i}] \quad (15)$$

$$\varepsilon_{L_{xz}} = \varepsilon^o_{L_{xz}} - [\kappa_{L_{xz}} + H(z-z_i)\,\alpha^D_{5i}] \quad (16)$$

$$\varepsilon_{L_{xy}} = \varepsilon^o_{L_{xy}} - \kappa_{L_{xy}} \quad (17)$$

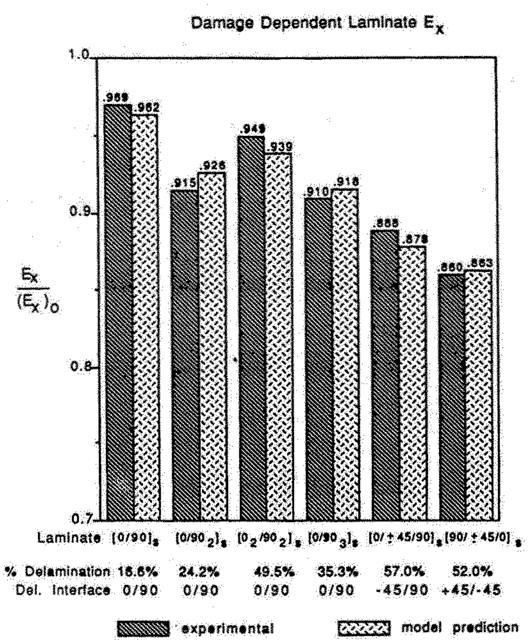

Fig. 10. Comparison of Experimental Results and Model Predictions of the Laminate Engineering Modulus, E_x, Degraded by Both Matrix Cracking and Delamination Damage.

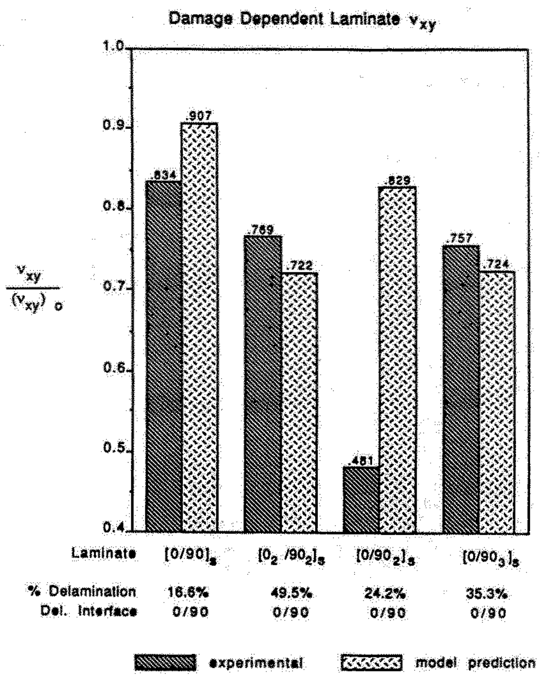

Fig. 11. Comparison of Experimental Results and Model Predictions of the Laminate Engineering Poisson's Ratio, ν_{xy}, Degraded by Both Matrix Cracking and Delamination Damage.

The above equations may be utilized to obtain the ply strains, and these results may be substituted into equation (2) to obtain the stresses in each ply [19]. Since the ply stresses determined by this procedure represent locally averaged values, they must be considered to be far-field stresses, so that stress intensity factors would be needed in order to determine matrix crack-tip stresses. This point will be discussed further in the section on damage evolution laws.

A computer code has been constructed to determine the effect of damage on the "far-field" ply stresses in composite laminates [19].

Predicted stresses are shown in Figs. 12 and 13 for a typical crossply laminate and a candidate quasi-isotropic layup with representative damage states. As evidenced from the results, the damage significantly affects the far-field ply stresses. Matrix cracks had a significant effect on ply stresses in the 90° plies in the cross-ply laminate. The quasi-isotropic laminate exhibited a small stress reduction in the ±45° plies, but showed a substantial stress reduction (fifteen percent verses one percent) in the 90° plies.

Damage Evolution Laws

Since the ply stresses determined by this procedure represent locally averaged values, they must be considered to be far-field stresses, so thata ISV evolution laws are generally of the form [19]:

$$\dot{\alpha}^M_{ij} = \dot{\alpha}^M_{ij}\ (\epsilon_{k\ell},\ T,\ \alpha^M_{k\ell},\ \alpha^D_{k\ell},\ K_I,\ K_{II},\ K_{III}) \tag{18}$$

and

$$\dot{\alpha}^D_{ij} = \dot{\alpha}^D_{ij}(\epsilon_{k\ell},\ T,\ \alpha^M_{k\ell},\ \alpha^D_{k\ell},\ K_I,\ K_{II},\ K_{III}) \tag{19}$$

where K_I, K_{II}, and K_{III} are the stress intensity factors, which relate the far-field stresses to the crack tip stresses for a given crack geometry. However, it is assumed that the geometry of both matrix cracking and delaminations is sufficiently independent of stacking sequence that the stress intensity factors may be treated as "material properties" and thus possess the same stress intensity factor dependence for all stacking sequences. Thus, they are encompassd implicitly in the material constants required to characterize damage evolution laws (18) and (19).

One approach to the formulation of the internal state variable evolutionary relationships is through micromechanical considerations. However, this approach is dependent on the availability of micromechanical solutions that can model the essential physical characteristics of the damage state. For the

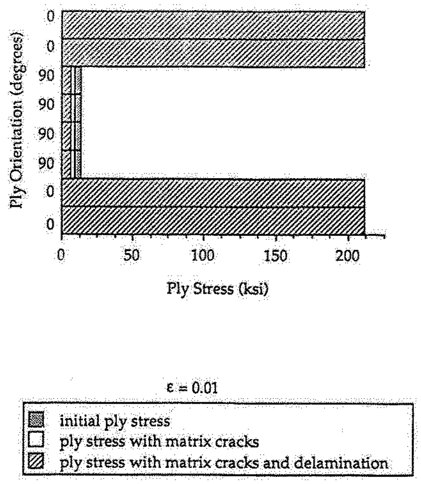

Fig. 12. Far Field Stresses in a $[0_2/90_2]$s Laminate

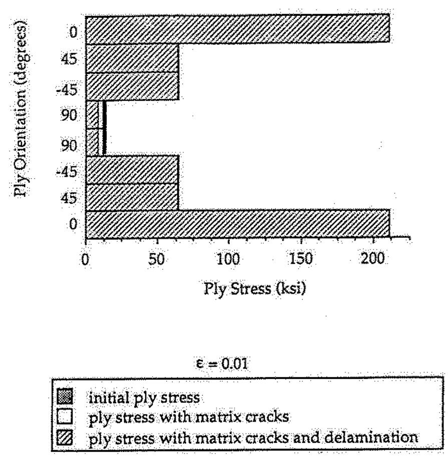

Fig. 13. Far Field Ply Stresses in a [0/±45/90]s Laminate.

problem of matrix cracks embedded in an orthotropic medium that is layered between two other orthotropic media, the solutions that are currently available are applicable only to very specific loading conditions and damage geometries. Therefore, the evolutionary equation proposed herein is phenomenological in nature. The form of the damage evolutionary relationship employed in this paper is based on the observation made by Wang, et al. [20] that for some materials the rate of damage surface evolution per load step, $\frac{dS}{dN}$, follows a power law in the energy release rate, G. Thus [21],

$$d\alpha^M_{ij} = \frac{d\alpha^M_{ij}}{dS} k_1 G^n dN \qquad (20)$$

The term $d\alpha^M_{ij}/dS$ reflects the changes to the internal state variable with respect to changes to the damage surfaces. $d\alpha^M_{ij}/dS$ can be obtained analytically from relationships describing the kinematics of the crack surfaces for given damage states and loading conditions, should such solutions exist. For transverse matrix cracks in crossply laminates, the average crack face displacement in the pure opening mode can be approximated by a solution obtained by Lee, et al. [22] for a medium containing an infinite number of alternating 0° and 90° plies. Thus, $d\alpha^M_{ij}/dS$ can be determined for crossply laminates subjected to uniaxial loading conditions. It has been found that for typical continuous fiber reinforced graphite/epoxy systems $d\alpha^M_{ij}/dS$ can be assumed to be constant for a given applied load until the damage state has reached an advanced stage of development. This assumption has facilitated the determination of the material parameters k_1 and n. The material properties for AS4/3501-6 graphite/epoxy are used in the calculations to enable the comparison of model prediction to experimental measurements made by Chou, et al. [24]. The material parameters for this polymeric composite system have been found to be

$$k_1 = 4.42 \quad \text{and} \quad n = 6.39 \qquad (21)$$

The damage history for a typical crossply layup has been predicted using the model. The model prediction for the damage state in the $[0_2/90_3]_s$ laminate fatigue loaded at a maximum stress amplitude of 26 ksi is shown in Fig. 14.

Good agreement is found between the model predictions and the experimental results.

To examine the amount of stress redistribution that occurs during the damage accumulation, the model was used to determine the axial stress in the 90° plies of the $[0_2/90_3]_s$ laminate fatigue loaded at three dirrerent stress amplitudes. Fig. 15 shows that for the stress amplitude of 38 ksi, the axial stress in the 90° plies after forty thousand cycles was less than fifty percent of the original stress level in the

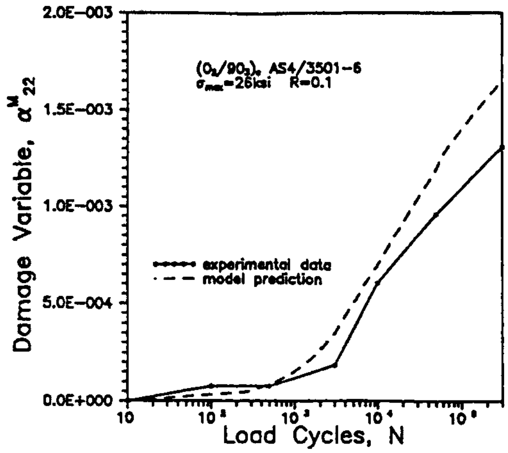

Fig. 14. Matrix crack damage in the 90° plies of a $[0_2/90_3]_s$ AS4/3501-6 laminate loaded at a stress amplitude of 26 ksi.

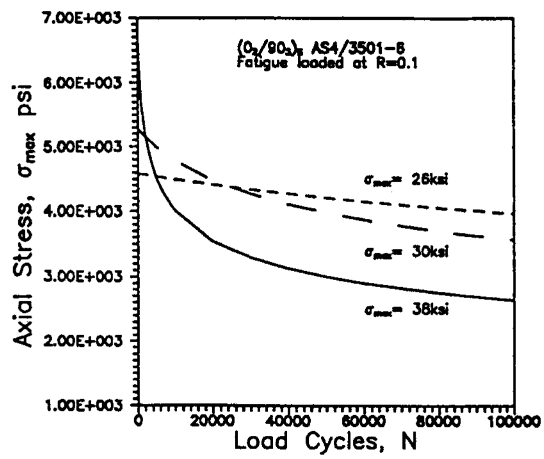

Fig. 15. Damage induced stress redistribution in the 90° plies of $[0_2/90_3]_s$ laminates subjected to constant stress amplitude fatigue loading.

undamaged laminate. These results demonstrate that the stress redistribution characteristics among the plies in the laminate are dependent on the loading conditions. These redistribution characteristics will affect the manner in which damage develops in the surrounding plies as well as eventual failure of the laminate.

Life Prediction

Usually, ultimate failure of laminated composites is caused by large scale fracture which is induced by fiber fracture at delamination sites. Therefore, there is not only a synergistic effect between matrix cracking and delamination, but also between delamination and the ultimate failure event. Since fiber fracture occurs very near the end of the component life, rather than model it with an additional internal variable, it is preferrable to simply treat it as the ultimate failure event and model it with a failure function. Typically, one of two approaches could be taken. A phenomenological approach would entail the modification of an existing failure function such as the Tsai-Wu failure function to account for the existing damage state. Alternatively, a fracture criterion could also be modified to account for the damage induced stress redistribution. The authors are pursuing this subject further at this time.

Conclusion

The authors have shown that by constructing local averages of the kinematic effects of microcracking it is possible to construct continuous internal variables which appear explicitly in a modified laminations theory for layered composites with damage. Comparisons of predicted stiffness loss as a function of damage state to experimental results lend credence to the model.

Because the lamination theory is damage dependent, it produces predicted stress redistribution as damage develops for a given fatigue load history. This predicteed stress redistribution in turn affects the evolution of damage, thus producing a life prediction model which can be used for any stacking sequence regardless of the load history applied to the component. However, while initial model comparisons to experiment are favorable, further research is suggested before the model is utilized in a design setting.

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