SENSITIVITY ANALYSIS OF A WING AEROELASTIC RESPONSE

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ABSTRACT

This paper describes the progress made in an ongoing research program on the shape sensitivity analysis of a wing aeroelastic response. As a first step, the aeroelastic response sensitivities were obtained using aerodynamic capabilities which are valid for high aspect ratio wings in subsonic, subcritical flow. For example, the sensitivity of the static aeroelastic responses to various shape parameters were obtained using Weissinger's L-Method to model the spanwise distribution of lift. Similarly, the sensitivity of the flutter response was obtained using modified classical strip theory. In these earlier studies, an equivalent plate analysis model for the wing structure was used. At present, efforts are being made to study the shape sensitivity of various static aeroelastic responses using a more realistic aerodynamic model. The formulation is quite general and accepts any aerodynamic analysis capability. It assumes that for a given shape and elastic deformation, the aerodynamic analysis will provide the distribution of the pressure and the pressure sensitivity derivatives with respect to the shape parameters of interest. The pressure from the aerodynamic code is represented as a double series in Chebyshev polynomials. The displacements of the wing are obtained using an iterative scheme. Equations have been derived to obtain the various global sensitivities in terms of local sensitivities. To illustrate the present methodology, an aerodynamic model based on lifting surface theory is being used and some preliminary results are presented.

INTRODUCTION

During the design phase of an engineering system, numerous analyses are conducted to predict changes in the characteristics of the system due to changes in design variables. Usually, this process entails perturbing each variable in turn, recalculating the characteristics, and evaluating the sensitivities with some sort of finite-difference process. The repeated analyses can drive the cost of design very high. An approach that has found increased interest recently in engineering design is analytical calculation of the sensitivity derivatives[1]. Typically, the analytical approach requires less computational resources than the finite-difference approach and is less subject to numerical errors (round-off or truncation). The analytical approach is best developed in parallel with the baseline analysis capability since it uses a significant portion of the numerical information generated during baseline analysis. In the design of modern aircraft, airframe flexibility is a concern from strength, control, and performance standpoints. To properly account for the aerodynamic and structural implications of flexibility, reliable aeroelastic sensitivity analysis is needed. Therefore, both structural and aerodynamic sensitivity analysis capabilities are necessary.

Structural sensitivity analysis methodology has been available for well over two decades for both sizing (thickness, cross-section properties) and shape (configuration) variables[2]. However, aerodynamic sensitivity analysis has been nonexistent until relatively recently. Some limited aerodynamic sensitivity analysis capability was developed for aircraft in subcritical compressible flow by Hawk and Bristow[3], but it only handled perturbations in the direction of the thickness of the wing (thickness, camber, or twist distribution). Yates[4] proposed a new approach that considers general geometry variations including planform for subsonic, sonic, and supersonic unsteady, nonplanar lifting-surface theory.

Aeroelastic sensitivity analysis methodology has also been available for more than two decades for structural sizing variables (see Haftka and Yates[5]). This is because changes in sizing variables exclusively affect the structural stiffness and mass distribution of the airframe and not its basic geometry. Therefore, structural sensitivity analysis capability is sufficient. However, the lack of development in aerodynamic shape sensitivity analysis explains why there are very few results in aeroelastic shape sensitivity analysis. In a notable exception, Haftka et al.[6] designed a sailplane wing under aeroelastic constraints and analyzed the design model with vortex lattice and finite element methods. A finite-

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difference aeroelastic sensitivity analysis capability is made possible by (1) devising a reduced order model to describe the wing static aeroelastic response and (2) using exact perturbation analysis to approximate changes in the vorticity vector with changes in the geometry.

Barthelemy and Bergen[7] demonstrated the feasibility of calculating analytically the sensitivity of wing static aeroelastic characteristics to changes in wing shape. Of interest also was the factor that the curvature of the aeroelastic characteristics was small enough that analytical sensitivity derivatives could be used to approximate them without costly reanalyses for large perturbations of the design variables. A brief description of this work will be given subsequently.

The dynamic aeroelastic phenomena is also of interest to designers and it would be advantageous to the aircraft designers to have a tool that can be used to predict the changes in flutter speed with the changes in basic shape parameters.

As is the case for static aeroelastic response, sensitivity calculations have only been available for structural sizing parameters. For examples, Rudisill and Bhatia[8] developed expressions for the analytical derivatives of the eigenvalues, reduced frequency and flutter speed with respect to structural parameters for use in minimizing the total mass. However, this method is limited because the structural parameters are sizing variables such as cross-sectional areas, plate thickness and diameters of spars.

Pedersen and Seyranian[9] examined the change in flutter load as a function of change in stiffness, mass, boundary conditions or load distribution. They showed how sensitivity analysis can be performed without any new eigenvalue analysis. The solution to the main and an adjoint problem provide all the necessary information for evaluating sensitivities. Their paper mainly focuses on column and beam critical load distributions.

In a recent study, Kapania, Bergen and Barthelemy[10] obtained the sensitivity of a wing flutter response to changes in its geometry. Specifically, the objective was to determine the derivatives of flutter speed and frequency with respect to wing area, aspect ratio, taper ratio, and sweep angle. The study used Giles[11,12] equivalent plate model to represent the wing structure. The aerodynamic loads were obtained using Yates[13] modified strip analysis to analyze flutter characteristics for finite span swept and unswept wings. It is noted that Yates modified strip theory was used quite recently by Landsberger and Dugundji[14], with a modification for camber effects given by Spielberg[15], to study the flutter and divergence of a composite plate.

At present, research is being carried out to study the sensitivity of various static aeroelastic responses to various shape parameters using a more realistic aerodynamic models. The formulation is designed to be quite general so that it is applicable with any aerodynamic code which, for a given geometry and structural deformations, provides aerodynamic pressures on the wing surface. To facilitate the calculation of the shape sensitivities of various quantities (required in aeroelastic analyses), the pressure distribution is first represented as a double series in Chebyshev polynomials. The displacements of the wing are being obtained using an iterative scheme. A formulation is given to obtain the various global sensitivities (i.e. including all interdisciplinary interactions) in terms of local sensitivities (i.e. the sensitivities obtained at the discipline level). To validate this more general formulation, sensitivity of the static aeroelastic response of an example wing is being obtained. The pressure distribution on the wing is being obtained using the code FAST[16]. Whenever possible, the present results are compared with previously available results.

MATHEMATICAL FORMULATION

This section gives the details of the mathematical formulations used in this study. For the sake of completeness and comparison, a brief description of the research work presented in Reference 7 is first given. Mathematical formulation used in the research currently being pursued is next described.

Structural Model:

In this research program, Giles[11,12] equivalent plate model is being used. This program has the capability to model aircraft composite wing structures with general planform geometry such as cranked wing boxes. The program, based on Ritz method, allows modelling of unsymmetric wing cross sections which can arise from airfoil camber or from having different thicknesses in the upper and lower cover skins.

The transverse deflection $W(x,y)$ of the wing is represented as

$$W(x,y) = \sum_{n=0}^{N} \sum_{m=2}^{M} C_{nm}(\frac{x}{x_{max}})^n(\frac{y}{y_{max}})^m \quad (1)$$

Here N and M were limited to 5 and 6 terms, respectively, in order to prevent numerical difficulties in the manipulation of the matrices, C_{nm} the set of unknown coefficients.

The deflection equation can also be written as

$$W(x,y) = \sum_{i=1}^{np} \gamma_i(x,y)\ C_i \quad (2)$$

where $\gamma_i(x,y)$ are the nondimensional displacement functions that satisfy the geometric boundary conditions for a cantilever plate. The displacement functions are nondimensional quantities in order to prevent numerical difficulties. The expressions for the strain energy, the potential of the applied loads and the kinetic energy can be easily written.

The principle of virtual work may be used to obtain the coefficients $C's$. For static problems, this results in a set of linear equations given by:

$$[K]\{C\} = \{Q\} \tag{3}$$

where $[K]$ is the stiffness matrix; $\{C\}$ is the vector of undetermined coefficients, and $\{Q\}$ is the vector of generalized forces.

The matrix $[K]$ and vector $\{Q\}$ are both dependent upon the vector of shape parameters $\{r\}$. The vector $\{Q\}$ for aeroelastic studies also depends upon the generalized displacement vector $\{C\}$. Note that the vector $\{Q\}$ will depend upon the aerodynamic model used. In this study, two different models are used: the Weissinger's L-method[17,18] and the lifting surface theory used in the program FAST[16]. This aerodynamic code can be considered to be a lifting surface theory as opposed to the Weissinger's L-method which is based on lifting line theory.

Barthelemy – Bergen Approach:

As stated, Weissinger's L-method[17], as implemented for computations by DeYoung and Harper[18], was used to represent the aerodynamic loads. It is valid for moderate-to-high-aspect-ratio wings that are symmetric with respect to the root chord, have a straight quarter-chord line over each semispan, and have no discontinuities in twist. The airfoil section properties are assumed known and the flight regime may be compressible although it must be subcritical. In this method, the flow around the wing is modeled by a lifting line of vortices bound at the wing quarter-chord line. A no-penetration boundary condition is specified at na control stations and that determines the spanwise distribution of vortex strength. In Weissinger's L-method the boundary conditions are enforced at the three-quarter-chord point of each station. In DeYoung-Harper modification, the boundary conditions are applied so as to account for lift-curve slopes of less than the theoretical value of 2π and also for effects of compressiblity.

In this method, a linear relationship between the vectors of local angles of attack and lift at the control station results:

$$\{\alpha\} = \frac{1}{2b}[\bar{A}]\{cc_\ell\} \tag{4}$$

Here $\{\alpha\}$ is a vector of angle of attacks along the wing at aerodynamic control points, b is the wingspan, $\{\bar{A}\}$ is the aerodynamics matrix that depends upon the airfoil properties, the Mach Number as well as the wing shape, and $\{cc_\ell\}$ is the vector of product of section lift coefficients and chord length at aerodynamic control stations.

The total lift developed by the full-span wing is then given by

$$\frac{nW}{2} = \frac{1}{2}bq\ \{u\}^T\ [V]\ \{cc_\ell\} \tag{5}$$

where q is the dynamic pressure, matrix $[V]$ is a diagonal matrix containing shape independent weights, $\{u\}$ is a vector whose each element is equal to unity, n is the load factor and W is the weight of the airplane.

The vector of generalized forces $\{Q\}$ can be written as

$$\{Q\} = [W]^T\ \{f\} \tag{6}$$

where $[W] = W_{ij}$, is the value of entry j of vector $\{w\}$ at the points of application of load i, $\{w\}$ = vector of displacement shape functions, and $\{f\}$ is the vector of applied forces and depends (implicitly) upon the generalized coordinates, and is given as

$$\{f\} = \frac{1}{4}\ bq\ [V]\ \{cc_\ell\} \tag{7}$$

The vector of angles of attack at the control points can be written as follows:

$$\{\alpha\} = \alpha_0\{u\} + \{\epsilon\} + \{\theta\} \tag{8}$$

where $\{\epsilon\}$ and $\{\theta\}$ are respectively, the vectors of pretwist and elastic angles of attack at the control points. The elastic twist can be written as

$$\theta_i = -\frac{\partial}{\partial x}w(x_i, y_i) \tag{9}$$

For consistency with the aerodynamic model, the elastic twist is measured at the three quarter chord. Then

$$\{\theta\} = -[W_x]\ \{C\} \tag{10}$$

Combining Eqs. (1), (4), and (7)-(10), as well as the trim equation (5), we can obtain the unknown angle of attack and the spanwise distribution of lift from:

$$\begin{bmatrix} [\bar{A}] + (qb^2/2)[W_x][K]^{-1}[W]^T[V] & -2b\{u\} \\ (b/2)\ \{u\}^T\ [V] & 0 \end{bmatrix} \begin{Bmatrix} \{cc_\ell\} \\ \alpha_0 \end{Bmatrix} = \begin{Bmatrix} 2\ b\{\epsilon\} \\ nW/2q \end{Bmatrix} \tag{11}$$

The sensitivity equations can be obtained by taking the derivative of above equation $w.r.t.$ the shape parameters. The details can be found in Ref. 7.

Present Approach:

The aerodynamic model used in Ref. 7 is a restricted model (i.e. restricted to large aspect ratio wings) and the formulation employed was specific to Weissinger's L-method. It is advantageous to use more realistic aerodynamics and to develop a formulation which is not specific to the aerodynamic capability being employed. Anticipating the availability of nonlinear aerodynamic models in the future, it is also desired to have a formulation that does not assume a linear dependance between the lift generated and the generalized coordinates and the initial angle of attack. This needs an iterative process to calculate the trim angle of attack to produce required lift.

The governing equations of motion for the aeroelastic analysis and the lift can be written as

$$[K(\{r\})]\ \{C\} = \{Q(\{r\},\ \alpha, \{C\})\} \tag{12}$$

$$\frac{nW}{2} = \int\int_\Omega p(x, y, \{r\}, \{C\})d\Omega \tag{13}$$

where $\{r\}$ = vector of shape variables, namely sweep, taper ratio, semispan, and aspect ratio; α is the angle of attack, and $\{C\}$ is the vectors of generalized displacements, n is the load factor, W is the weight of the airplane, and $p(x,y)$ is the pressure distribution on the wing, Ω is the wing surface area.

The vector of generalized forces can be obtained as:

$$Q_i = \int\int_{\Omega} p(x,y)\ \gamma_i(x,y)\ dx\ dy \qquad (14)$$

where $\gamma_i(x,y)$ is the ith displacement function used in the structural model (Eq. 2)

To facilitate both the integration and subsequent sensitivity calculations, a co-ordinate transformation (see Fig. 1) was used to simplify the integration limits. This was accomplished using the following transformation:

$$x(\eta,\xi) = \sum_{j=1}^{4} N_j(\eta,\xi)x_j \qquad (15)$$

$$y(\eta,\xi) = \sum_{j=1}^{4} N_j(\eta,\xi)y_j \qquad (16)$$

where $N_j(\eta,\xi)$ are the shape functions and the x_j and y_j are the co-ordinates of the four corner points of the wing. The shape functions are given as

$$N_i(\eta,\xi) = (1+\xi\xi_i)\ (1+\eta\eta_i)/4 \qquad (17)$$

where η_i and ξ_i are the coordinates of the node i in $\eta-\xi$ system. Note that this transformation will change the domain of the wing to a square ($-1 \leq \eta \leq 1\,; -1 \leq \xi \leq 1$).

As a first step to obtain the generalized forces, the pressure distribution on the wing was represented as

$$p(\eta,\xi) = \sum_{j=1}^{M} \beta^j(\eta,\xi)\ a^j \qquad (18)$$

where a^j can be considered as the generalized pressure coefficients and $\beta^j\ (\eta,\xi)$ are some known interpolation functions of η and ξ. A large number of interpolating polynomials are available in the literature[19,20]. In this study, a tensor product of Chebyshev polynomials is used.

The pressure distribution can thus be written as

$$p(\eta,\xi) \ = \ \sum_{q=1}^{Q}\ \sum_{p=1}^{P}\ a_{pq}\ T_p(\eta)\ T_q(\eta) \qquad (19)$$

Here, $T_p(\cdot)$ is the Chebyshev polynomial of order p.

The coefficients a_{pq} can be easily obtained if the value of the pressure coefficients at the zeros of the Chebyshev polynomials are known.

The integral for a generalized force Q_i, then becomes

$$Q_i = \int_{-1}^{1}\int_{-1}^{1}\Big(\sum_{q=1}^{Q}\sum_{p=1}^{P} a_{pq}\ T_p(\eta)T_q(\xi)\Big) \gamma_i(\eta,\xi)|J(\eta,\xi)|\ d\eta\ d\xi \qquad (20)$$

where $|J(\eta,\xi)|$ = Jacobian of the coordinate transformation. The generalized force Q_i can be written as

$$Q_i = \sum_{j=1}^{M} A_{ij}\ a^j \qquad (21)$$

In matrix form;

$$\{Q\} = \ [A]\{a\} \qquad (22)$$

where a typical term A_{ij} is given as

$$A_{ij} = \int_{-1}^{1}\int_{-1}^{1} \beta^j(\eta,\xi)\gamma_i(\eta,\xi)\ |J(\eta,\xi)|\ \ d\eta\ d\xi \qquad (23)$$

Similarly, the lift equation can be written as:

$$\frac{nW}{2} \ = \ \sum_{j=1}^{M}\ a^j\ L^j \ = \ \{L\}^T\{a\} \qquad (24)$$

where

$$L^j = \int_{-1}^{1}\int_{-1}^{1} \beta^j(\eta,\xi)\ |J(\eta,\xi)|\ \ d\eta\ d\xi \qquad (25)$$

Aeroelastic Response:

The aeroelastic response was obtained in an iterative fashion. In that, the pressure distribution on the wing is first obtained by assuming the wing to be rigid and having an angle of attack of 1° (throughout the span). The pressure distribution thus obtained is used to obtain the vector of generalized forces (Eq. 22) which in turn is used to obtain the vector of generalized displacements (Eq. 12). The elastic displacements are superimposed on the rigid wing and a new pressure distribution on the wing is obtained. This pressure distribution is then used to obtain the generalized displacements (using Eqs. 8 and 12). This process is repeated till a converged value is achieved for the total lift on the wing (obtained using Eq. 14). The trim angle of attack is obtained by dividing the total required lift by the converged value of the lift obtained for an angle of attack of unity. This is possible because a linear aerodynamic model is used. For a nonlinear aerodynamic model, an iterative process will be needed to obtain the trim angle of attack also .

Sensitivity Analysis:

Equations 12, 13, 22, and 24 can be used to perform the shape sensitivity analysis of static aeroelastic response. Taking derivatives of the equilibrium and the trim equation, w.r.t. the shape variable r_ℓ (namely sweep, aspect ratio, wing area, taper ratio), we obtain

$$[K]\{\frac{dC}{dr_\ell}\} + \left[\frac{dK}{dr_\ell}\right]\{C\} \ = \ \left\{\frac{dQ}{dr_\ell}\right\} \qquad (26)$$

$$\frac{d(nW/2)}{dr_\ell} = \left\{\frac{dL}{dr_\ell}\right\}^T \{a\} + \{L\}^T \left\{\frac{da}{dr_\ell}\right\} = 0 \qquad (27)$$

Note that the derivative of lift is zero, because we need to maintain the total lift acting on the wing, to be same.

The vector $\left\{dQ/dr_\ell\right\}$ can be obtained as

$$\frac{dQ_i}{dr_\ell} = \sum_{j=1}^{M}\left(\left[\frac{\partial A_{ij}}{\partial r_\ell}\right]a^j + A_{ij}\frac{da^j}{dr_\ell}\right) \qquad (28)$$

where

$$\frac{da^j}{dr_\ell} = \frac{\partial a^j}{\partial r_\ell} + \sum_{n=1}^{np}\left(\frac{\partial a^j}{\partial C_n}\frac{dC_n}{dr_\ell}\right) + \frac{\partial a^j}{\partial \alpha}\frac{d\alpha}{dr_\ell} \qquad (29)$$

where $\partial a^j/\partial r_\ell$ = local sensitivity of the aerodynamic generalized pressure coefficients and can be obtained while performing the aerodynamic analysis; $\partial a^j/\partial C_n$ is the derivative of the generalized pressure coefficient w.r.t. to a generalized displacement C_n, and $d\alpha/dr_\ell$ is the derivative of the trim angle of attack w.r.t. r_ℓ.

In matrix form, the global sensitivity of generalized forces become

$$\begin{aligned}\left\{\frac{dQ}{dr_\ell}\right\} &= \left[\frac{\partial A}{\partial r_\ell}\right]\{a\} + [A]\left\{\frac{\partial a}{\partial r_\ell}\right\} \\ &+ [A]\left[\frac{\partial\{a\}}{\partial\{C\}}\right]\left\{\frac{dC}{dr_\ell}\right\} + \frac{d\alpha}{dr_\ell}[A]\left\{\frac{da}{d\alpha}\right\}\end{aligned} \qquad (30)$$

The sensitivity of the generalized displacements, therefore, becomes

$$\begin{aligned}\left[[K] - [A]\left[\frac{\partial\{a\}}{\partial\{C\}}\right]\right]\left\{\frac{dC}{dr_\ell}\right\} &= \left[\frac{\partial A}{\partial r_\ell}\right]\{a\} + [A]\left\{\frac{\partial a}{\partial r_\ell}\right\} \\ &+ \frac{d\alpha}{dr_\ell}[A]\left\{\frac{da}{d\alpha}\right\} - \left[\frac{dK}{dr_\ell}\right]\{C\}\end{aligned} \qquad (31)$$

In this equation, all the terms on R.H.S. are known, except for $d\alpha/dr_\ell$. This can be obtained by considering the sensitivity of the lift equation, Eq. 27. This equation can be written as:

$$\begin{aligned}\left\{\frac{\partial L}{\partial r_\ell}\right\}^T\{a\} &+ \{L\}^T\left\{\frac{\partial a}{\partial r_\ell}\right\} + \{L\}^T\left[\frac{\partial\{a\}}{\partial\{C\}}\right]\left\{\frac{dC}{dr_\ell}\right\} \\ &+ \frac{d\alpha}{dr_\ell}\{L\}^T\left\{\frac{da}{d\alpha}\right\} = 0\end{aligned} \qquad (32)$$

The required sensitivity derivatives can then be obtained by simultaneously solving the sets of Eqs. (31) and (32). For the sake of brevity, the details of calculating various matrices in Eqs. (31) and (32) are not being given here.

It is noted that the sensitivity equations can be rewritten in a form that is somewhat similar to Sobieski's global sensitivity equations[21].

$$\left\{\begin{matrix}\dfrac{\partial\{a\}}{\partial r_\ell} \\ \dfrac{\partial\{C\}}{\partial r_\ell} \\ \left\{\dfrac{\partial L}{\partial r_\ell}\right\}^T\{a\}\end{matrix}\right\} = \begin{bmatrix} [I] & -\left[\dfrac{\partial\{a\}}{\partial\{C\}}\right] & -\dfrac{d\{a\}}{d\alpha} \\ -[K]^{-1}[A] & [I] & [0] \\ -\{L\}^T & 0 & 0 \end{bmatrix} \left\{\begin{matrix}\dfrac{d\{a\}}{dr_\ell} \\ \dfrac{d\{C\}}{dr_\ell} \\ \dfrac{d\alpha}{dr_\ell}\end{matrix}\right\} \qquad (33)$$

where the vector on the L.H.S. and the matrix on the R.H.S. are completely known.

NUMERICAL RESULTS

This section describes the numerical results obtained. These were obtained for a wing shown in Fig. 2. The results for some aeroelastic sensitivities (w.r.t. shape parameters) are first presented using lifting line (LL) type of aerodynamic theories. Some recently obtained results using lifting surface (LS) type of theory are subsequently presented.

Figure 3 shows the variation of the divergence dynamic pressure with respect to the sweep angle using lifting line theory. In Fig. 3, the results are also shown for the sensitivity of the dynamic divergence pressure with respect to the quarter-chord sweep angle. The results for shape sensitivities of other aeroelastic characteristics (lift at tip section, trim angle of attack, rolling moment, induced drag) with respect to other shape variables (surface area, aspect ratio, taper ratio) are given in Ref. 7.

Figure 4 shows the variation of the flutter speed with respect to surface area. The baseline configuration used for this analysis is also given in the figure. The sensitivity of the flutter speed with respect to the surface area is also given in Fig. 4. The results for sensitivity of flutter speed and the corresponding reduced frequency with respect to other shape variables is given in Ref. 10.

Results for shape sensitivity of static aeroelastic response are being obtained presently using a more realistic aerodynamics. The displacements of the wing are being obtained in a conventional iterative fashion.

Figure 5 shows the variation of the trim angle of attack with the sweep angle. For the sake of comparison, the results obtained from both lifting line and lifting surface theories are shown in Fig. 5. It is seen that, for the case of rigid wings, the lifting line (LL) and lifting surface (LS) theories yield results that have similar trends. However, for the case of elastic wing, the trends are similar for positive sweep angles only. The two sets of results are diverging from each other for negative sweeps (sweep forward). The reasons for this discrepancy are not clear at this stage. The results for trim angle of attack, as obtained using lifting line and lifting surface theories, for various wing configurations are given in Table 1. The same discrepancy between the LL and LS results that was observed in Fig. 5 for elastic swept forward wings can also be seen in the results presented in Table 1.

At present, the results are being obtained for the shape sensitivity of the aeroelastic responses.

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Table 1: Rigid and Elastic Trim Angle of Attack Comparisons
Lifting surface vs. Lifting Line ($q = 4000 N/m^2, M = 0$)

Sweep	Aspect Ratio	Taper Ratio	Rigid Wing L.S.	Rigid Wing L.L.	Elastic Wing L.S.	Elastic Wing L.L.
-20	7.50	0.50	6.625	6.75	6.299	5.36
-10			6.359	6.50	5.879	5.66
0			6.238	6.41	5.842	5.97
10			6.246	6.44	6.221	6.37
20			6.394	6.62	6.830	6.99
-20	9.50	0.75	6.298	6.45	5.991	4.10
-10			6.039	6.22	5.449	4.90
-20		1.00	6.373	6.53	6.048	3.90
-10			6.122	6.31	5.425	4.85
-20	11.50	0.50	5.991	6.15	5.655	3.36
-10			5.722	5.91	5.202	4.38
0			5.607	5.82	5.069	5.20

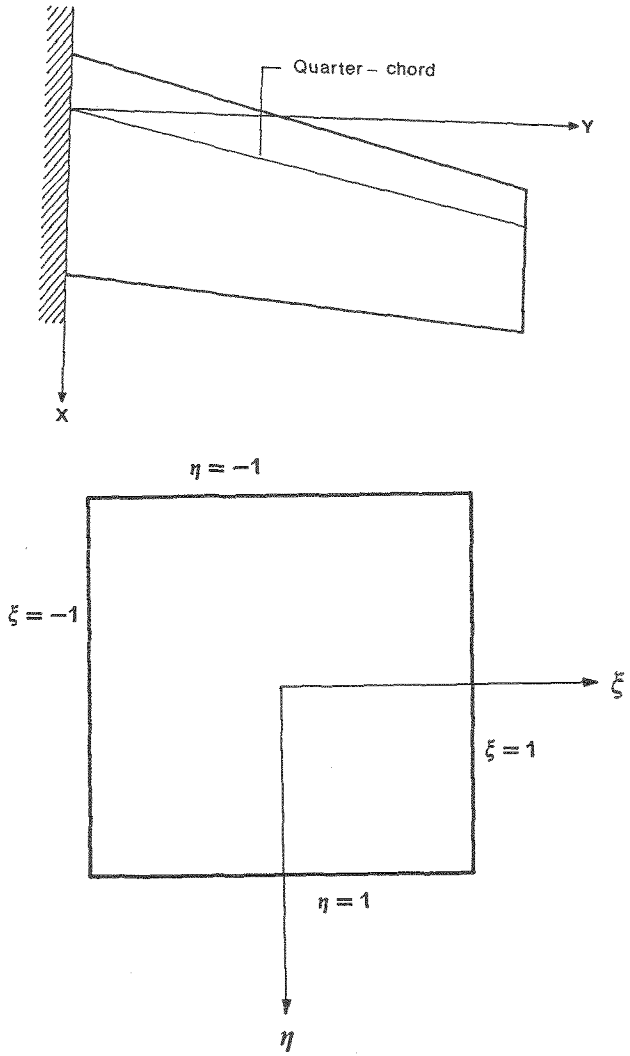

Fig. 1 Coordinate Transformation for the Wing.

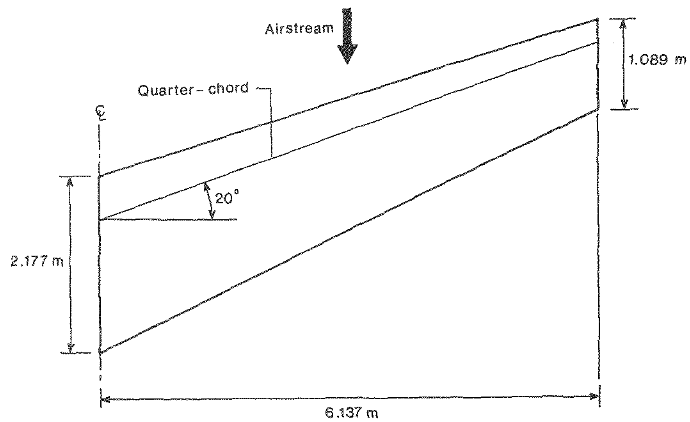

Fig. 2 Baseline Configuration for the Wing.

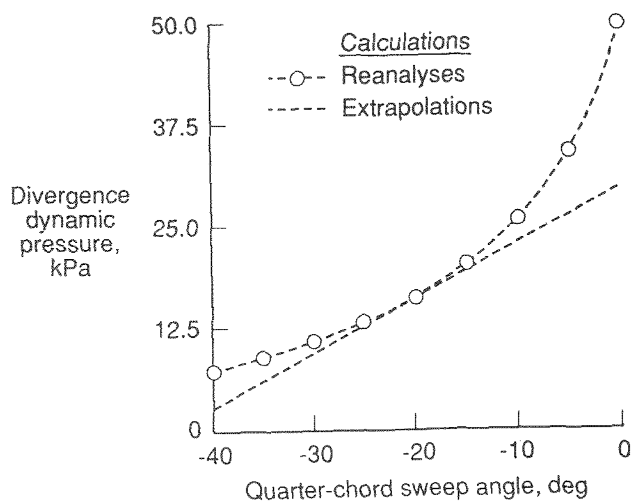

Fig. 3 Sensitivity of Divergence Dynamic Pressure to Sweep Angle.

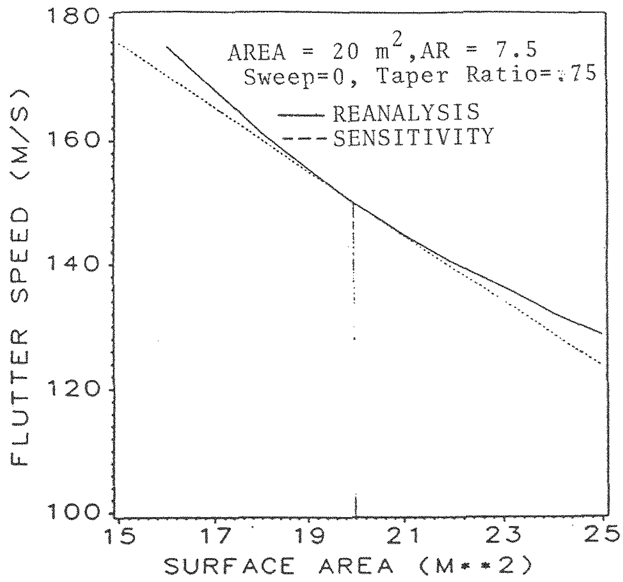

Fig. 4 Sensitivity of Flutter Speed to Surface Area.

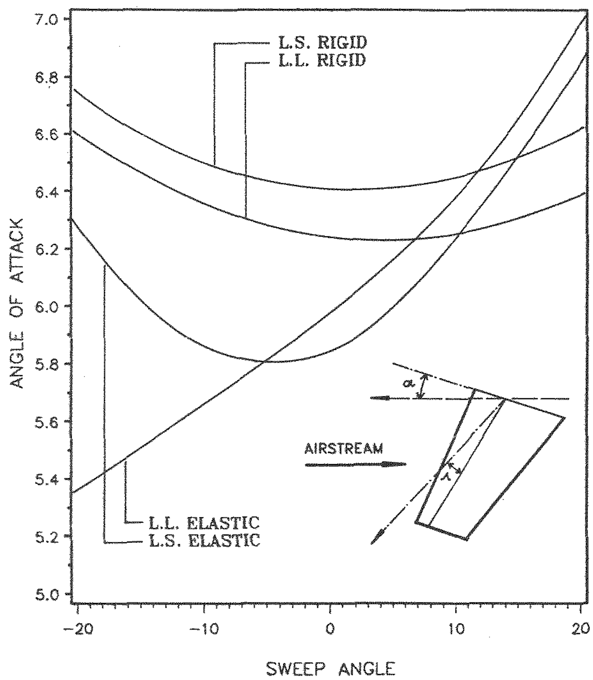

Fig. 5 Variation of Trim Angle of Attack with Sweep Angle.