On Calculation of Predicted Tracer Line using FIFO Structure

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ABSTRACT

An improved predicted tracer liner (PTL) was presented in [1] by Zhu and Gong. We address problem of the implementation of the improved formula using First-in-First-out (FIFO) structure in this paper. Number of numerical examples are given to show the differences of three versions of PTL principle formulaes, where the inertial nevigation (IN) data required by them are generated by an integrator which numerically integrates the Euler angular kinetical equations. The calculation of characteristic point on the tracer line by interpolation are also investigated.

Key Words: Avionic System, Airborne Weapon, Sight System,Numerical Algorithm

1. Introduction

In a recent paper [1], Zhu and Gong presented an improved PTL principle formula under the assumption that the attacker is in circular motion at constant speed. Their main idea is to give the differences between the attitudes of the attacker at the moment the projectile is shouted and that at the moment it is observed by integrating Euler angular kinetical equation,and thus express straightward the projectile vector in the attacker body coordinate frame at the observing moment without employing inertial coorinate frame as an intermediate means, as the derivation of the traditional PTL formula requires. At first glance, the structure of the improved algorithm seems a little more involved, since it deals with numerical solution of a set of nonlinear ordinary differential equations which has no explicit form unless some additional assumptions are made. However ,as pointed in [1], such solution of the differential equations are in fact unnecessary when the improved scheme is executed in the airborne mission computer (MC), especially in the environment of bus-based avionic system, for the required backword differences of attacker attitudes can readily obtained from historical IN data available in the bus of the system.

There are several reasons for which we go on our reserch on the improved PTL formula in [1]. First of all, the improved principle formula is logically more reasonable and cover wider range than any alternate version so far available, while in view of the development of modern integrated avionic system, the algorithmetic structure of the improved scheme are far from being too complex. Secondly, even a crude expression for the characteristic point on PTL are extremly difficult to find since the improved scheme is not expressed in completely analytical form. therefore a practical interpolation algorithm has to be developed, which is a topic addressed in [1]. At last,the suggestion that the improved PTL scheme executed by employing First-in-First-out (FIFO) structures has not been verified in terms of numerical algorithm and software development, which significant step for many noval theoretical results before they are effectively applied.

an approximate version of the improved PTL principle formula was also published in [1] under some extra assumptions. It turns out that in the case that the historical IN data are available, the approximate version is no longer appreciated in that its algorithmic structure is not less involved than its presise version.

the layout of this paper is organized as follows: In Section 2 we briefly review the main results presented in [1] since they have not imported in English; the problem of the calculation of characteristic point on PTL using binary tree is addressed in Section 3; We investigate considerable number of numerical examples in Section 4; And Section 5 is a brief conclusion of this paper.

2. A Brief Review

The object of this paper is to briefly review the main results of [1], we starts with assuming that:

A1) The attacker is travelling in circular motion at constant speed;

A2) The gun turret of the attacker is fixed;

A3) The angle of attack and slide angle of the attacker are all small, and remain constant within the time of fllight of the nominal projectile;

A4) The differences of the attacker attitudes are small within the time interval under consideration.

In addition , it is also assumed that

A5) The disturbances on the ballistical parametres due to the change of the projectile height can be neglected.

Then by the kinetic relationship showm in Fig.1, we immediately have

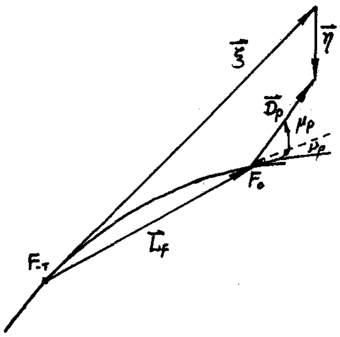

FIG.1 Geometry of PTL

$$\bar{L}_F + \bar{D}_P = \bar{\xi} + \bar{\eta} \qquad (2.\ 1)$$

where in terms of IN data measured at the observing time

$$\bar{L}_F = \bar{V}_1 T - (\bar{\omega}_f \times \bar{V}_1)\frac{T^2}{2} \qquad (2.\ 2)$$

Now decompositing Eqn.(2.1) in the attacker body coordinatr frame at the observing moment, and taking A3) and A4) into account, it is routine to show that

$$\begin{aligned} R_1 &= D_P \mathrm{COS}\mu_P \mathrm{COS}\nu_P \\ &= (V_{AV} - V_1)T - \\ &\quad - (b_\varphi \mathrm{SIN}\theta + V_1(\beta\omega_{Y_1} + \alpha\omega_{Z_1}))\frac{T^2}{2} \end{aligned} \qquad (2.\ 3)$$

$$\begin{aligned} R_2 &= D_P \mathrm{SIN}\mu_P \\ &= -V_{AV} T \Delta\theta(T) - (1 - \frac{V_{AV}}{V_{01}})V_1 \alpha T + \\ &\quad + ((\omega_{Z_1} - \beta\omega_{X_1})V_1 - b_\varphi \mathrm{COS}\theta \mathrm{COS}\gamma)\frac{T^2}{2} \end{aligned} \qquad (2.\ 4)$$

$$\begin{aligned} R_3 &= D_P \mathrm{COS}\mu_P \mathrm{SIN}\nu_P \\ &= -V_{AV} T \Delta\varphi(T) - (1 - \frac{V_{AV}}{V_{01}})V_1 \beta T + \\ &\quad + ((\omega_{Y_1} - \alpha\omega_{X_1})V_1 - b_\varphi \mathrm{SIN}\gamma \mathrm{COS}\theta)\frac{T^2}{2} \end{aligned} \qquad (2.\ 5)$$

where

$$V_{AV} = V_{01} - C_H D K_{V_{AV}}(C_H D, V_{01}) \qquad (2.\ 6)$$

$$b_\varphi = g\left(\frac{V_{AV}}{V_{01}}\right)^2 g_\varphi(C_H D, V_{01}) \qquad (2.\ 7)$$

are given by Aero Exterior Ballistics [3], and the differences of the attacker attitudes are governed by the well-known Euler angular kinetic equation [4]

$$\begin{aligned} \frac{d\gamma}{dt} &= \omega_{X_1} - (\omega_{Y_1}\mathrm{COS}\gamma - \omega_{Z_1}\mathrm{SIN}\gamma)\mathrm{tg}\theta \\ \frac{d\varphi}{dt} &= \frac{(\omega_{Y_1}\mathrm{COS}\gamma - \omega_{Z_1}\mathrm{SIN}\gamma)}{\mathrm{COS}\theta} \\ \frac{d\theta}{dt} &= \omega_{Y_1}\mathrm{SIN}\gamma + \omega_{Z_1}\mathrm{COS}\gamma \end{aligned} \qquad (2.\ 8)$$

If we assume in addition that

$$\frac{d\gamma}{dt} \approx 0, \quad \omega_{X_1} \approx 0,$$

then the solution of (2.8) can be approximated by

$$\begin{cases} \Delta\varphi(T) = (\omega_{Y_1}\mathrm{COS}\gamma - \omega_{Z_1}\mathrm{SIN}\gamma)\dfrac{T}{\mathrm{COS}\theta} \\ \Delta\theta(T) = (\omega_{Y_1}\mathrm{SIN}\gamma + \omega_{Z_1}\mathrm{COS}\gamma)T \end{cases} \qquad (2.\ 9)$$

and we thus have an approximate version of the PTL principle fomula. the detailed derivation of the above expressions can be found in [1]. therefore by vertue of the explicit geomatric relationship, it follows

$$\begin{cases} \nu_P = \mathrm{tg}^{-1}\left(\dfrac{R_3}{R_1}\right) \\ \mu_P = \mathrm{tg}^{-1}\left(\dfrac{R_2 \mathrm{COS}\nu_P}{R_1}\right) \\ D_P = (R_1^2 + R_2^2 + R_3^2)^{\frac{1}{2}} \end{cases} \qquad (2.\ 10)$$

Remarks: In fact, when the improved PTL principle formula is implemeted especially in the environment of bus-based avionic system, the IN data required by the proceding process may be measured, recorded and then in two copies of FIFO list before the moment the nominal tracer point is calculated. This makes our improved schadual very simple to execute.

Secondly, it is an interesting fact that under the conditions described at the beginning of this section,the improved PTL principle formula or the associated operating versions can be processed in a parallel manner, for the calculation of the tracer point nominally lunched at any moment does not depend upon that of other points.

It is also pointed that the above results are applicable to locate the laser axis in air combat trainer where the projectile is replaced by a beam of laser.

To conclude this Section, we point out that the main idea in [1] applies to the calculation of general tracer line with slight change ,say for instance, the terms associated with the attacker accelerate being expressed by integral with respect time of flight of the nominal projectile, rather than simply multiplied by it. However, it is noted that such generalization may make the principle formula impractically involved.

3. Interpolation of the Characteristic Point

Recall that we introduce the backward differences of the attacker attitudes, namely $\Delta\theta(T)$ and $\Delta\varphi(T)$, in Eqns.(2.5) and (2.6). In principle, they are given by the numerical integral of the Euler equation in closed form does not exist. This results in that an analytical expression, even a crude one, is impossible to given. thus one has to calculate it by interpolation.

Fortunately, such interpolation turns out to be very practical,for when it is displayed on the head-up-display (HUD), the tracer line is none than a collection of several straight segment. Thus the calculation of the characteristic point on PTL may be completed by linear interpolation,namely

$$\mu_p(D)=\frac{\mu_p(i+1)-\mu_p(i)}{D_p(i+1)-D_p(i)}(D-D_p(i))+\mu_p(i) \quad (3.\ 1)$$

$$\nu_p(D)=\frac{\nu_p(i+1)-\nu_p(i)}{D_p(i+1)-D_p(i)}(D-D_p(i))+\nu_p(i) \quad (3.\ 2)$$

$$i=0,1,2,\ldots,N-1$$

where $\mu_p(i)$, $\nu_p(i)$ and $D_p(i)$ stand for elevation, azimuth and range of the tracer point nominally shouted at the moment $-iT$, respectly. They are obtained from Eqn.(2.13). D denoted the measured (and usually then filtered) ranger of the actual target. N denotes number of tracer point on PTL and usually is taken to be 6.

Following the general idea in parallel process, we apply the binary search whose flow chart is shown in Fig.2, when we determine which segment of the displayed tracer line the actual target range, lies in.

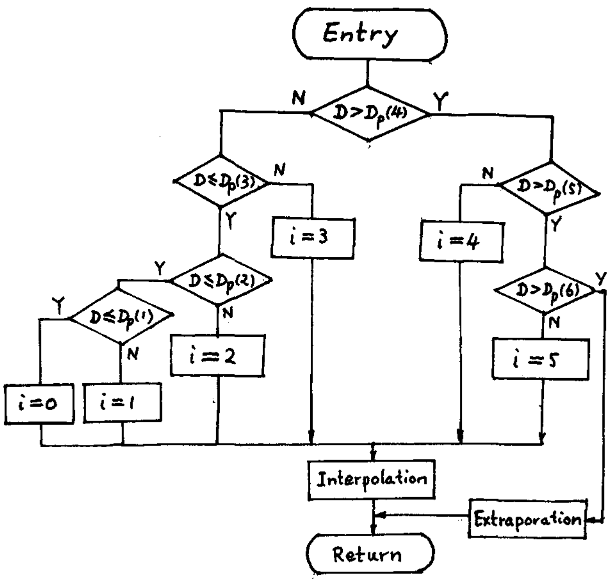

FIG.2 Flowchart for Characteristic Point Calculation

Taking the fact that D is always positive into account, one can easily evaluate that the average test times as follows:

$$E(t)=\frac{2}{3}+\frac{3}{2}+\frac{4}{5}=2.9 \text{ (times)},$$

while if we search segment by segment, then

$$E(t)=\sum_{i=1}^{6}\frac{i}{6}=3.5\text{(Times)},$$

We could obtain more benifit from the binary search if the number of the tracer point were larger.

Thus in the scheme of this paper, in addition to an average 2.9 times of test, once of division, twice of multiplication, and five times of adddition are needed to completation the characteristic point.

4. Numerical Results

All the examples in this paper are completed by a general-purpose software developed by the authors. The software is programed in FORTRAN 77 and the only way to impletement a FIFO queue is to use arrays. This might be a little unconvenient. However this will not matter much since all the operations on the queues here are in-and out-queue and the length of the queues is fixed. The structure of the software is shown in Fig.3 .

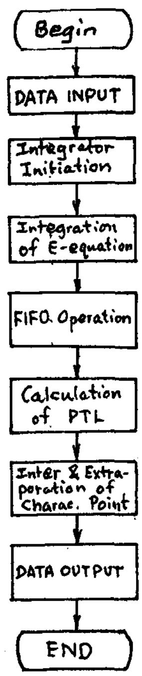

FIG.3 Flow Chart of The Improved PTL Calculation

The software can be used to evaluate the principle errors of every version of PTL operating formulaes Large number of examples are calculated. The purpose of this paper is to give some typic examples to shown the differences between the improved formula and tranditional formula in terms of linear errors and angular errors, respectly. We also examine those between two versions improved principle formula. The common conditions of flight and projectile are listed below:

$V_0 = 750\ m/s$,

$V_1 = 250\ m/s$,

$C_H D = 2400$ m,

$K_{V_{AV}} = 0.135$,

$g_{\varphi} = 1.721$,

$\alpha = 2^{\circ}$,

$\beta = 1^{\circ}$,

$\varphi_{\theta} = 10^{\circ}$,

$\omega_{Y_t} = 0.08\ s^{-1}$.

4.1 The Attacker Is Circling on a Horizontal Plane

Firstly we examine the case that the attacker is moving on horizontal plane, namely

$\omega_{Z_t} = 0.0, \quad \omega_{X_t} = 0.0, \quad \theta_{\theta} = 0.0.$

The attitudes of the attacker listed in all figures is in sampled at 1.5 s before the nominal projectile is observed.

The linear errors and angular errors of the traditional principle formula (PTL) are illustrated in Fig.4 and Fig.6, respectly, and those of the approximate improved principle formula (AIPF) in Fig.5 and Fig.7, with different conditions.

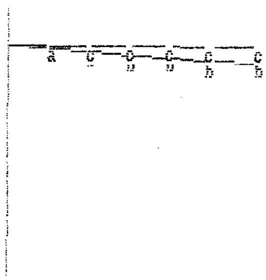

FIG.4
Linear Errors of TPF

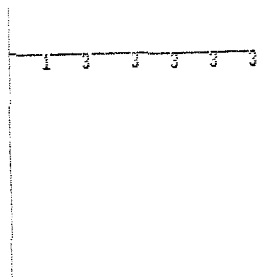

FIG.5
Linear Errors of AIPF

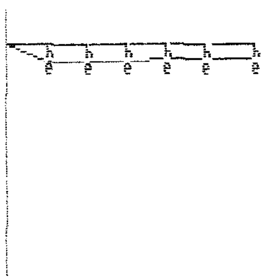

FIG.6
Angle Errors of TPF

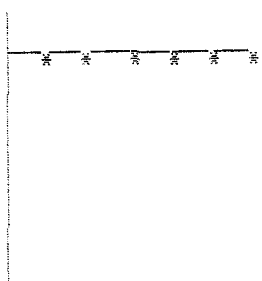

FIG.7
Angle Errors of AIPF

It is seen that the attacker azimuth imposed little effect upon the calculation for both TPF and AIPF. Both formulaes work very well under the conditions listed under the bottom of Fig.4–7, except that R of TPF contains a very small linear item due to the assumption that

$\beta = 0.0$

4.2 The Effect of $\omega_{X_t} \neq 0$

Next, we examine the effect due to ω_{X_t} not equal to zero. The results for TPF are shown in Fig.8 and FIG.10 for $\omega_{X_t} = 0.075 s^{-1}$. The analogue results for AIPF are illustrated in Fig.9 and Fig.11, respectly.

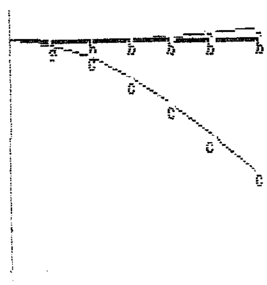

FIG.8
Linear Errors of TPF

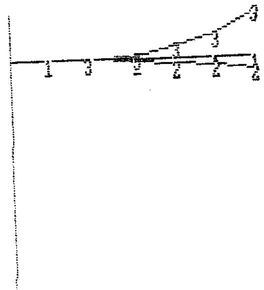

FIG.9
Linear Errors of AIPF

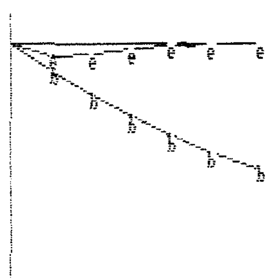

FIG.10
Angle Errors of TPF

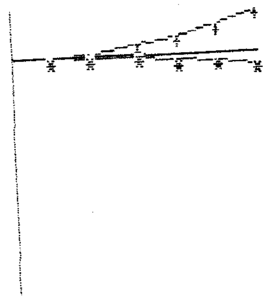

FIG.11
Angle Errors of AIPF

It is seen that the effectness of ω_{X_t} on both TPF and AIPF is raletively heavy, but AIPF seems to works a litter bettle than TPF.

4.3 The Effect of $\omega_{Z_t} \neq 0$

Now let us see how effect the accuracy of both the TPF and AIPF formulaes. The results for the TPF for

$\omega_{Z_t} = 0.06\ s^{-1}$

are shown in Fig.12 and Fig.14. while the analogue results for the AIPF in Fig.13 andFig.15.

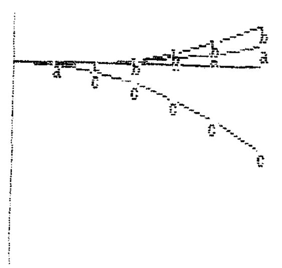

FIG.12
Linear Errors of TPF

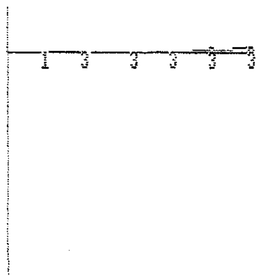

FIG.13
Linear Errors of AIPF

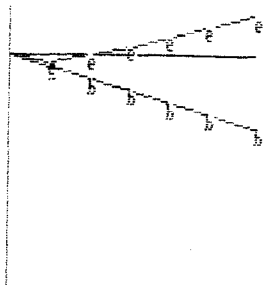

FIG.14
Angle Errors of TPF

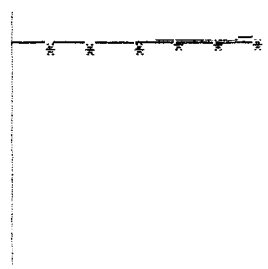

FIG.15
Angle Errors of AIPF

It is shown that a relatively large ω_{z_1} may effect significantly the accuracy of TPF. As a comparision, AIPF seems have much improvement in accuracy.

4.4 The Effect of $\theta_0 \neq 0$

From Fig.16 and Fig.18, it is seen that the pitch of attacker effects significantly upon the accuracy of the TPF. It is not strange, for to have the following hold [3],

$$\frac{d\varphi}{dt}\mathrm{SIN}\gamma - \frac{d\theta}{dt}\mathrm{COS}\gamma = -\omega_{z_1}$$

$$\frac{d\varphi}{dt}\mathrm{COS}\gamma - \frac{d\theta}{dt}\mathrm{SIN}\gamma = \omega_{Y_1}$$

It is required that $\theta = 0$. The results for AIPF are illustrated in Fig.17 and Fig.19 it is seen that θ has a relatively small impact upon the AIPF.

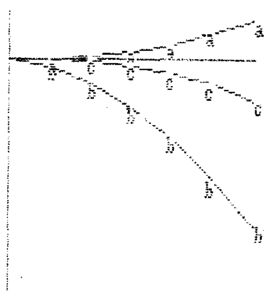

Fig.16
Linear Errors of TPF

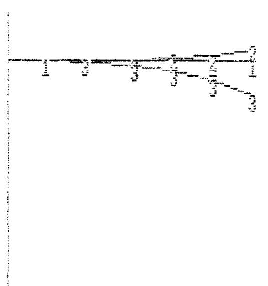

Fig.17
Linear Errors of AIPF

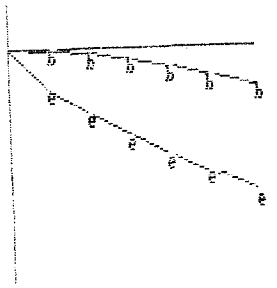

Fig.18
Angle Errors of TPF

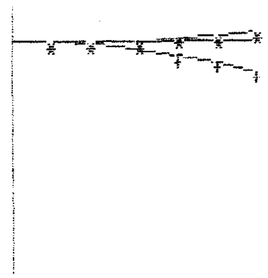

Fig.19
Angle Errors of AIPF

Remark: It is obvious that if θ is close to $\pi/2$, the problem of numerical stability will occur. The singularity is caused by the incompletity of the mathematical description of the rigid kinetical relationship. Fortunately, this problem occurs only in extremely rare case.

5. Conclusion

The problem of the calculation of the improved PTL scheme in [1] using FIFO structure is examined. It is shown that the improved scheme is by itself applicable if the bus-based avionic system is employed. However there is room for its simplication. For example, it is shown by experience that the elevation μ_p and the azimuth ν_p of the tracer point are small and thus the revelent triangular functions can be reasonably appoximated. Another improvment may be made by imploying a more accurate exterior ballistic algorithm, which is preferrable in a practical operating flight software (OFP). It is expected that such improvements will make our scheme more attacting in view of practice. A better interpolation algorithm for the characteristic point on the improved tracer line is also addressed,since there is no analytical expression for it. Thus such study is an important step for the utilization of the improved PTL scheme.Considerable examples are given to illustrated the principle differences of the other versions of the PTL formula.

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Notations

T Time of flight of the nominal projectile

F_{-T} Position of the attacker when the nominal projectile is fired

F_0 Position of the attacker when the nominal projectile is observed

$\bar{L}_s$ Shift vector of attacker within T

$\bar{\xi}$ Projectile vector

$\bar{\eta}$ Gravity drop vector

$\bar{D}_P$ Projectile vector relative to the observing point

μ_P Elecvation of D

ν_P Azimuth of D

$\bar{\omega}_f$ Rotating rate of the attacker

α Angle of attack of the attacker

β Slide angle of the attacker

θ Pitch of the attacker

φ Yaw of the attacker

γ Roll of the attacker

V_0 Muzzle velocity of the projectile

V_{01} Absolute velocity of the projectile

V_{AV} Average velocity of the projectile during the time of flight

V_f Air speed of the attacker

D Target range