# THE AEREL FLUTTER PREDICTION SYSTEM

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## Abstract

The AEREL system contains subprograms for determining analytical displacement modes, numerical values of aerodynamic transfer functions, analytical approximations to these, eigenvalues and eigenvalue derivatives. The approximations are combinations of simple functions fitted to given values, which can be calculated by programs based on the Advanced Doublet Element method, an extension of the Characteristic Box method, strip theory or piston theory or obtained in some other way. Eigenvalues are determined by Newton iteration for increasing flow density by using natural frequencies as initial approximations or a routine based on complex integration for determining these. Control laws may be included.

## 1 INTRODUCTION

The AEREL system is primarily intended for flutter analysis and is written in Fortran. It has been used for the transport aircraft SAAB 340, the canard fighter SAAB 39 Gripen, and several flutter models tested in wind tunnels[1].

When the system is applied to an aircraft, this is represented by a configuration of trapezoidal panels with two edges in the free-stream direction. The configuration is otherwise arbitrary. For each panel, analytical displacement modes are determined by first subtracting rigid-body contributions for trailing edge or leading edge control-surfaces from given data and then fitting a linear combination of suitable functions.

The aerodynamic transfer functions can be calculated for subsonic flow by a subprogram based on the previously employed[1] but unpublished ADE or Advanced Doublet Element method. The advanced velocity potential[2,3] and doublet-type solutions with constant density on panel elements is used in this. The ADE method is more attractive than the Lifting Line Element method[3] or the equivalent Double Lattice method[4,5], since simpler kernel functions are involved and the resulting influence coefficients can be calculated almost exactly. The method permits in addition the use of a modified Kutta condition.

For supersonic flow, a corresponding subprogram applicable to a canard configuration formed by two pairs of trapezoidal panels is included. The method employed, which is called the CHB or Characteristic Box method, is basically the method of Stark[6], who allowed for the downwash singularity at a subsonic leading edge. An approximate extension makes the program applicable to a canard configuration[7].

The strip theory program may be useful for treating a wing with control-surface with tab, while the piston theory program may be utilized for achieving the appropriate asymptotic behavior of the approximate transfer functions.

As a wind gust hits different parts of an airplane at different times, the resulting forces do not appear simultaneously. The simple functions in the approximations to the aerodynamic transfer functions are therefore designed such that the time delays can be allowed for.

The subprogram included for flutter and gust analysis shall solve the set of algebraic equations that results from Laplace transformation of the equations of motion[8], which in turn implies that a nonlinear eigenvalue problem shall be solved. This is achieved by a routine based on the iterative Newton-Raphson method[9]. Given eigenvalues for a density lower than that considered usually form the initial approximations, which also can be found by complex integration.

## 2 AIRCRAFT MODELLING

A configuration of trapezoidal panels representing a transport aircraft is shown in Fig. 1.

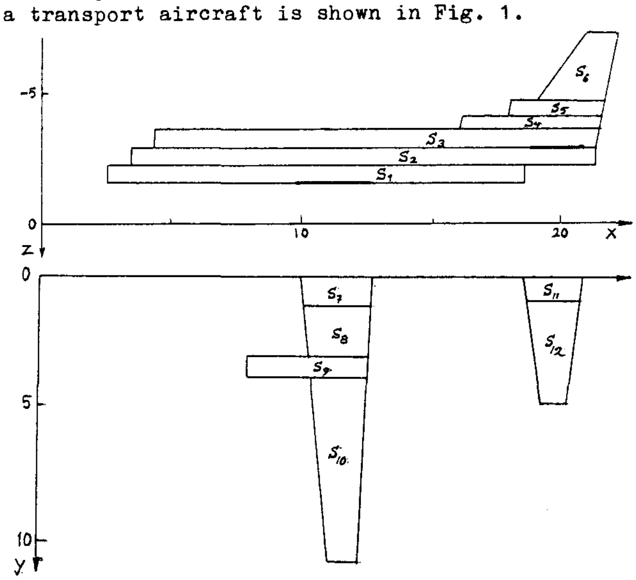


Fig. 1 Panel configuration for a transport aircraft. The wing and stabilizer dihedrals are 7 and 15 degrees respectively.

The configuration is defined by the coordinates of the corners of the panels. These are to be given in a global coordinate system as input to the program system. By giving them in the same order as the corners appear when the panel contour is followed around the panel, a unique normal direction is defined for each panel.

In addition to the global system, a local system x, y, z for each panel is defined. The origin of the local system lies at the center of one of the side edges, the x axis is directed in the same direction as the global x axis, and the y axis lies in the plane of the panel. The global x and z axes shall lie in the symmetry plane of the aircraft and the x axis shall have the free-stream direction.

We let $e_x^K$, $e_y^K$, and $e_z^K$ be unit vectors in the directions of the local axes for the $K^{th}$ panel, which is called $S_K$.

## 3 EQUATIONS OF MOTION

### Basic relations

It is of interest to consider a disturbed motion relative to a specified rectilinear steady flight. The steady flight is represented by the motion of an inertial coordinate system, the global system, which moves with the constant velocity U in the negative x direction relative to the undisturbed atmosphere.

The equations of motion may be derived on the basis of Cauchy's first law[10]. Applied to an aircraft in steady flight, this yields

$$\sigma \underset{\sim}{f} + \operatorname{div} \underset{\sim}{T}^0 = 0 \tag{1}$$

where $\sigma$ is the density of the aircraft, $\underset{\sim}{f}$ the force due to gravity on unit mass, and $\underset{\sim}{T}^0$ the steady-state stress tensor. Using this relation, a corresponding application of Cauchy's law to the disturbed motion results in

$$\sigma \underset{\sim}{a} - \operatorname{div} \underset{\sim}{T} = 0 \tag{2}$$

where $\underset{\sim}{a}$ is the acceleration field and $\underset{\sim}{T}$ the contribution to the stress tensor due to the disturbance.

A finite number of equations of motion may be derived by forming scalar products of weighting fields $\underset{\sim}{h}_m$ with the left hand member of Eq. (2), by integrating the products over the aircraft, and transforming the integral of $\underset{\sim}{h}_m \cdot \operatorname{div} \underset{\sim}{T}$ by means of Green's theorem. Applying this procedure to some part of the aircraft, e. g. a wing, we may write the resulting equations as

$$\int_V \left[\sigma \underset{\sim}{h}_m \cdot \underset{\sim}{a} + \operatorname{tr}(\nabla \underset{\sim}{h}_m \cdot \underset{\sim}{T})\right] dV = \int_S \underset{\sim}{h}_m \cdot \underset{\sim}{t}\, dS + \int_I \underset{\sim}{h}_m \cdot \underset{\sim}{t}\, dS \tag{3}$$

where V is the aircraft part. The boundary of this is S+I and S is that part of the boundary that is exposed to the airstream. I is the internal part of the boundary and $\underset{\sim}{t}$ is the contribution to the contact force on unit area of S or I due to the disturbance.

On S, the tangential component of $\underset{\sim}{t}$ may be deleted and the normal component (in the direction into the aircraft) equals the perturbation pressure p of the airstream. Modelling the aircraft as described, we may thus write Eq. (3) as

$$\int_V \left[\sigma \underset{\sim}{h}_m \cdot \underset{\sim}{a} + \operatorname{tr}(\nabla \underset{\sim}{h}_m \cdot \underset{\sim}{T})\right] dV + qL^2 \sum_K \int_{S_K} h_m \Delta p\, dx dy = \int_I \underset{\sim}{h}_m \cdot \underset{\sim}{t}\, dS \tag{4}$$

where $h_m$ is the component of $\underset{\sim}{h}_m$ in the local z direction and $\Delta p$ the jump in p across $S_K$ in the same direction. The pressure p and the coordinates x, y, and z are dimensionless and referred to the free-stream dynamic pressure q and a reference length L respectively. Since the field $\underset{\sim}{h}_m$ is zero outside the aircraft part considered, the second term in Eq. (4) yields contributions only from this part.

### Linear approximation

The displacement $\underset{\sim}{s}$ of a material point relative to the inertial system due to the disturbance may be small so that an approximation to it can be found in the form of a linear combination of given fields. Taking these fields identical to the weighting fields and dimensionless, we write

$$\underset{\sim}{s} = L \sum_n \underset{\sim}{h}_n q_n \tag{5}$$

where the quantities $q_n$ are undetermined coefficients. These depend only on the time t, which is dimensionless and referred to L/U.

The pressure jump $\Delta p$ is considered linearly related to $\underset{\sim}{s}$ so that

$$\Delta p = \sum_n \Delta p_n \tag{6}$$

where $\Delta p_n$ corresponds to the $n^{th}$ term of the sum in Eq. (5).

The stress tensor $\underset{\sim}{T}$ depends on the deformation and the deformation velocity at the current time as well as all earlier times. For simplicity, it is here written as

$$\underset{\sim}{T} = \sum_n L \left[\underset{\sim}{T}_n^1 q_n + (U/L) \underset{\sim}{T}_n^2 q_{nt}\right] \tag{7}$$

where $\underset{\sim}{T}_n^1$ and $\underset{\sim}{T}_n^2$ are given tensors determined by the fields $\underset{\sim}{h}_n$.

Substituting the expressions (5), (6), and (7) into Eq. (4), defining mass matrix elements by

$$M'_{mn} = \int_V \sigma \underset{\sim}{h}_m \cdot \underset{\sim}{h}_n\, dV \tag{8}$$

stiffness matrix elements by

$$S'_{mn} = \int_V \operatorname{tr}(\nabla \underset{\sim}{h}_m \cdot \underset{\sim}{T}_n^1)\, dV \tag{9}$$

damping matrix elements by

$$D'_{mn} = \int_V \operatorname{tr}(\nabla \underset{\sim}{h}_m \cdot \underset{\sim}{T}_n^2)\, dV \tag{10}$$

and aerodynamic matrix elements by

$$K_{mn}(t) = \frac{L^2}{S_r} \sum_K \int_{S_K} h_m \Delta p_n\, dx dy \tag{11}$$

where $S_r$ is a reference area, and dividing each term by a reference mass $M_r$, the square of a reference circular frequency $\omega_r$, and L, we obtain the equations

$$\sum_n \left[M_{mn} v^2 q_{ntt} + D_{mn} v q_{nt} + S_{mn} q_n + (v^2/\mu) K_{mn}(t)\right] = \frac{1}{L M_r \omega_r^2} \int_I \underset{\sim}{h}_m \cdot \underset{\sim}{t}\, dS \tag{12}$$

where

$$M_{mn} = M'_{mn}/M_r \tag{13}$$

$$D_{mn} = D'_{mn}/(M_r \omega_r) \tag{14}$$

$$S_{mn} = S'_{mn}/(M_r \omega_r^2) \tag{15}$$

$$v = U/(\omega_r L) \tag{16}$$

$$1/\mu = \rho S_r L/(2M_r) \tag{17}$$

and $\rho$ the free-stream density. The equations (12) correspond to those derived in Ref. 8.

## Indicial functions

In addition to aerodynamic forces due to the displacement of the aircraft, aerodynamic forces due to a gust may appear. We consider a "frozen" gust with transversal velocity $U e_g w_g(X)$ in a direction determined by the unit vector $e_g$.

The factor $w_g(X)$ depends only on the distance $X$ in the flight direction from the point where the gust starts to the point considered. The global and local coordinates of this point are $x' = x + x_K'$ and $x$ respectively and $x_K'$ the global coordinate of the center of the root chord of $S_K$. The root chord shall be smaller than $2x_K'$ for all panels.

Since the origin of the global system is supposed to meet the gust at t=0, the boundary condition for the gust-generated dimensionless potential $\phi_g$ reads

$$\phi_{gz} = \begin{Bmatrix} e_z^K \cdot e_g w_g(t-x') & 0 < x' < t \\ 0 & t < x' \end{Bmatrix} \quad \text{on } S_K \tag{18}$$

The gust generates a pressure jump $\Delta p_g$ and a corresponding aerodynamic matrix element

$$K_{mg}(t) = \frac{L^2}{S_r} \sum_K \int_{S_K} h_m \Delta p_g \, dxdy \tag{19}$$

For expressing the aerodynamic matrix elements in terms of $q_n$, we introduce indicial potentials $\phi_n^r$ and $\phi_g^I$. The former shall satisfy

$$\phi_{nz}^r = \begin{Bmatrix} h_{nx} H(t) & \text{for } r=1 \\ h_n H(t) & \text{" } r=2 \end{Bmatrix} \quad \text{on } S_K \tag{20}$$

and the latter

$$\phi_{gz}^I = e_z^K \cdot e_g H(t-x') \quad \text{on } S_K \tag{21}$$

where $H(t)$ is the Heaviside unit step function. Like $\phi_g$, the potentials $\phi_n^r$ and $\phi_g^I$ are dimensionless and referred to UL.

The indicial potentials yield via the linearized Bernoulli equation indicial pressure jumps $\Delta p_n^r$ and $\Delta p_g^I$ and by means of these we define indicial functions by

$$I_{mn}^r(t) = \frac{L^2}{S_r} \sum_K \int_{S_K} h_m \Delta p_n^r \, dxdy \tag{22}$$

and

$$I_{mg}(t) = \frac{L^2}{S_r} \sum_K \int_{S_K} h_m \Delta p_g^I \, dxdy \tag{23}$$

The aerodynamic matrix elements may then be expressed as

$$K_{mn}(t) = \int_0^t \left[ I_{mn}^1(t-\tau) q_{n\tau} + I_{mn}^2(t-\tau) q_{n\tau\tau} \right] d\tau \tag{24}$$

and

$$K_{mg}(t) = \int_0^t I_{mg}(t-\tau) w_{g\tau} \, d\tau \tag{25}$$

## Laplace transformation

Using the notation $\bar{f}$ for the Laplace transform of f(t) with respect to the dimensionless time t and p for the transform parameter, the result of transforming Eq. (24) and (25) is

$$\bar{K}_{mn} = A_{mn}(p) \bar{q}_n \tag{26}$$

and

$$\bar{K}_{mg} = G_m(p) \bar{w}_g \tag{27}$$

where

$$A_{mn}(p) = p \bar{I}_{mn}^1 + p^2 \bar{I}_{mn}^2 \tag{28}$$

and

$$G_m(p) = p \bar{I}_{mg} \tag{29}$$

These functions may be called aerodynamic transfer functions. They are analytic functions of p, which is complex in general.

Eq. (24) may be used for calculation of the aerodynamic coefficients $K_{mn}(t)$ for arbitrary variation of $q_n$ with time, e. g. harmonic variation. Limiting the study to this case and to the behavior of $K_{mn}(t)$ for large values of t, we substitute $e^{p\tau}$ for $q_n(\tau)$. It is then seen that for $t \to \infty$,

$$K_{mn}(t) \to e^{pt} \int_0^t \left[ p I_{mn}^1(\tau) + p^2 I_{mn}^2(\tau) \right] e^{-p\tau} d\tau$$

$$\to A_{mn}(p) e^{pt} \tag{30}$$

and for $p = i\omega$ (with $\mathrm{Im}\{\omega\} < 0$) we get

$$K_{mn}(t) \to A_{mn}(i\omega) e^{i\omega t} \tag{31}$$

The ordinary unsteady aerodynamic coefficients, which are often considered defined only for real $\omega$, are thus seen to be identical to $A_{mn}(i\omega)$.

## 4 MODELLING OF TRANSFER FUNCTIONS

### Theory

For subsonic flow, the indicial functions may behave qualitatively as shown in Fig. 2

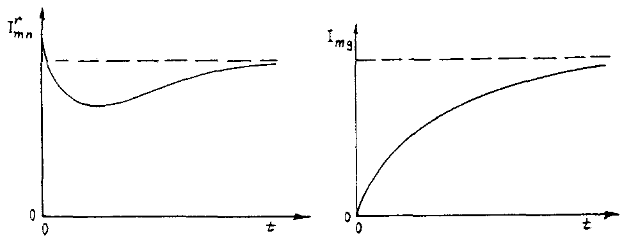


Fig. 2 Qualitative behavior of indicial functions for subsonic flow

In order to reproduce the behavior at t=0 and for $t \to \infty$, we use functions of the form

$$f_j(t) = \begin{cases} 1 & j = 1 \\ \delta(t) & j = 2 \\ t^{j-3} e^{-t} & j > 2 \end{cases} \tag{32}$$

in approximations for $I^r_{mn}(t)$, $\delta(t)$ being the Dirac delta function, and functions of the form

$$g_j(t) = \begin{cases} 1-e^{-t} & j = 1 \\ j^{j-1}e^{-t} & j > 1 \end{cases} \qquad (33)$$

in approximations for $I_{mg}(t)$.

It is observed, however, that a gust hits different parts of the aircraft at different times and that the different time delays must be allowed for. The approximations are therefore written as

$$I^r_{mn}(t) = \sum_k \sum_j f_j((t-T_k)/C)H(t-T_k)a^r_{jk} \qquad (34)$$

and

$$I_{mg}(t) = \sum_k \sum_j g_j((t-T_k)/C)H(t-T_k)b_{jk} \qquad (35)$$

where $T_k$ and $C$ are arbitrary parameters.

Laplace transformation of Eq. (34) and (35) and substitution into Eq. (28) and (29) yields

$$\sum_k \sum_j e^{-pT_k}\, pC\bar{f}_j(pC)(a^1_{jk} + pa^2_{jk}) = A_{mn}(p) \qquad (36)$$

and

$$\sum_k \sum_j e^{-pT_k}\, pC\bar{g}_j(pC)b_{jk} = G_m(p) \qquad (37)$$

where

$$p\bar{f}_j(p) = \begin{cases} 1 & j = 1 \\ p & j = 2 \\ p/(1+p)^{j-2} & j > 2 \end{cases} \qquad (38)$$

and

$$p\bar{g}_j(p) = \begin{cases} 1 & j = 1 \\ p/(1+p)^{j-1} & j > 1 \end{cases} \qquad (39)$$

It has been found[11] that the variation with time of indicial functions is almost the same for functions corresponding to different normal velocity distributions. The coefficient $a^2_{jk}$ in Eq. (36) may therefore be deleted.

The remaining coefficients $a^1_{jk}$ and $b_{jk}$ are determined in the AEREL system by the method of least squares on the basis of values of $A_{mn}(i\omega)$ and $G_m(i\omega)$ for a number of real values of $\omega$. The parameters $C$ and $T_k$ can be determined rather easily by trial and error and use of a plotting program.

## Subprograms

The system contains two subprograms for aerodynamic transfer function modelling. One of these works essentially as described above and yields close approximations both for $A_{mn}(i\omega)$ and $G_m(i\omega)$. This is illustrated in Fig. 3. The curves represent the approximation and the circles the given values of the transfer function.

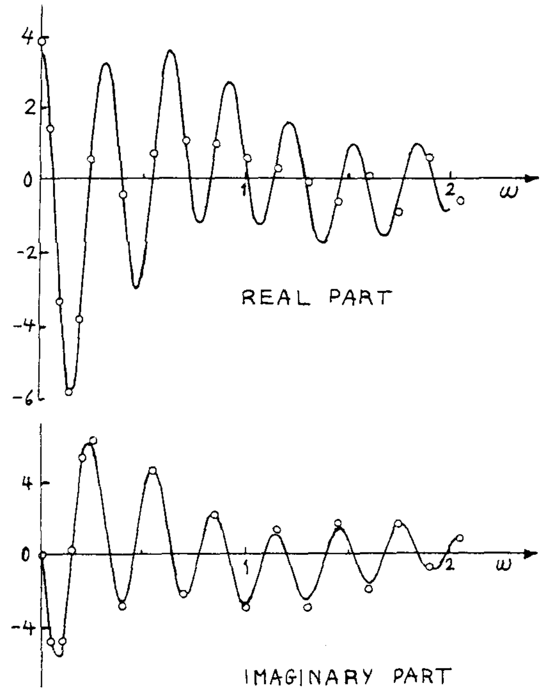


Fig. 3 Transfer function for the yawing moment on a transport aircraft due to a spanwise gust.

The second program offers the possibility of employing other functions instead of $pCf_j(pC)$ in Eq. (36). One of the options available implies that this function can be replaced by $pC_jf_3(pC_j)$, which means that several parametrs, $C_j$, instead of one, $C$, are to be determined. The resulting approximation will be of the same kind as those of R. T. Jones[12] and W. P. Jones[13].

For supersonic flow, the second program offers the possibility of using the functions described in Ref. 14 for this case. Fig. 4 shows that close agreement can be obtained by means of these functions.

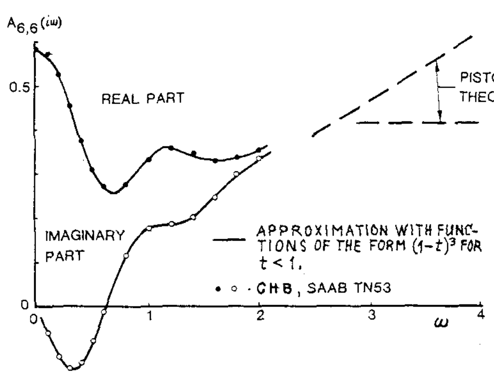


Fig. 4 Transfer function for a control-surface mode of a cropped delta wing. M=1.08 .

## Basic equations

The dimensionless perturbation velocity potential $\phi_n$ satisfies in the linearized theory the wave equation. In terms of the inertial local coordinates for $S_K$ this reads

$$\nabla^2\phi_n - M^2(\partial/\partial x + \partial/\partial t)^2\phi_n = 0 \tag{40}$$

where M is the Mach number.

Laplace transformation applied to Eq. (40) and the boundary conditions for $\phi_n$ and $\phi_g$ yields

$$\nabla^2\bar{\phi}_n - M^2(\partial/\partial x + p)^2\bar{\phi}_n = 0 \tag{41}$$

$$\bar{\phi}_{nz} = (h_{nx} + ph_n)\bar{q}_n \quad \text{on } S_K \tag{42}$$

and

$$\bar{\phi}_{gz} = \underset{\sim}{e}^K_z \cdot \underset{\sim}{e}_g \, e^{-px'}\bar{w}_g \quad \text{on } S_K \tag{43}$$

where $\bar{w}_g$ is the transform of $\bar{w}_g(X)$ with respect to the dimensionless distance X.

The right hand members of Eq. (42) and (43) are seen to be proportional to the unknown or arbitrary factors $\bar{q}_n$ and $\bar{w}_g$. In order to find general solutions, we may therefore calculate transfer functions that correspond to unit values of these factors. In terms of transfer functions $\bar{\Phi}_n$, $\bar{\Phi}_g$, $\Delta P_n$, and $\Delta P_g$ for the potentials and pressure jumps, we may write

$$\bar{\phi}_n = \bar{\Phi}_n\bar{q}_n \qquad \bar{\phi}_g = \bar{\Phi}_g\bar{w}_g \tag{44}$$

$$\Delta\bar{p}_n = \Delta P_n\bar{q}_n \quad \text{and} \quad \Delta\bar{p}_g = \Delta P_g\bar{w}_g \tag{45}$$

where

$$\Delta P_n = -2(\Delta\bar{\Phi}_{nx} + p\Delta\bar{\Phi}_n) \quad \text{and} \quad \Delta P_g = -2(\Delta\bar{\Phi}_{gx} + p\Delta\bar{\Phi}_g) \tag{46}$$

and the transfer functions $A_{mn}(p)$ and $G_m(p)$ can be written

$$A_{mn}(p) = \frac{L^2}{S_r}\sum_K \int_{S_K} h_m \Delta P_n \, dx dy \tag{47}$$

and

$$G_m(p) = \frac{L^2}{S_r}\sum_K \int_{S_K} h_m \Delta P_g \, dx dy \tag{48}$$

## Elementary solution

Equation (41) is satisfied by the elementary solutions

$$\bar{\phi} = (4\pi R e^{pT})^{-1} \tag{49}$$

where

$$R = (\rho^2 + Bz^2)^{1/2} \tag{50}$$

$$\rho = ((u-x)^2 + B(v-y)^2)^{1/2} \tag{51}$$

$$T = M(M(u-x) \pm R)/B \tag{52}$$

and

$$B = 1-M^2 \tag{53}$$

The so called advanced velocity potential[2] $\psi$ is related to the ordinary potential by

$$\psi(x',y,z,t) = \phi(x',y,z,t+x')$$

Laplace transformation of this function, which is used in the subsonic case, yields the corresponding transfer functions

$$\Psi_n = \bar{\Phi}_n e^{px'} \quad \text{and} \quad \Psi_g = \bar{\Phi}_g e^{px'} \tag{54}$$

The solution that corresponds to the - sign in Eq. (52) is not relevant for subsonic flow, and an elementary solution of doublet type for the advanced potential and $M < 1$ is therefore defined by

$$\bar{\psi} = \frac{\partial}{\partial z}(4\pi R e^{p\tau})^{-1} \tag{55}$$

where

$$\tau = (u-x + MR)/B \tag{56}$$

Derivation of the elementary solution (55) yields the kernel functions

$$Y(\underset{\sim}{u}-\underset{\sim}{x}) = \frac{\partial^2}{\partial y \partial z}(4\pi R e^{p\tau})^{-1} \tag{57}$$

and

$$Z(\underset{\sim}{u}-\underset{\sim}{x}) = \frac{\partial^2}{\partial z^2}(4\pi R e^{p\tau})^{-1} \tag{58}$$

where $\underset{\sim}{u}$ and $\underset{\sim}{x}$ are position vectors with components u, v, 0 and x, y, z in the local system for $S_K$.

Both signs in Eq. (52) are relevant for supersonic flow, and

$$\bar{\phi} = (2\pi R)^{-1}\cosh(pMR/B)e^{pM^2(u-x)/B} \tag{59}$$

is therefore a useful elementary solution for the velocity potential and $M > 1$.

## The ADE method

A subprogram based on the ADE method, the Advanced Doublet Element method, is included in the AEREL system. In this method, the jump in $\Psi_n$ across a panel is approximated by dividing the panel into small elements and replacing the jump across the $k^{th}$ element by an undetermined constant $\Psi_{kn}$. This results in a set of equations containing influence coefficients $W_{jk}$ equal to the advanced normal velocity at a control point on the $j^{th}$ element due to a unit jump across the $k^{th}$ element.

The jump in $\Psi_n$ across the wake of a panel is approximated in a simpler way. Since

$$\Delta P_n = -2\Delta\Psi_{nx}e^{-px'} \tag{60}$$

and since there is no pressure jump, $\Delta\Psi_n$ is independent of x on the wake. It is sufficient, therefore, to divide the wake into streamwise strips and to replace $\Delta\Psi_n$ by a single constant for each strip. The ADE method is thus similar to the Vortex Lattice method[15].

A sufficient number of equations for determining a unique solution is obtained by applying the Kutta condition. The program is written in such a way, however, that a modified condition can be used. The potential jump across a wake strip is put equal to the product of the jump across the panel element at the trailing edge and an arbitrary constant.

For the transport aircraft that was modelled as shown in Fig. 1, it was found that a more satisfactory solution was obtained by using a smaller value than the default value, which is unity, for the arbitrary constant for the strips behind the panels $S_1$ and $S_2$.

Table 1 Side force Y, yawing moment N, and rolling moment L due to steady yaw angle $\beta$.

| | PHOBOS | ADE | Wind tunnel |
|---|---|---|---|
| | M=0.63 | M=0.63 | M=0.15 |
| $Y/(qS_r\beta)$ | 1.00 | 1.05 | 1.01 |
| $N/(qS_rL\beta)$ | 0.20 | 0.21 | 0.18 |
| $L/(qS_rL\beta)$ | -0.19 | -0.19 | -0.17 |

Table 1 shows that the results obtained by the ADE method for the side force, the yawing moment, and the rolling moment due to steady yaw are in good agreement with those from the PHOBOS program[16], in which many elements on the actual body surface are used, and from measurements.

The influence coefficient for a control point $\underset{\sim}{x}_j$ on $S_J$ and an element $\Delta_k$ on $S_K$ may be written

$$W_{jk} = \lim_{\underset{\sim}{x} \to \underset{\sim}{x}_j} (\underset{\sim}{e}_z^J \cdot \underset{\sim}{e}_y^K) \int_{\Delta_k} Y(\underset{\sim}{u}-\underset{\sim}{x}_j)dudv + (\underset{\sim}{e}_z^J \cdot \underset{\sim}{e}_z^K) \int_{\Delta_k} Z(\underset{\sim}{u}-\underset{\sim}{x}_j)dudv \quad (61)$$

and the equations for the constants $\Delta\Psi_{kn}$ and $\Delta\Psi_{kg}$ as

$$\sum_k W_{jk}\Delta\Psi_{kn} = (h_{nx}+ph_n)e^{px'}\Big|_{\underset{\sim}{x}=\underset{\sim}{x}_j} \quad j=1,2, \dots \quad (62)$$

and

$$\sum_k W_{jk}\Delta\Psi_{kg} = \underset{\sim}{e}_z^J \cdot \underset{\sim}{e}_g \quad j=1,2, \dots \quad (63)$$

The control points are centers of the elements, which are bounded by chords and constant percent chord lines. These intersect the chords at equidistant points. The distance from the leading edge to the first point like the distance from the trailing edge to the last point is one quarter of the distance between the points.

The formulas derived and employed for calculation of the influence coefficients cannot be shown or described here. They are included in or can be found as special cases of those derived for the Polar Coordinate method[17]. Simpler and more direct numerical quadrature routines have been included, however, for use when the control point is located at a large distance from the element forming the quadrature region.

If $\Delta_k$ is an element located downstream of the element $\Delta_{k-1}$, the value of $\Delta P_n$ at the leading edge of $\Delta_k$ is approximately equal to

$$\Delta P_{kn} = -(2\Delta x_k)(\Delta\Psi_{kn} - \Delta\Psi_{k-1,n})e^{-px_k'} \quad (64)$$

where $x_k'$ is the global x coordinate of the center of the leading edge of $\Delta_k$ and $\Delta x_k$ the mean chord of $\Delta_k$. Replacing $\Delta_{k-1,n}$ by zero if the element $\Delta_k$ lies at the leading edge, the program calculates $\Delta P_{kn}$ by Eq. (64) and the aerodynamic transfer functions $A_{mn}(p)$ by

$$A_{mn}(p) = \frac{L^2}{S_r} \sum_K \sum_k h_{km} \Delta P_{kn} \Delta x_k \Delta y_k \quad (65)$$

where $h_{km}$ is the value of $h_m$ at the center of the leading edge of $\Delta_k$ and $\Delta y_k$ the span of $\Delta_k$. The method employed for $G_m(p)$ is analogous.

## The CHB method

Aerodynamic transfer functions for supersonic flow may be calculated by a subprogram based on the CHB method, the Characteristic Box method[6], which is included in the AEREL system.

Using characteristic coordinates s and t defined by

$$x = x_0 + \beta(s+t)/M \quad (66)$$

and

$$y = (-s+t)/M \quad (67)$$

where $\beta = (M^2-1)^{1/2}$, the transfer function for the potential jump across a planar wing can be expressed for $p=i\omega$ as

$$\Delta\phi(x,y) = -\int_0^s\int_0^t W_n(u,v)\cos(2\omega r/\beta)\cdot e^{-i\omega M(s-\sigma+t-\tau)} \frac{d\sigma d\tau}{\pi Mr} \quad (68)$$

where $W(u,v)$ is the downwash transfer function,

$$r = ((s-\sigma)(t-\tau))^{1/2} \quad (69)$$

$$u = x_0 + \beta(\sigma+\tau)/M \quad (70)$$

and

$$v = (-\sigma+\tau)/M \quad (71)$$

The integral is evaluated in the CHB method by dividing the wing plane by Mach lines defined by

$$s = s_j = j\delta \quad j = 0, 1, 2, \dots$$

and

$$t = t_k = k\delta \quad k = 0, 1, 2, \dots$$

and replacing the downwash $W_n(u,v)$ in the boxes formed by these lines by constants. For boxes cut by a subsonic leading edge, these constants are determined by a special procedure, but for boxes lying within the wing boundaries they are known and given by

$$a_{jk} = W_n(x_0+(j+k)\delta\beta/M, \ (-j+k)\delta/M) \quad (72)$$

At a node $(s,t) = (j\delta,k\delta)$, the potential jump is then approximately equal to

$$\Delta\phi_{jk} = \frac{-\delta}{\pi M} K(0,0) \sum_{\mu=1}^{j} \sum_{\nu=1}^{k} a_{\mu\nu} \tilde{K}(j-\mu,k-\nu) \quad (73)$$

where

$$\tilde{K}(j,k) = K(j,k)/K(0,0) \qquad (74)$$

$$K(j,k) = \int_0^1\int_0^1 \cos(2\omega\delta R/\beta)\, e^{-i\omega\delta M(j+u+k+v)/\beta}\, \frac{du\,dv}{R} \qquad (75)$$

and

$$R = ((j+u)(k+v))^{1/2} \qquad (76)$$

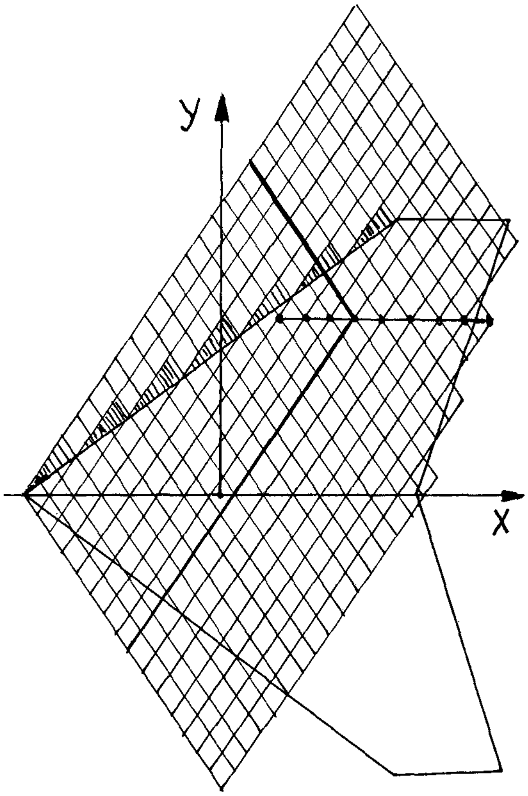


Fig. 5 Wing in characteristic lattice. I=3.

In case of a subsonic leading edge with sweep angle $\varphi$, the program can be applied for Mach numbers given by

$$M = (1 + ((I-1)/(I+1)\tan\varphi)^{1/2} \qquad (77)$$

for I = 2, 3, 4,, .... As shown in Fig. 5, the leading edge intersects in these cases the Mach lines $s = s_j$ at nodes only and divides the I boxes lying between these nodes into two parts. The downwash on the parts upstream of the edge is approximated by the product of a suitable function and a constant such that the potential at the node on the edge becomes zero. The mean value of the downwash approximation is then calculated for the boxes cut by the edge and inserted for corresponding constants in Eq. (73).

The CHB program has been extended[7] in such a way that a canard configuration consisting of two parallel symmetric wings each formed by two trapezoidal panels can be treated. Since there is no upstream influence in supersonic flow, the potential jump across the canard and its wake is first determined without regard to the presence of the main wing. The downwash due to this potential jump is then calculated approximately by a two-dimensional method at points on the main wing and subtracted from the given normal velocity of this wing, whereupon the potential jump across the main wing is calculated by the CHB program.

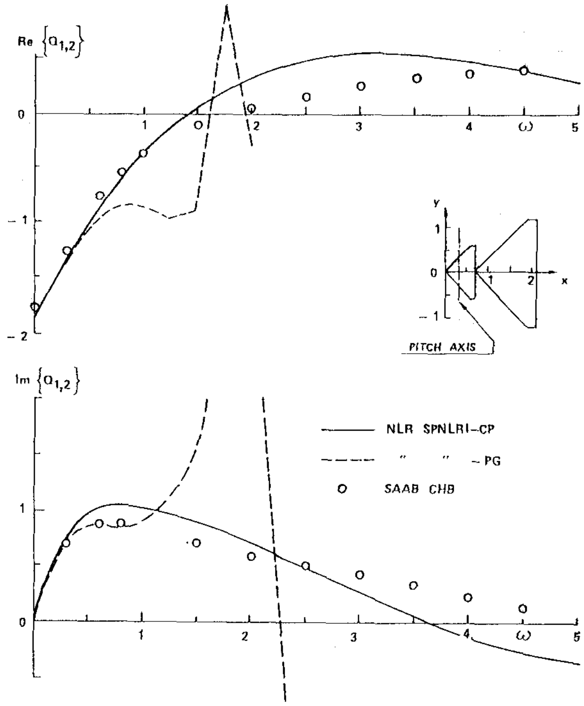


Fig. 6 Transfer function for lift on the main wing due to pitch of the canard. M=1.054 .

The lift on the main wing of a canard configuration due to oscillations in pitch of the canard has been calculated[7] by the method described. It is compared in Fig. 6 to corresponding results kindly supplied by Hounjet[18]. The agreement is seen to be surprisingly good.

## 6 SOLUTION OF EQUATIONS

### Laplace transformation

Applied to the complete aircraft, Eq. (12) gives a set of equations for determining the unknown generalized coordinates $q_n$. We take gust excitation into account by replacing $K_{mn}(t)$ in the equation by $K_{mn}(t)+K_{mg}(t)$ and write a set of equations for $n_s$ terms in the displacement approximation (5) as

$$\sum_{n=1}^{n_s} F_{mn} = C_m - (v^2/\mu)K_{mg} \quad m = 1,\ 2,\ \dots\ n_s \qquad (78)$$

where

$$F_{mn} = M_{mn}v^2 q_{ntt} + D_{mn}vq_{nt} + S_{mn}q_n + (v^2/\mu)K_{mn} \qquad (79)$$

$C_m$ stands for the term on the right hand side of Eq. (12) and represents a generalized force exerted by a control-surface actuator.

Having determined the generalized coordinates, an internal shear or moment component at some section separating two aircraft parts can be obtained by applying Eq. (12) to one of the parts and taking the weighting field $h_m$ equal to an appropriate rigid-body field. The term on the right hand side of Eq. (12) is then a dimensionless generalized force representing the desired shear or moment. It may be called $S_m(t)$ and is given by

$$S_m(t) = \sum_{n=1}^{n_s} F_{mn} + (v^2/\mu)K_{mg} \quad m=n_s+1, n_s+2, \dots \qquad (80)$$

The integrations involved here, i. e. for $m > n_s$, are confined to the aircraft part considered, and since the integrals (9) and (10) are zero for $h_m$ equal to a rigid-body field there are no damping or stiffness terms involved in this expression.

Applying Laplace transformation to the equations and letting the initial values be zero, we get

$$\sum_{n=1}^{n_s} Q_{mn}\bar{q}_n = \bar{c}_m-(v^2/\mu)G_m\bar{w}_g \quad m=1,2,\ldots,n_s \tag{81}$$

and

$$\bar{S}_m = \sum_{n=1}^{n_s} Q_{mn}\bar{q}_n+(v^2/\mu)G_m\bar{w}_g \quad m=n_s+1,n_s+2,\ldots \tag{82}$$

where

$$Q_{mn} = M_{mn}(vp)^2+D_{mn}vp+S_{mn}+(v^2/\mu)A_{mn}(p) \tag{83}$$

Solving the generalized coordinates from the set (81) and substituting them into Eq. (82) yields

$$\bar{S}_m = (v^2/\mu)\Big[G_m-\sum_{j=1}^{n_s}\frac{1}{D}\sum_{n=1}^{n_s} Q_{mn}C_{nj}G_j\Big]\bar{w}_g + \\ + \sum_{j=1}^{n_s}\frac{1}{D}\sum_{n=1}^{n_s} Q_{mn}C_{nj}\bar{c}_j \tag{84}$$

where D is the determinant of the matrix $Q=[Q_{mn}]$. D and the quantities $C_{nj}$ form the inverse matrix $Q^{-1}=[C_{nj}]/D$.

Inverse Laplace transformation gives

$$S_m(t) = (v^2/\mu)(K_{mg} - \int_0^t W_m(t-\tau)w_g(\tau)d\tau) + \\ + \sum_{j=1}^{n_s}\int_0^t R_{mj}(t-\tau)c_j(\tau)d\tau \tag{85}$$

where $W_m(t)$ and $R_{mj}(t)$ are weighting functions defined by

$$W_m(t) = \frac{1}{2\pi i}\int_C (X_m(p)/D(p))e^{pt}dp \tag{86}$$

and

$$R_{mj}(t) = \frac{1}{2\pi i}\int_C (Y_{mj}(p)/D(p))e^{pt}dp \tag{87}$$

where

$$X_m(p) = \sum_{j=1}^{n_s} Y_{mj}(p)G_j(p) \tag{88}$$

and

$$Y_{mj}(p) = \sum_{n=1}^{n_s} Q_{mn}(p)C_{nj} \tag{89}$$

The integrals (86) and (87) can be evaluated on a line from $p=\sigma-i\infty$ to $p=\sigma+i\infty$ where $\sigma$ has a value large enough for all eigenvalues, zeros of D(p), to lie to the left of the line.

However, the line may be moved or deformed into a curve such that there are $\nu_s$ eigenvalues, $p=p_\nu$, to the right of the line or curve. The equations (86) and (87) then transform into

$$W_m(t) = \sum_{\nu=1}^{\nu_s} (X_m(p_\nu)/D'(p_\nu))e^{p_\nu t} + \\ + \frac{1}{2\pi i}\int_C (X_m(p)/D(p))e^{pt}dp \tag{90}$$

and

$$R_{mj}(t) = \sum_{\nu=1}^{\nu_s} (Y_{mj}(p_\nu)/D'(p_\nu))e^{p_\nu t} + \\ + \frac{1}{2\pi i}\int_C (Y_{mj}(p)/D(p))e^{pt}dp \tag{91}$$

where $D'(p_\nu)$ is the derivative of the determinant at $p=p_\nu$.

The integrals in Eq. (90) and (91) may be zero if there are no singularities to the left of the line or curve.

## Flutter program

Flutter is a dynamic instability which exists if the real part of any $p_\nu$ is positive. The flutter problem is therefore mainly a problem of determining the eigenvalues, i. e. a problem of solving the characteristic equation

$$D(p) = 0 \tag{92}$$

Based on the Newton-Raphson method, a program for this purpose is included in the AEREL system. The method, which has earlier been described[9], is iterative and predicts simply that

$$p' = p - D(p)/D'(p) \tag{93}$$

is an improved approximation provided p is a value that deviates not too much from the true eigenvalue.

The program developed is such that a selected number of eigenvalues can be generated for some selected Mach numbers as functions of the altitude or as functions of the stagnation pressure of an isentropic flow for each Mach number. Aerodynamic transfer functions shall be available on a file for one or a few Mach numbers and for a number of reduced frequencies for each Mach number. Spline interpolation is included for interpolation in the Mach number and the modelling described is utilized in the program for rapid aerodynamic matrix generation for complex values of p.

In flutter investigations, it is common practice to let the fields $h_n$ in the displacement approximation be natural modes determined in a ground vibration test or by a finite-element calculation. The associated natural frequencies $\omega_n$ and the generalized masses $M_{nn}$ are then also given. Measured data for the modes can be used as input to subprograms for determining analytic modes and aerodynamic transfer functions, while the natural frequencies and the generalized masses form input data to the flutter program.

It is favourable to use natural modes in the displacement approximation both because the associated natural frequencies yield suitable initial approximations to the eigenvalues and because a a good displacement approximation is obtained in that way.

Since one wants to know the eigenvalues for a number of altitudes, it is natural to start at a high altitude, where the natural frequencies yield useful initial approximations, and to proceed to lower altitudes by using the results for the previous altitude as initial approximations. Only a few iterations are then reqired for each altitude.

The values required for $D(p)$ and $D'(p)$ in Eq. (93) are determined in the program by using Gaussian elimination and triangularization for $D(p)$ and the difference formula

$$D'(p) = (D(p+\Delta)-D(p-\Delta))/2\Delta \qquad (94)$$

for the derivative; $\Delta$ being any complex number with small modulus.

When running the program for a number of altitudes, it sometimes happens that the eigenvalues generated (for one and the same altitude) are not all different but that some are identical.. This can usually be avoided by using smaller altitude steps, but another possibility exists.

The alternative possibility is to use a routine based on complex integration by means of which the number of eigenvalues within arbitrarily selected rectangular parts of the p plane can be determined[14].

Since the modelling of the aerodynamic transfer functions is made by using simple analytical functions, it is easy to calculate their derivatives with respect to the frequency parameter p. Extensions for calculation of derivatives of eigenvalues were therefore easy to include in the program. These can be employed for calculation of derivatives both with respect to the flow density and to design variables. The latter are needed in optimization studies by the OPTSYS system[19] in which flutter constraints are being included.

Divergence

The flutter program also predicts divergence, if such an instability exists. This appears from Fig. 7, which shows how the eigenvalues for a fin with rudder and tab vary in a particular case for increasing flow density. One of the eigenvalues is seen to become real, first negative and then positive, as the density increases, which implies divergence.

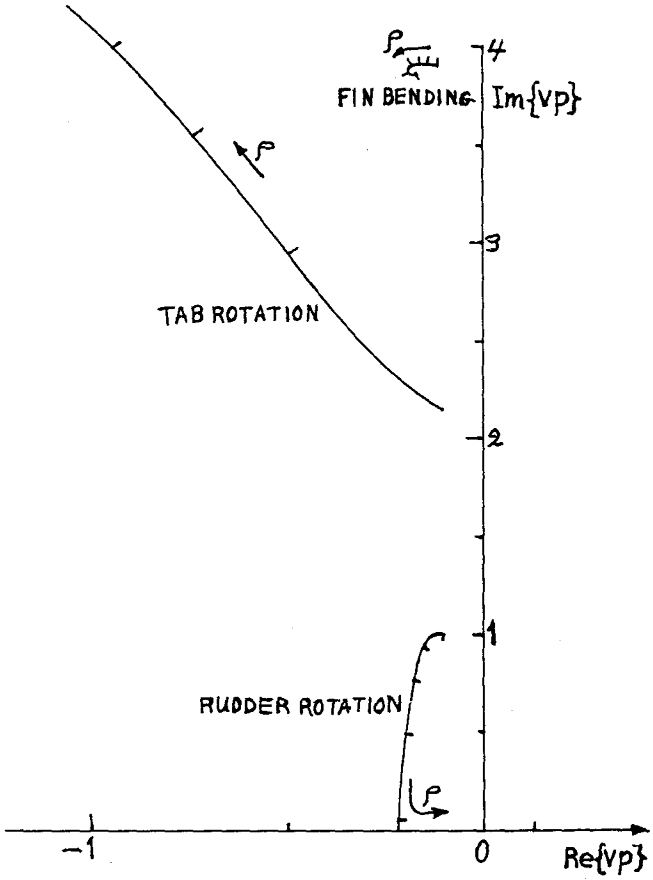


Fig. 7 Eigenvalue loci for a fin with rudder and tab. M=0.5.

7 ACKNOWLEDGEMENT

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