

VISCOELASTIC MODELLING OF
CONTINUOUS CARBON FIBRE
REINFORCED THERMOPLASTICS

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Abstract

In the present paper a computer programme has been developed which simulates the creep behaviour of continuous carbon fibre reinforced composites and by using a repetitive procedure finds the most appropriate combination of spring-dashpots elements for describing this behaviour. The programme is also capable to find the retardation as well as the relaxation spectrum of the material under investigation. The method is illustrated by application to experimental data on a [+45]_s continuous carbon fibre reinforced Polyamide composite.

1. Introduction

Advanced engineering thermoplastic composites employing carbon fibre reinforcement form the basis for many contemporary engineering structural designs, especially designs related to aerospace applications such as rocket motor cases, military and commercial aircraft, space vehicles and other as well as, designs related to automobile components and pressure vessels.

The carbon fibre reinforced compounds also offer a reduced coefficient of thermal expansion, lower mold shrinkage, increased thermal conductivity, increased tensile strength, improved creep resistance, superior wear resistance, outstanding toughness and high strength-to-weight ratios. Apart from these attractive properties, FRPs offer improved resistance to fatigue failure, improved corrosion resistance, while their properties can be "tailored" to meet

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the desired properties for a specific design application.

These structural composites are often subjected to sustained loads at ambient and higher temperatures and hence for long-time design situations it is important to understand their creep behaviour. Polymeric matrices of these materials impart a viscoelastic type of behaviour to the composite.

The viscoelasticity of polymeric composites is often described by models consisting of springs and dashpots connected in series and/or in parallel. The elasticity is represented by the springs, the viscosity by the dashpots. In this way, an arbitrary, stress-strain-time relation can be represented by increasing the number of spring-dashpots elements. Although it is possible to simulate the viscoelastic behaviour of polymeric composites by various combinations of spring and dashpots, the number and complexity of these combinations is usually too great. The least amount of complexity is introduced if the material is linearly viscoelastic which implies that, at any instant, the magnitude of a time-dependent response is directly proportional to the magnitude of the applied time-dependent input. Often this concept is discarded in favour of using a simple model, but with the assumption that the retardation or relaxation time is not the same for all parts of the material. There is a spread, or distribution, of relaxation times which can be measured and which can sometimes be associated with physical effects in the material such as stretching or rotation of primary bonds and displacement or entanglement of molecular chains.

Also, Laplace transformation can be taken of the time-dependent variables to obtain an associated elasticity problem in the transformed variables the solution of which when transformed back to the real time variables, gives the desired results. However, this technique includes several difficulties such as, the basic necessity of having to solve the associated elasticity problem, as well as, the manner in which the viscoelastic stress-strain law is to be introduced into the associated problem.

That viscoelastic properties are important for carbon-fibre polymeric composites has been previously demonstrated and reported ¹⁻⁹. Also, creep behaviour of other reinforced plastics has been studied in ¹⁰⁻¹⁶.

The class of materials investigated in this paper are [+45]s carbon fibre laminates with a time-dependent polyamide matrix.

2. Background

If we consider n Voigt elements connected in series plus an equilibrium spring then we have the so called "Generalized Voigt Model". This model is shown in Figure 1. If we now consider this model subjected to a constant stress test (Creep), then the total strain ϵ is equal to the sum of strains developed on the individual elements.

That is:

$$\epsilon = \epsilon_e + \sum_{i=1}^n \epsilon_i \quad (1)$$

where:

$$\epsilon_e = \sigma_0 / E_s \quad (2)$$

$$\epsilon_i = \sigma_0 / E_i (1 - e^{-t/\tau_i}) \quad (3)$$

ϵ_i is the strain developed on each individual Voigt element and τ_i the retardation time given by:

$$\tau_i = \eta_i / E_i \quad (4)$$

Combining equations (1) (2) and (3), the total strain is obtained:

$$\epsilon(t) = \left\{ (1/E_s) + \sum_{i=1}^n (1 - e^{-t/\tau_i}) / E_i \right\} \sigma_0 \quad (5)$$

Thus, one can associate or postulate a function of the form:

$$E_i = E_i(\eta_j) = E_i(\tau_j) \quad (6)$$

that becomes continuous as the number of elements increase without bound. A convenient form is found to be:

$$1/E_i = L(\tau_i) \Delta_i \tau / \tau_i \quad (7)$$

where $\Delta_i \tau$ is the *i*th increment in retardation time.

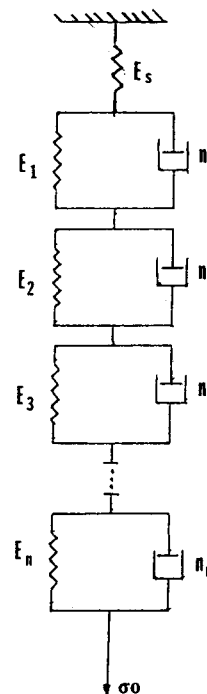


Fig. 1 The Generalized Voigt Model

Assuming an infinite array of Voigt elements (i.e. $n \rightarrow \infty$ and $\Delta_i \tau \rightarrow 0$), equation (5) becomes:

$$\epsilon(t) = \left\{ (1/E_s) + \int_0^{\infty} L(\tau) (1 - e^{-t/\tau}) / \tau \, d\tau \right\} \sigma_0 \quad (8)$$

in which $L(\tau)$ is known as the "Retardation Spectrum". Thus, instead of having to determine an infinite number of experimental constants E_i and η_i , the problem has been

traded for one of determining a suitable function $L(\tau)$. In the present case, the assumed form for $L(\tau)$ is:

$$L(\tau) = c (\tau_0/\tau)^n e^{-\tau_0/\tau} \quad (9)$$

so that upon substitution into (8), there results

$$\epsilon(t) = \left[(1/E_g) + \int_0^\infty c (\tau_0/\tau)^n e^{-\tau_0/\tau} (1 - e^{-t/\tau}) \cdot \tau^{-1} d\tau \right] \sigma_0 \quad (10)$$

The result of the above integration is:

$$\epsilon(t) = \left[D_e - \frac{D_e - D_g}{(1 + (t/\tau_0))^n} \right] \sigma_0 \quad (11)$$

and

$$L(\tau) = \frac{D_e - D_g}{\Gamma(n)} (\tau_0/\tau)^n e^{-\tau_0/\tau} \quad (12)$$

where τ_0 depends on the temperature of the experiment.

In all the above relations the following definitions must be considered:

τ_0 = is the overall retardation time of the model which can be determined through a repetitive computer procedure which minimize the sum of differences between experimental values and theoretical predictions.

c = is a constant which can be determined from the boundary conditions of the problem.

n = is a parameter which can be determined from the slope of the strain-time curve through the transition region between glassy and rubbery behaviour.

$\Gamma(n)$ = is the Gamma function.

D_g , D_e = are the Creep Compliances in the glassy and rubbery phase respectively.

3. Experimental

Specimens of rectangular shape cut from the Carbon-Polyamide (C/PA) plates, were used for the tests. The specimens were 24.7 mm in width and 1.65 mm in thickness. The length of the specimens was ta-

ken long enough to assure a uniform tensile field along the gauge area of the specimen. Tensile tests in creep were carried out at various steps of temperature (i.e. 80°, 90°, 100°, 110°, 120°, 130°, 140°, 150°, 160° and 165°C), to obtain the extension modulus as well as, the lateral contraction ratio.

The test equipment contained a simple lever loading frame with dead weights for the creep test which assured an instantaneous, but not hammering, loading and unloading of the specimen. The load applied was small enough so that it did not produce excessive deformations. Since the lateral contraction of the specimen for this loading is insignificant it may be assumed that, by keeping the applied load constant, the applied stress to the specimen is independent of time. This stress, then, was calculated from the applied load and the cross-sectional area in the unstretched specimen. Strains, in both longitudinal and transverse direction were measured as functions of time at equal intervals in a logarithmic scale. The time was measured from the instant when the load was applied. Inertia effects were avoided by not taking reading until 15 seconds after application of the load.

4. Results and Discussion

From what we have already presented, and taking into account the boundary conditions of the problem, it may be easily derived that the creep compliance $m(t)$ is given by the following relation.

$$m(t) = D_g + (D_e - D_g) \left(1 - \frac{1}{(1 + t/\tau_0)^n} \right) \quad (13)$$

The above relation, as well as, relation (12) are the basic relations for the present application.

A characteristic curve showing the variation of $m(t)$ with time is shown in Fig. 2. In the same Figure, the technique applied for the determination of the exponent $n = \tan \theta$, is also shown.

The same variation for different temperatures is shown in Figs. 3-5. From these

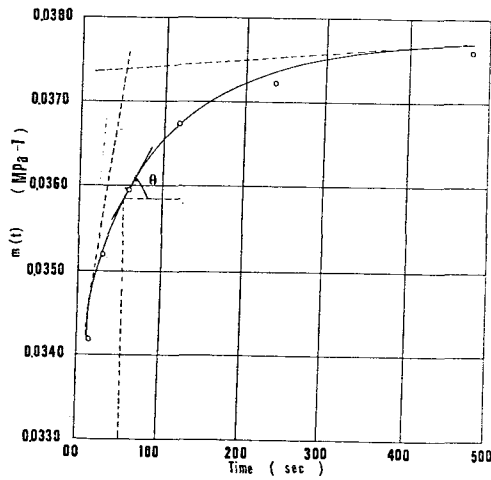


Fig. 2 Typical curve showing the variation of $m(t)$ with time.

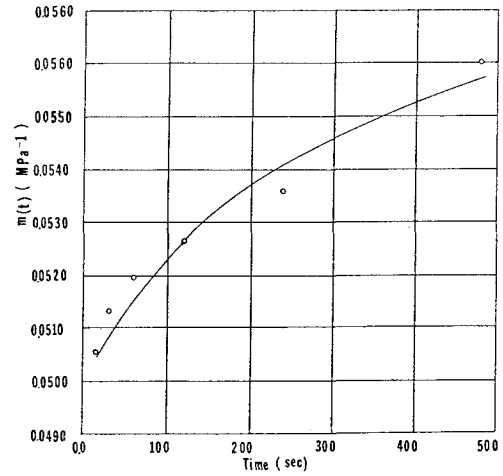


Fig. 5 Variation of $m(t)$ with time for $T = 165^\circ\text{C}$.

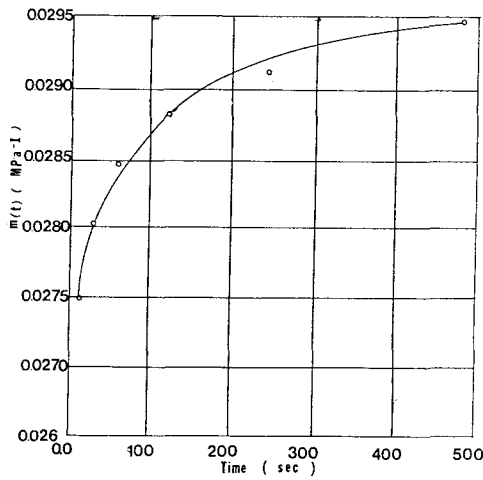


Fig. 3 Variation of $m(t)$ with time for $T = 80^\circ\text{C}$.

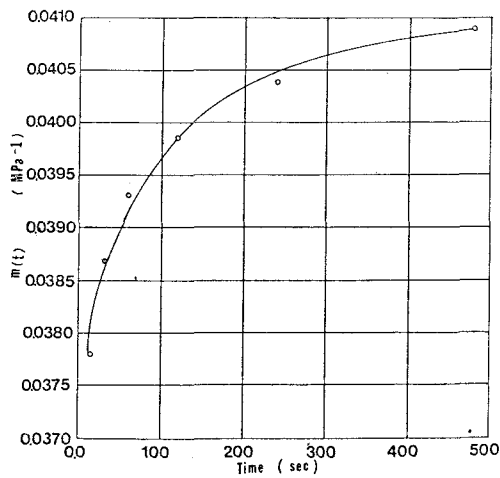


Fig. 4 Variation of $m(t)$ with time for $T = 120^\circ\text{C}$.

figures it becomes clear that there is a very good agreement between experimental results and theoretical predictions. Also, with increasing temperature, a steeper transition from lower to higher values is observed.

The variation of the viscoelastic behaviour with temperature is better shown in Figs. 6-7 where the retardation spectra $L(\tau)$ as derived from the computer programme are plotted as a function of time for various temperatures.

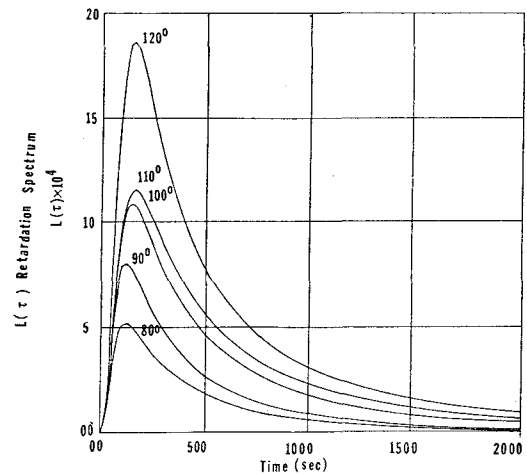


Fig. 6 Variation of retardation spectra as derived from the computer programme as a function of time for various temperatures in the range $80-120^\circ\text{C}$.

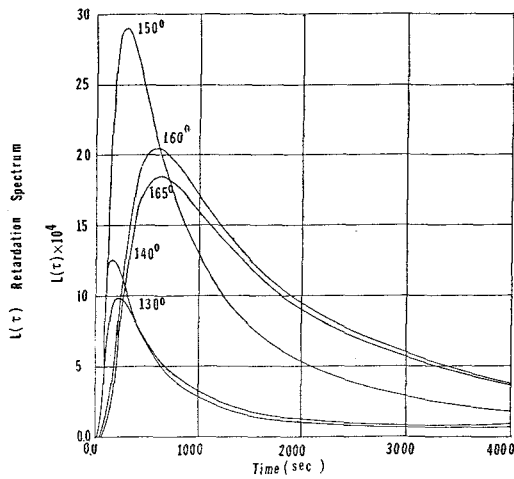


Fig.7 Variation of retardation spectra as derived from the computer programme as a function of time for various temperatures in the range 130 to 165°C.

In the temperature range 80-120°C, an increase in temperature is accompanied by a shift of the respective spectra to higher values, while the time corresponding to the peak-value of the spectral function L remains almost unchanged (see fig.6). This means that in this temperature range, the relation between the elastic and viscous component of the material remains unaffected. The same conclusion may be derived from Fig.8 where the characteristic retardation times are plotted against temperature. From this Figure, it may be observed that in the range 80° - 120°C the characteristic time remains almost constant, while for higher temperatures, i.e. 130 - 165°C, an increase of τ is observed which is reflected to a simultaneous shift to higher values and to the right of the respective spectra, as shown in Fig.7.

Finally, the variation of the extension modulus as it was calculated during the creep experiments of the C/PA materials, is shown in Fig. 9. As it was expected, a decreasing variation of E with temperature is observed.

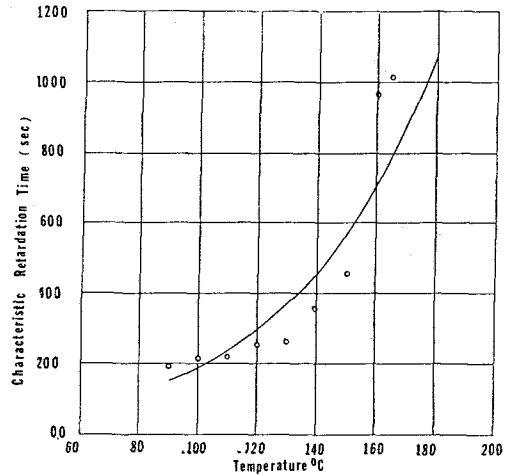


Fig.8 Variation of the characteristic retardation time as a function of temperature.

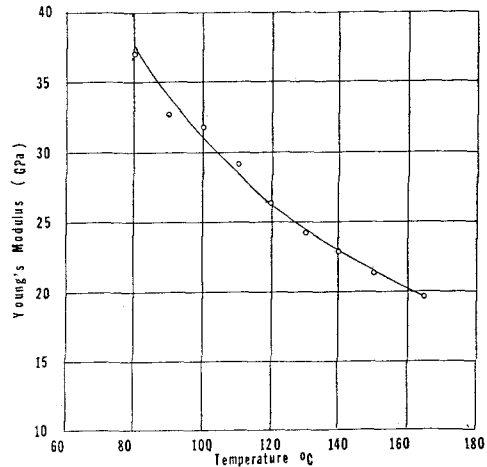


Fig.9 Variation of the extension modulus with temperature.

5. Conclusions

In the present paper, a theoretical viscoelastic model based on a computer programme which simulates the creep behaviour of continuous fibre-reinforced polymers has been developed. A carbon-polyamide [+45]_s laminate was chosen to illustrate the application of the method. A good agreement between experimental results and the theoretical predictions is observed. Also, the method may be applied for the derivation of the retardation spectra of the material under investigation.

6. Acknowledgments

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