SUBSONIC STEADY, UNSTEADY AERODYNAMIC CALCULATION FOR WINGS AT HIGH ANGLE OF ATTACK

Zheng-yin Ye* Yong-nian Yang** Ling-cheng Zhao**
Northwestern Polytechnical University
Xi'an, People's Republic of China

Abstract

A new numerical method——Potential Difference Method(PDM) is presented in this paper to calculate the nonlinear steady, unsteady aerodynamic force on wings with sharp edge separation at high angle of attack in subsonic flow. The method is based on integral technique to solve the velocity potential equation. For the arbitrary motion of wings, the method can give the varying shapes of the vortex rolling up and velocity field directly in the time domain. Numerical results are compared with experimental data, and are shown to be satisfactory.

Introduction

To obtain good aerodynamic performance in modern aircraft design, wings with separated vortices shed from sharp leading edge or side edge at high angle of attack are widely used. As the separated vortices roll up over the wing and induce additional lift forces, the flow displays nonlinear characteristics. Therefore, the separation flow must be properly modeled.In the last two decades, many numerical methods[1-4] to calculate the nonlinear loads on wings at high angle of attack have been developed. In these methods, most attention has been paid to modifications of the Vortex Lattice Method(VLM) because of its well-known advantages. Recent advances in techniques to get exact solutions of the Euler equations and the full Navier-Stokes equations with finite difference method are limited to low angle of attack and configurations which are not too complicated. Besides, they consume large amount of computation time. So, in the present,the evaluation of the aerodynamic loads of wings with sharp edges at moderate and high angle of attack will still require the use of approximate methods such as Nonlinear Vortex Lattice Method (NLVLM). Unfortunately, most of the NLVLM, up to now, are applicable only to the incompressible flow. Some works[3,4] about the extension of steady NLVLM to the subsonic flow have been done by using the Prandtl-Glauert transformation. Recently, Kandil and Yates[5,6] obtained the nonlinear loads on wings with separated flow at high angle of attack in subsonic and transonic flow with the NLVLM based on integration of the nonlinear equations. But their results are limited to steady flows. Compared with subsonic steady or imcompressible unsteady flows, the calculation of subsonic unsteady flows is much more complicated where the delay time and varying shapes of the wake must be considered.

This paper gives a numerical method to calculate the nonlinear airloads of wings with separated vortices at high angle of attack in subsonic unsteady flows. Because the time domain is used, the fully unsteady motion can be simulated. If steady flow is concerned, the method becomes the NLVLM.As the velocity field is obtained directly, the errors in Green's Function Method[7] using finite difference of velocity potential are avoided.

Basic Equations

The full nonlinear equation for the velocity potential $\Phi = U_\infty(x+\varphi)$ is used here

$$\nabla^2\varphi - \frac{1}{a_\infty^2}\frac{d^2\varphi}{dt^2} = \psi \qquad (1)$$

where φ is the perturbation potential and ψ includes all the nonlinear terms.

$$\frac{d}{dt} = \frac{\partial}{\partial t} + U_\infty\frac{\partial}{\partial x} .$$

By applying Green's Theorem to Eq.(1), the following equation is obtained

* Doctor of Engineering, Department of Aircraft Engineering.
** Professor, Department of Aircraft Engineering.

$$
\begin{aligned}
4\pi\varphi = & -\iint_{\sigma}\{\mathbf{n}\cdot[\nabla\varphi]^{\theta}-\frac{U_{\infty}}{a_{\infty}^{2}}n_{x}[\frac{d\varphi}{dt}]^{\theta}\}\frac{1}{r_{\beta}}d\sigma \\
& +\iint_{\sigma}\{\mathbf{n}\cdot\nabla(\frac{1}{r_{\beta}})-\frac{U_{\infty}^{2}}{a_{\infty}^{2}}n_{x}\frac{\partial}{\partial x}(\frac{1}{r_{\beta}})\}[\varphi]^{\theta}d\sigma \\
& -\iint_{\sigma}\{\mathbf{n}\cdot\nabla\theta-\frac{U_{\infty}}{a_{\infty}^{2}}n_{x}(1+U_{\infty}\frac{\partial\theta}{\partial x})\} \\
& \cdot[\frac{\partial\varphi}{\partial t}]^{\theta}\frac{1}{r_{\beta}}d\sigma-\iiint_{v}[\psi]^{\theta}\frac{1}{r_{\beta}}dv
\end{aligned} \tag{2}
$$

where $[\;]^{\theta}=[\;]|_{t=t-\theta}$;
$r_{\beta}=\sqrt{(x-x_{p})^{2}+\beta^{2}[(y-y_{p})^{2}+(z-z_{p})^{2}]}$

$\theta=\frac{1}{a_{\infty}\beta^{2}}[r_{\beta}+M_{\infty}(x-x_{p})]$; $\beta^{2}=1-M_{\infty}^{2}$

To obtain a simpler expression, the generalized Prandtl-Glauert transformation is used:

$$
\begin{aligned}
& X=x/\beta L\ ;\quad Y=y/L\ ;\quad Z=z/L\ ; \\
& T=U_{\infty}t/L\ ;\quad \phi=\varphi/L\ ;
\end{aligned} \tag{3}
$$

Then, Eq.(2) becomes :

$$
\begin{aligned}
4\pi\phi(P\cdot T) = & -\iint_{\Sigma}[\frac{\partial\phi}{\partial N}]^{\Theta}\frac{1}{R}d\Sigma+\iint_{\Sigma}[\phi]^{\Theta}\frac{\partial}{\partial N}\frac{1}{R}d\Sigma \\
& -\iint_{\Sigma}[\frac{\partial\phi}{\partial T}]^{\Theta}\frac{\partial\hat{\Theta}}{\partial N}\frac{1}{R}d\Sigma+\iiint_{v}[Q]^{\Theta}\frac{1}{R}dv
\end{aligned} \tag{4}
$$

where $R=[(X-X_{p})^{2}+(Y-Y_{p})^{2}+(Z-Z_{p})^{2}]^{1/2}$
$\Theta=[R+M_{\infty}(X-X_{p})]M_{\infty}/\beta$
$\hat{\Theta}=[R-M_{\infty}(X-X_{p})]M_{\infty}/\beta$

$$
\begin{aligned}
Q=-2M_{\infty}^{2}\{ & \frac{1}{\beta^{2}}\phi_{x}\phi_{xx}+\frac{1}{\beta}\phi_{y}\phi_{xy}+\frac{1}{\beta}\phi_{z}\phi_{xz} \\
& +\frac{1}{\beta^{2}}\phi_{x}\phi_{xT}+\phi_{Y}\phi_{YT}+\phi_{z}\phi_{zT} \\
& +\frac{k-1}{2}[\phi_{T}+\frac{1}{\beta}\phi_{x}][\frac{1}{\beta^{2}}\phi_{xx}+\phi_{YY}+\phi_{zz}]\}
\end{aligned}
$$

In this paper, only thin wings are dealt with. So Eq.(4) becomes

$$
\begin{aligned}
4\pi\phi(P\cdot T)= & \iint_{S}[\Delta\phi]^{\Theta}\frac{\partial}{\partial N}\frac{1}{R}ds-\iint_{S}[\frac{\partial\Delta\phi}{\partial T}]^{\Theta}\frac{\partial\hat{\Theta}}{\partial N}\frac{1}{R}ds \\
& +\iiint_{V}[Q]^{\Theta}\frac{1}{R}dv
\end{aligned} \tag{5}
$$

As finite difference method will be used in calculating some nonlinear terms in [Q], the following transformation is used

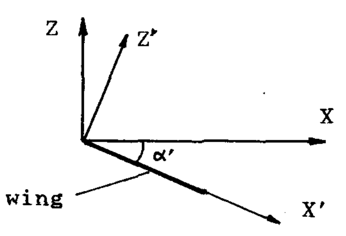

Fig.1 Coordinates transformation

$$
\begin{aligned}
X' &= X\cos\alpha' - Z\sin\alpha' \\
Z' &= X\sin\alpha' + Z\cos\alpha' \\
Y' &= Y \\
tg\,\alpha' &= \beta\, tg\,\alpha
\end{aligned}
$$

where α is the angle of attack.

Then, the operator

$\nabla_{p}=\frac{\partial}{\partial X'_{p}}\vec{i}+\frac{\partial}{\partial Y'_{p}}\vec{j}+\frac{\partial}{\partial Z'_{p}}\vec{k}$ is applied to Eq.(5)

$$
\begin{aligned}
4\pi\vec{V}_{p}(P\cdot T)= & \nabla_{p}\iint_{S}[\Delta\phi]^{\Theta'}\frac{\partial}{\partial N}\frac{1}{R}ds \\
& -\nabla_{p}\iint_{S}[\frac{\partial\Delta\phi}{\partial T}]^{\Theta'}\frac{\partial\hat{\Theta}'}{\partial N}\frac{1}{R}ds \\
& +\nabla_{p}\iiint_{V}[Q]^{\Theta'}\frac{1}{R}dv
\end{aligned} \tag{6}
$$

where $\vec{V}_{p}(P,T)=\nabla_{p}\phi(P,T)$;

$\Delta\phi=[\phi]_{upper}-[\phi]_{Lower}$;

$\Theta'=[R+M_{\infty}(X'-X'_{p})\cos\alpha'+M_{\infty}(Z'-Z'_{p})\sin\alpha']M_{\infty}/\beta$

$\hat{\Theta}'=[R-M_{\infty}(X'-X'_{p})\cos\alpha'-M_{\infty}(Z'-Z'_{p})\sin\alpha']M_{\infty}/\beta$

Eq.(6) can only be solved by numerical methods. The wings and wakes are divided into many lattices. For simplicity, $[\Delta\phi]$ in each lattice is considered to be constant at every moment. The improper integrals in Eq.(6) are more difficult to deal with than those in Green's Function Method.

At first, Eq.(6) is changed to

$$
\begin{aligned}
4\pi\vec{V}_{p}(P\cdot T)= & \iint_{S}[\Delta\phi]^{\Theta'}\nabla_{p}[\frac{\partial}{\partial N}\frac{1}{R}]ds \\
& -\iint_{S}[\Delta\dot{\phi}]^{\Theta'}\nabla_{p}\Theta'\frac{\partial}{\partial N}\frac{1}{R}ds \\
& -\iint_{S}[\Delta\dot{\phi}]^{\Theta'}\nabla_{p}[\frac{\partial\hat{\Theta}'}{\partial N}\frac{1}{R}]ds \\
& +\iint_{S}[\Delta\ddot{\phi}]^{\Theta'}\nabla_{p}\Theta'\frac{\partial\hat{\Theta}'}{\partial N}\frac{1}{R}ds \\
& +\iiint_{V}[Q]^{\Theta'}\nabla_{p}(\frac{1}{R})dv-\iiint_{V}[\dot{Q}]^{\Theta'}\nabla_{p}\Theta'\cdot\frac{1}{R}dv
\end{aligned} \tag{7}
$$

where the dots over the $[\Delta\phi]$ mean the derivates with respect to time.

The first term in the right side of Eq.(7) has the same expression as that in the VLM for incompressible flow or compressible steady flow, so it is calculated in the way of the VLM. Other terms can be treated by a limiting process indicated in the Appendix.

Finally, the improper integrals of the terms which involve $[\Delta\phi]$, $[\Delta\dot{\phi}]$ and $[\Delta\ddot{\phi}]$ can be written respectively as

$\iint_{S}\nabla_{p}[\frac{\partial}{\partial N}\frac{1}{R}]ds$ evaluated in the way of the VLM

$\iint_{S'}\{\nabla_{p}\Theta'\frac{\partial}{\partial N}\frac{1}{R}+\nabla_{p}[\frac{\partial\hat{\Theta}'}{\partial N}\frac{1}{R}]\}ds$

$$\xRightarrow{\varepsilon\to 0}\quad 2\pi\frac{M_\infty}{\beta}\vec{K}-2\pi\frac{M_\infty^2}{\beta}\cos\alpha'\vec{I}\quad(Z\to 0^+)$$

$$2\pi\frac{M_\infty}{\beta}\vec{K}+2\pi\frac{M_\infty^2}{\beta}\cos\alpha'\vec{I}\quad(Z\to 0^-)$$

$$\iint_{S'}[\nabla_p\Theta'\cdot\frac{\partial\hat{\Theta}'}{\partial N}\cdot\frac{1}{R}]ds'\xRightarrow{\varepsilon\to 0}0 \qquad (8)$$

In addition, to determine the flow field uniquely, the following boundary conditions must be used.

1. $\phi \longrightarrow 0$, when $R \longrightarrow \infty$, this condition is satisfied automatically.

2. $\frac{DF}{DT}=0$ where $F(X,Y,Z,T)=0$ is the equation of wing surface.

3. Kutta conditions at leading, side and trailing edges, from which the separated vortex sheds.

4. No pressure difference on the wake surfaces.

Method of Numerical Calculation

In the time domain, the variation history of angle of attack is divided into discrete changes corresponding to time steps. The problem is then solved at each time step where the solution of the preceding time step serves as the initial condition for the present time step.

This approach can treat problems where the flow unsteadiness starts from a steady flow or starts impulsively from rest. In this paper, the latter case is dealt with.

At first, the wing is divided into many lattices as in Ref.[2], and planar triangular panels are used to model the nonplanar and twisted wake surfaces. For $T<0$, the wing is at rest. At $T=0$, the wing starts impulsively from rest at some angle of attack. At the same time the $[\Delta\phi]$ distribution must have values to satisfy the boundary condition on the wing surface. Meanwhile, vortices shed from the wing sharp edges appear and move downstream with the local velocity.

At $T=1\Delta T$, a strip of wake lattices appears around the sharp edges of the wing and will move with the local velocity in the flow.The $[\Delta\phi]$ distribution on the wing surface must be altered by considering the effect of the wake. The values of $[\Delta\phi]$ on the wake are those of $[\Delta\phi]$ on the appropriate wing lattices near sharp edges at $T=0$, With the increase of time, the wake lattices increase. The sketches of wake at $T=0$, $T=1\Delta T$, $T=2\Delta T$ are shown in Fig.2.

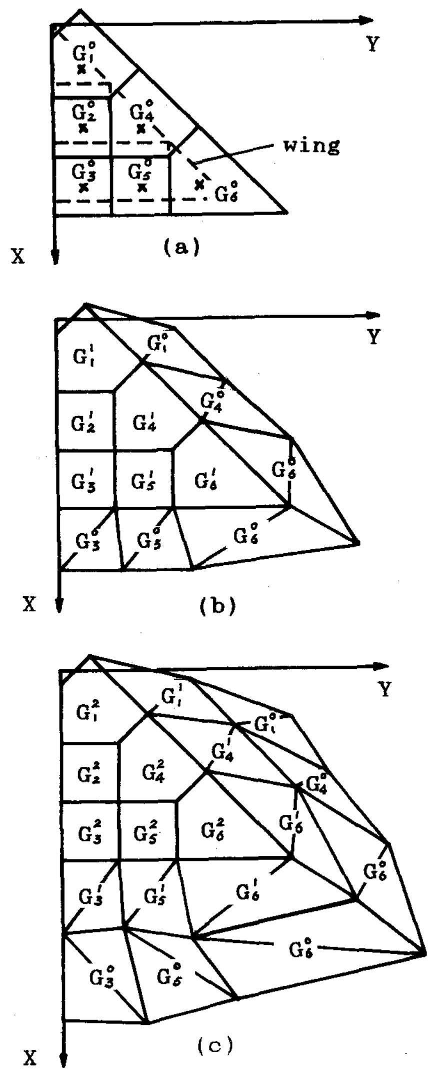

Fig.2 The sketches of developing wake and the lattices on the wing (a) $T=0$; (b) $T=1\Delta T$; (c) $T=2\Delta T$; where G_i^j means the ith G at jth time step

Unlike the VLM in incompressible flow and compressible steady flow or the Green's Function Method[7] where the shape of wake doesn't change and no separation appears, the nonlinear unsteady model of wings with separation at high angle of attack is more difficult in considering the delay-time and varying shape of the wake. In this paper, when the delay-time is less than one ΔT, the numerical method of Ref.[7] is used. When the delay-time is greater than one ΔT, the delay-time is changed into $n\,\Delta T$ approximately, where n is an integer.

For further simplicity,the integrals in Eq.(7) except the first term are treated in the following way :

$$\iint_{S_j} F_{ij}\, ds \doteq \overline{F}_{ij} \cdot \Delta s_j \quad ,$$

$$\iiint_{V_j} A_{ij}\, dv \doteq \overline{A}_{ij} \cdot \Delta v_j$$

where S_j and V_j are discrete surface element and volume element respectively. The $\overline{F}_{ij}$ and $\overline{A}_{ij}$ are calculated at the central points of jth surface element or jth volume element.

Pressure Coefficient

Because of the presence of separated flow over the wing, the velocity component in Y′ direction and the nonlinear terms in pressure coefficient expression must be taken into account in calculating the lift coefficient.

The pressure coefficient in oxyz coordinates is

$$C_p = -\frac{2}{U_\infty}\left(\varphi_x + \frac{1}{U_\infty}\varphi_t\right) - \frac{\varphi_x^2+\varphi_y^2+\varphi_z^2}{U_\infty^2}$$

$$+\frac{1}{U_\infty^2 a_\infty^2}(\varphi_t + 2\varphi_t\varphi_x U_\infty + U_\infty^2\varphi_x^2) + 0(\varphi^3)$$

and becomes in OX′Y′Z′ coordinates

$$C_p = -2[\Phi_T + (\Phi_{X'}\cos\alpha' + \Phi_{Z'}\sin\alpha')\frac{1}{\beta}]$$

$$-[\Phi_{X'}^2 + \Phi_{Y'}^2 + \Phi_{Z'}^2]$$

$$+M_\infty^2[\Phi_T^2 + 2\Phi_T\frac{\Phi_{X'}\cos\alpha' + \Phi_{Z'}\sin\alpha'}{\beta}]$$

then

$$\Delta C_p = C_{pU} - C_{pL}$$

$$= -\frac{2}{\beta}\cos\alpha'[\Phi_{X'U} - \Phi_{X'L}] - 2(\Phi_{TU} - \Phi_{TL})$$

$$-[\Phi_{X'U}^2 - \Phi_{X'L}^2] - [\Phi_{Y'U}^2 - \Phi_{Y'L}^2]$$

$$+M_\infty^2[(\Phi_{TU}^2 - \Phi_{TL}^2)$$

$$+2\frac{\Phi_{TU}\Phi_{X'U} - \Phi_{TL}\Phi_{X'L}}{\beta}\cos\alpha'] \tag{9}$$

To calculate the last term, an approximation is used

$$\Phi_{TU} = \frac{\Delta}{2}\Phi_T \quad , \quad \Phi_{TL} = -\frac{\Delta}{2}\Phi_T$$

So

$$\Delta C_p = -\frac{2}{\beta}\Delta\Phi_{X'}\cos\alpha' - 2\Delta\Phi_T - 2\Delta\Phi_{X'}\Phi_{X'm}$$

$$-2\Delta\Phi_{Y'}\Phi_{Y'm} + M_\infty^2\cdot\frac{2\cos\alpha'}{\beta}\Delta\Phi_T\Phi_{X'm} \tag{10}$$

The numerical method in Ref.[2] is used to calculate the ΔC_p of Eq.(10). As the method in Ref.[2] is applicable only to incompressible flow, additional terms of $[\Delta\Phi_{X'}]$ must be taken into account because of the existence of $[\Delta\dot{\Phi}]$ in subsonic flow.

Numerical Results

When the wing starts impulsively from rest at some angle of attack, its wake develops. After a certain number of ΔT, the wake shape and $[\Delta\Phi]$ distribution on the wing approach steady states, then the steady results are obtained.

In Fig.3, some numerical results at different Mach number for a rectangular wing with aspect ratio AR=1 are plotted. At M_∞=0.0, experiment data are also shown in the figure. The rectangular wing has a sharp side edge, where the separated vortex sheds from.

Fig.4 shows the comparision of calculated results with the experimental data at M_∞=0.6 and 0.8. The wing is a delta wing with AR=1, and is divided into 5×5 lattices. Separation occurs at leading edge.

Fig.5 shows the comparision of calculated results with the experimental data at M_∞=0.7. The wing is also a delta wing but with AR=1.865, it is divided into 6×6 lattices. Separation occurs at leading edge.

Finally, in Fig.6, the normal force coefficient of a rectangular wing with AR=1 oscillating in pitching about midchord at M_∞=0.5 are plotted versus nondimensional time. The wing is divided into 4×4 lattices. separation occurs at side edge.

When the nonlinear term [Q] in Eq.(7) is taken into account,the lift coefficient is somewhat smaller than that without [Q] term. This result agrees with the conclusion in Ref.[5].

Conclusion

The numerical method presented in this paper can calculate the nonlinear unsteady airloads on wings with sharp edge separation at high angle of attack in subsonic flow. It can be applied to wings with complex planforms and gives the velocity field directily. Because time domain is used, fully unsteady motion of wings can be treated. The numerical results calculated by the present method are shown to be satisfactory. Furthermore,the thickness of wings can be taken into account in the method.

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Appendix

The Improper Integrals in Eq.(7)

When the integration area is in the lattice containing the collocation point P, improper integrals arise in Eq.(7).

To solve the problem, the area is changed into two parts S_1 and S_2 as shown in Fig.(7). The improper integral is substituted by the integration on the half-spherical surface S_2.

At first, the two terms including $[\Delta\phi]$ are considered.

$$-\iint_S \{\nabla_P \Theta' \frac{\partial}{\partial N}\frac{1}{R}+\nabla_P[\frac{1}{R}\frac{\partial\hat{\Theta}'}{\partial N}]\}\,ds \qquad \text{(A-1)}$$

For the first term in (A-1)

$$\nabla_P \Theta' = \frac{M_\infty}{\beta}\nabla_P R - \frac{M_\infty^2}{\beta}\cos\alpha'\vec{I} - \frac{M_\infty^2}{\beta}\sin\alpha'\vec{K}$$

then

$$\nabla_P \Theta' \frac{\partial}{\partial N}\frac{1}{R} = \frac{M_\infty}{\beta}\nabla_P R\cdot\frac{\partial}{\partial N}\frac{1}{R} - \frac{\partial}{\partial N}\frac{1}{R}\cdot\frac{M_\infty^2}{\beta}[\cos\alpha'\vec{I}+\sin\alpha'\vec{K}] \qquad \text{(A-2)}$$

For the second term in (A-1)

$$\nabla_P[\frac{1}{R}\frac{\partial\hat{\Theta}'}{\partial N}] = \frac{M_\infty}{\beta}\nabla_P[\frac{1}{R}\frac{\partial R}{\partial N}] - \nabla_P\{\frac{1}{R}\frac{M_\infty^2}{\beta}[\cos\alpha'\frac{\partial(X'-X_P')}{\partial N} + \sin\alpha'\frac{\partial(Z'-Z_P')}{\partial N}]\} \qquad \text{(A-3)}$$

As the thin wing is on the $OX'Y'$ surface

$$\frac{\partial}{\partial N}(X'-X_P')=0$$

$$\nabla_P[\frac{1}{R}\frac{\partial\hat{\Theta}'}{\partial N}] = \frac{M_\infty}{\beta}\nabla_P[\frac{1}{R}\frac{\partial R}{\partial N}] - \nabla_P[\frac{1}{R}\frac{M_\infty^2}{\beta}\sin\alpha'\frac{\partial(Z'-Z_P')}{\partial N}] \qquad \text{(A-4)}$$

The expression (A-1) becomes

$$-\iint_S \frac{M_\infty}{\beta}\{\nabla_P R\cdot\frac{\partial}{\partial N}\frac{1}{R}+\nabla_P[\frac{1}{R}\frac{\partial R}{\partial N}]\}\,ds + \iint_S \frac{M_\infty^2}{\beta}\frac{\partial}{\partial N}\frac{1}{R}\cdot\cos\alpha'\,ds\,\vec{I} + \iint_S \frac{M_\infty^2}{\beta}\frac{\partial}{\partial N}\frac{1}{R}\cdot\sin\alpha'\,ds\,\vec{K} + \iint_S \frac{M_\infty^2}{\beta}\nabla_P[\frac{1}{R}\sin\alpha'\frac{\partial(Z'-Z_P')}{\partial N}]\,ds \qquad \text{(A-5)}$$

The present integral is on the thin wing, then

$$\frac{\partial}{\partial N}\frac{1}{R} = -\frac{Z'-Z_P'}{R^3}$$

The third and the fourth terms in (A-5) become

$$\iint_S \frac{M_\infty^2}{\beta}\cdot\frac{(X'-X_P')\vec{I}+(Y'-Y_P')\vec{J}}{R^3}\sin\alpha'\,ds$$

This integral is zero on S_2 due to the antisymmetry of the integrand.

To integrate the first and second terms, it is found that in S_2, there exists the relation $\frac{\partial}{\partial N} = -\frac{\partial}{\partial R}$, then, the first integral in S_2 becomes

$$\frac{M_\infty}{\beta}\iint_{S_2}\frac{2}{R^2}\frac{\vec{R}}{R}\,ds = -2\pi\frac{M_\infty}{\beta}\vec{K}$$

The second integral in S_2 becomes

$$\iint_{S_2}\frac{M_\infty^2}{\beta}\frac{\partial}{\partial N}\frac{1}{R}\cos\alpha'\,ds\,\vec{I}$$

Let the radius of the half-sphere $\varepsilon \rightarrow 0$, above results are still hold.

So, the improper integral including $[\Delta\phi]$ terms on the surface S_2 is reduced to

$$-\iint_{S_2}\{\nabla_P\Theta'\frac{\partial}{\partial N}\frac{1}{R}+\nabla_P[\frac{1}{R}\frac{\partial\hat{\Theta}'}{\partial N}]\}\,ds$$

$$\xrightarrow{\varepsilon\to 0}\quad \begin{array}{ll} -2\pi\frac{M_\infty}{\beta}\vec{K}+2\pi\frac{M_\infty^2}{\beta}\cos\alpha\,\vec{I} & (Z=0^+) \\ -2\pi\frac{M_\infty}{\beta}\vec{K}-2\pi\frac{M_\infty^2}{\beta}\cos\alpha\,\vec{I} & (Z=0^-) \end{array} \qquad \text{(A-6)}$$

In the same way, the integral including the [$\Delta\phi$] term is found to be approaching zero on S_2 when $\varepsilon\to 0$.

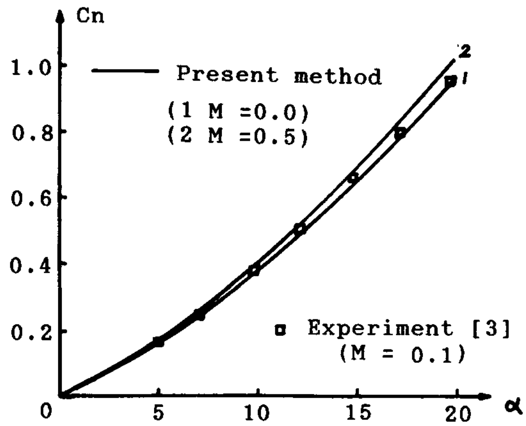

Fig.3 Normal force coefficient versus angles of attack (rectangular wing AR=1)

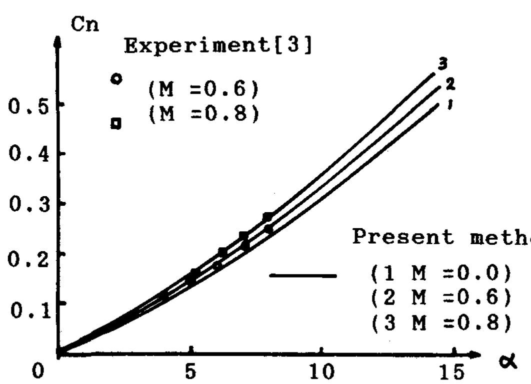

Fig.4 Normal force coefficient versus angles of attack (delta wing AR=1.07)

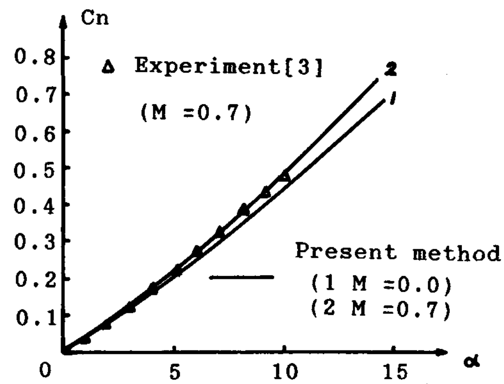

Fig.5 Normal force coefficient versus angles of attack (delta wing AR=1.865)

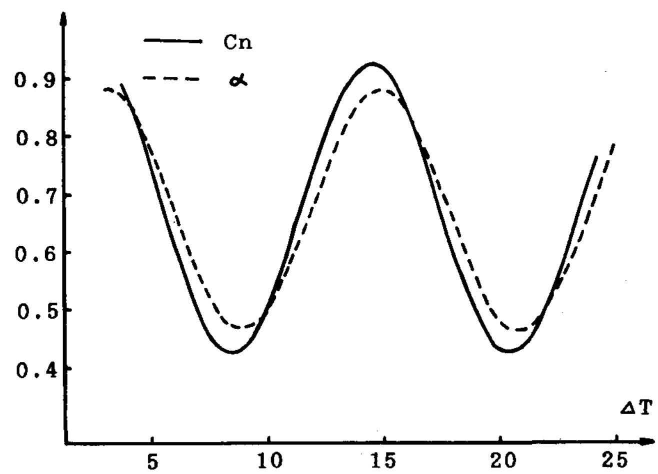

Fig.6 Normal force coefficient and angle of attack versus nondimensional time $\alpha = 15° + 4° \sin\frac{\pi}{6}T$

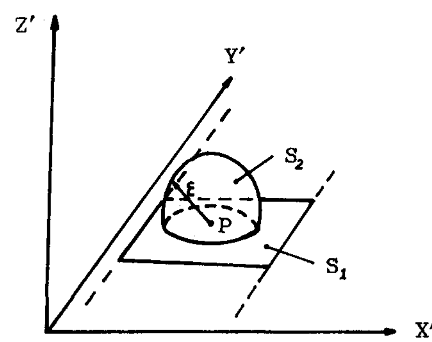

Fig.7 The divided integration area