OPTIMIZATION OF NONLINEAR AEROELASTIC TAILORING CRITERIA

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ABSTRACT

Good analysis capabilities presently exist within the separate aerodynamics, structure/dynamics, and control disciplines. However, new approaches and higher level of sophistication in analytical methodology are required to solve the design problem which often includes significant nonlinearities and interaction between the design disciplines.

An integrated system with a mathematical optimization technique and automated information exchange is needed for rapid optimal design of aircraft and spacecraft. In order to analyze all disciplines simultaneously, a powerful methodology which exists in the field of multi-level optimization will be utilized. The multi-level optimization technique facilitates the development of superior engineering system design with minimal turn around time. It is based on the implicit function theorem. The present method develops the system sensitivity equations in a new form using the partial (local) sensitivity derivatives of the output with respect to the input for each part of the system.

For example, typical aerodynamic, structure and control derivatives of low aspect ratio wing configurations can be computed to minimize root bending moment for the flexible wing with a given geometry (fixed airfoil shape, taper and aspect ratios). In this process, uniquely defined aerodynamic/control/elastic structure coupled derivatives must be generated for the design optimization problem.

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This paper addresses detailed sensitivity derivative computation procedures of individual disciplines as well as a technique for solving global multi-level optimization problems. Results are presented for a flexible wing configuration at Mach number of 0.9, angle of attack of 0.0 and dynamic pressure of 1.054 psi.

NOMENCLATURE

 C_{τ} = lift coefficient

 C_{M} = root bending moment coefficient

 $C_{L_s} = \partial C_{L}/\partial \delta$

 $C_{M_{\xi}} = \partial C_{M} / \partial \delta$

L = lift force (lbs.)

M = root bending moment (in-lbs.)

 M_{∞} = free stream Mach number

Q = free stream dynamic pressure (psi)

 α = angle of attack (degrees)

 δ_{1-4} = control surface deflection (degrees)

t_{ij} = Vector element or composite layer thickness

= Vector ply orientation or composite

element layer orientation

 σ_i = ith element or composite element layer

stress

 μ_i = Real function of ith design variable

INTRODUCTION

Aeroelasticity, the coupling between aerodynamic and structural elasticity, has traditionally been treated as a problem in aircraft design that must be overcome by adding stiffness (which imposes weight penalties) to the structure. Examples of aeroelasticity problems are aileron reversal, divergence and flutter. Recently, through the use of advanced composite structure, design methods have been developed to control the problems of aeroelasticity while increasing aerodynamic performance where nonlinear aerodynamic effects are dominant. The use of flexibility and control surface is normally accentuated by requirements for high speed vehicle's agility and high maneuverability conditions. At the same time, the bending or torsional forces acting on the wing can be extremely high and structural forces to hold the body together can be well beyond its specified limitation. Therefore, the minimization of the wing bending moment force during a manuever by selecting the right combination of control surface deflections is an important task.

The optimal design requires a significant degree of interaction among various disciplines. Although good analysis and optimization capabilities exist within the separate disciplines involved, the interaction and iterations required to resolve conflicting design requirements and the transformation of data between disciplines slows down the design cycle time considerably. Recent software development has tended toward large scale multidisciplinary systems which promise improved airframe design through the use of mathematical optimization techniques and automated information exchange between the coupled disciplinary areas.

It would be very idealistic to believe that engineers can fully understand the coupling effects and relationships between disciplinary areas in the preliminary design phase. To provide an early knowledge of the aeroelastic effect and to account for correct behavior of aerodynamic sensitivity derivatives for nonlinear structural coupling phenomenon, a reliable code that can calculate the aeroelastic behavior of the structure (including composite material) due to aerodynamic loads must be utilized. An example of this behavior is the aeroelastic deformation of a wing under airload which will yield a lower lift near the tip than a rigid or stiff wing. Although the theory of design sensitivity has not been fully developed, this effort uses the recent encouraging results in method of design sensitivity analysis and its numerical implemntation for nonlinear system. Therefore . Therefore. one of objectives of this paper is to show how the nonlinear aerodynamic characteristic (CFD as a main tool), structure, control and their products can be utilized for interdisciplinary problems as shown in figure 1.

Traditionally, CFD analysis has been used to determine aerodynamic trends for various design configurations. Sobieski's approach to the coupled optimization problem can be performed if this classical CFD analysis is extended one step further,

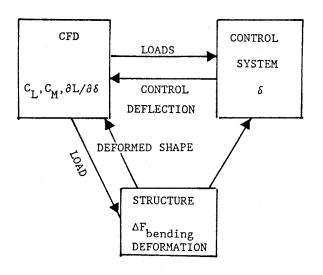


Figure 1. A typical coupled problem schematic between aerodynamics, control and structure.

by computing the sensitivity derivatives. ³ They can be computed easily without additional complexity. One may use the sensitivity derivatives to screen the dominant independent variables which directly or indirectly relate to the design parameters. Also, the sensitivity derivatives can be used to determine a trend of over all picture. Finally, they are used in a optimization technique directly. Thus, a multidisciplinary optimization method can be used as a subsystem optimization technique to reduce the number of dependent variables in the multi-level optimization problem. ⁴

Another objective is introduction of a new approach in computation of global sensitivity derivatives with respect to independent (input) variables for complex internally coupled system, while avoiding the high computational cost and time consuming classical cyclic analysis performed on the entire system.

APPROACH

In order to accomplish a practical task, a system of computer programs is integrated to perform multi-level optimization with linear decomposition technique. The technique can be used for analysis of large engineering systems by decomposition of a large task into a set of smaller self contained subtasks that can be solved concurrently. Each discipline is augmented by its sensitivity analysis to account for the coupling from other subtasks.

In the current study, full potential/aero elastic code is used to calculate aerodynamic sensitivity derivatives with respect to control surface deflection angles. This approach will allow identification of these derivatives for the transonic flight regime where nonlinear aerodynamic effects are very significant. The generated aerodynamic forces are then transformed to the Rockwell's new Rapid Structural Optimization Program (RSOP) for generation of structural sensitivity derivatives. The structural flexiblility is corrected furthur by performing subsystem optimization for change in ply orientation as well as the structural sizing parameters. Each computation will be performed subject to satisfing the strength, strain and buckling requiements. The generated sensitivity derivatives are then used by the optimizer for determination of optimal design. Figure 2 illustrates the architecture of this design system.

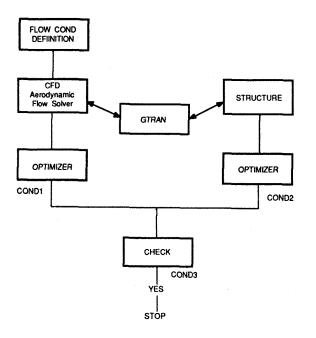
DESIGN CRITERIA

A low aspect ratio wing wind tunnel model was selected for this study. The wing configuration and airfoil section profile is shown in figure 3. Note that there are two bumps in the lower section which are due to the leading edge and trailing edge control surface actuators.

The fuselage section of the wind tunnel model has been modified for simplification of CFD computation. The modification, however, is taking account for 3-D body effects.

The control surface definition is shown in figure 3. There are four semi-span control surfaces. They are leading edge inboard(LEI), leading edge outboard(LEO), trailing edge inboard(TEI) and trailing edge outboard(TEO). Also, aerodynamic and structural sensitivity derivatives evaluation stations are shown. Although it is possible to analyze at all structural influence coefficient points or aero/structural nodal points

which are defined by CFD or FEM grid, only 4 stations are selected for simplicity. In this figure, station 1 is considered as the root section of the wing.



COND 1 Areodynamically adjusted delta

COND 2 Structurally adjusted delta

COND 3 COND1 - COND2 is Less than preset criterion

Figure 2. Optimization flow chart.

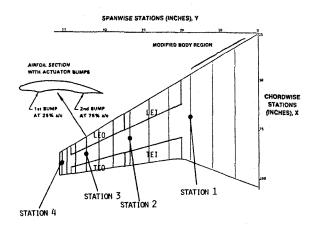


Figure 3. A fighter wing configuration with control surfaces. Body region has been modified.

The finite element model of wing torque box, control surfaces, and carry through structure is created using the RCADS mesh generator module of the RSOP system, and is depicted in Figure 4. The FEM model has the following characteristics: number of nodes = 860, number of elements = 2733, and number of nodal degrees of freedom = 2512. Membrane elements are used to model the upper and lower covers of the wing. They are modeled with four layers to represent a standard balanced tape layup (0/+45/-45/90). The substructure is modeled using isotropic shear and rod elements.

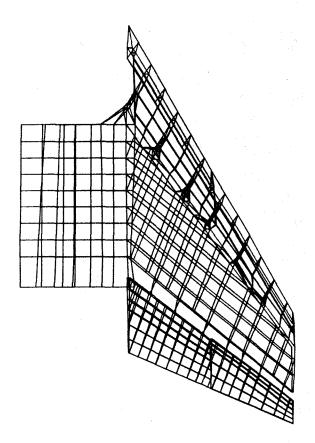


Figure 4. FEM model.

CFD/AEROELASTIC ROLE IN OPTIMIZATION

One of the objectives of this paper is to show how CFD and products of CFD can be utilized for interdisciplinary problems. The nature of CFD analysis is to compute the aerodynamic flow and forces for a given object shape. Therefore, the input shape geometry is the most important factor that determines flow characteristics.

The present 3-D full potential /aeroelastic code (Ref 5, 6, 7) which was developed by Rockwell International, NAA, can handle aerodynamics, control, and aeroelasticity simultaneously. aeroelastic methodology of this paper has three modules: 1) a full potential based flow solver, 2) a structural response model based on generalized model functions, and 3) a grid package for update of geometry and grid at each time level. The first module, the full potential solver, employs the following concepts: 1) internal Newton iterations for time accuracy and computational efficiency, enabling large courant number for steady/unsteady computations; 2) implicit boundary conditions consistent with approximate factorization scheme; 3) flux biasing techniques based on sonic conditions for treatment of hyperbolic regions with shocks and sonic surfaces; 4) local time liniarization for density; 5) unsteady wake treatment; and 6) nonreflecting outer boundary conditions. The second module (aeroelastic model) includes both static and dynamic options with control surfaces deflection capability by solving the generalized structural equations of motion.

With proper assumptions of structure and thermal effects (chordwise rigidity, zero heat flux to the structure---etc), the minimization of wing bending force problem can be focused on the local

level. This method seems to be very simple compared with multiply defined optimization methods.

FORMULATION OF PROBLEM

A simple but practical problem is set here. This problem is minimization of a given wing's bending force at the root chord by changing control surface deflections, ply thicknesses, and ply orientations. This problem can be formulated in the following forms. Define that $\delta_{\rm LEI}=\delta_1$, $\delta_{\rm LEO}=\delta_2$, $\delta_{\rm TEI}=\delta_3$, $\delta_{\rm TEO}=\delta_4$ and if a function exists, then,

$$M_{\text{bending}} = f(\delta_1, \delta_2, \delta_3, \delta_4, t, \phi, L)$$
 (1)

where M_{bending} = the root bending moment,

 δ_1 - δ_4 = control surface deflection,

t = thickness,

 ϕ = ply orientation angle,

L = lift force.

and the required task is;

OBJECTIVE:

Minimize (Mbending)

CONSTRAINTS:

 $L \ge L_{required}$

VARIABLES:

 δ_1 , δ_2 , δ_3 , δ_4 , t, ϕ

Often, a proper function, f, in Eqn (1) is difficult to identify. One advantage of using the CFD method is that f can be reconstructed by the results of CFD analyses. Suppose that f can be expressed in a simple piecewise linear function for a range of δ changes, then

$$^{\text{M}}_{\text{bending}} = ^{\text{M}}_{0} + (\overset{4}{i} = 1 - \frac{\partial f}{\partial \delta_{i}} - \Delta \delta_{i})$$
 (2)

where
$$M_0 = (M_{bending})_{initial}$$
 with $\Delta \delta_i = 0$

Eqn (2) can be expanded to more terms when the structural effect is included. The following forms represent the fundamental formula of aerodynamic and structure objective equations which will be used in this analysis.

$$\begin{split} \mathbf{L} &= \mathbf{L}_0 + \widetilde{\mathbf{A}} \left(\underline{i} \underline{\Sigma}_1 - \frac{\partial \widetilde{\mathbf{f}}}{\partial \delta_i} - \Delta \delta_i \right) + \\ &\qquad \qquad \widetilde{\mathbf{B}} \left(\underline{j} \underline{\Sigma}_1 - \frac{\partial \widetilde{\mathbf{f}}}{\partial \delta_i} - \Delta t_j \right) \end{split} \tag{3}$$

$$M_{\text{bending}} = M_0 + \tilde{A} \left(\frac{\Delta}{i} - \frac{\partial \tilde{f}}{\partial \tilde{\delta}} - \Delta \delta_i \right) + \tilde{A} \left(\frac{\Delta}{i} - \frac{\partial \tilde{f}}{\partial \tilde{t}} - \Delta t_j \right)$$

$$\tilde{B} \left(\frac{\Delta}{j} - \frac{\partial \tilde{f}}{\partial \tilde{t}} - \Delta t_j \right)$$
(4)

where $\tilde{f} = \tilde{f}(L)$ and $\tilde{f} = \tilde{f}(M_{bending})$.

Coefficients A and B are unknown but will be evaluated. The reader may note that the second term of Eqns (3) and (4) is aerodynamically related and the last term of theses equations is structurally related. In general, this type of objective equation can be generated if individual contributing analysis (CA) is expressible in piecewise linear gunction and optimization range is not too large. The global sensitivity equation to be used by the optimizer can be derived from the local sensitivity equations obtained from each contributing analysis;

$$\begin{bmatrix} 1 & -\frac{\partial L}{\partial u} \\ -\frac{\partial u}{\partial L} & 1 \end{bmatrix} \begin{bmatrix} \frac{dL}{dt} & \frac{dL}{d\delta} \\ \frac{du}{dt} & \frac{du}{d\delta} \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial t} & \frac{\partial L}{\partial \delta} \\ \frac{\partial u}{\partial t} & \frac{\partial u}{\partial \delta} \end{bmatrix}$$
(5)

where u is the local deformation and $\partial L/\partial u$ is the local aero force changes due to deformation. Similarly, $\partial u/\partial t$ is the structural deformation and $\partial L/\partial \delta$ is aerodynamic change due to control surfaces. $\partial L/\partial t$ and $\partial u/\partial \delta$ in the above Eqn (5) are zero.

The procedure to compute these derivatives is shown below;

- 1. Both rigid and flexible CFD analyses are performed for the clean case (δ_i =0.0).
- 2. Flexible CFD analysis is performed for each δ deflection (4 cases). In this case, 3 δ values are chosen (-5.0, 0.0, +5.0) to capture possible nonlinearlity.
- L, M_{bending}, u are computed at the specified stations which are shown in figure 3.
- The RSOP program is used independently for structural analysis at the same stations.
- 5. Steps 1-4 are used to construct the matrix of the local sensitivities for the contributing analyses (CA), left hand side of equation (5), and the matrix of local sensitivities to design parameters (right hand side).

MULTIDISCIPLINARY OPTIMIZATION PROCEDURE

A. MINIMUM WEIGHT DESIGN

A multidisciplinary mathematical optimization module which is supported by the previous derivative sensitivity analysis program was exercised for minimum weight design, subject to strength, buckling, deflection and strain constraints. The final derivative sensitivity as well as the suboptimized ply thicknesses are passed to the multilevel optimization capability for further design convergance. The design variables consist of element gauges, and ply orientation.

B. STRESS CONSTRAINT

The element stresses are updated during optimization by simply modified Taylor series expansion method, shown below: 9

$$\begin{split} \sigma_{i} &= \sigma_{io}^{} + \underset{i}{\overset{\mathbb{N}}{=}}_{1} [\left(\partial \sigma / \partial t \right)_{io}^{} (t_{i}^{} - t_{io}^{}) \mu_{i}^{}] \\ &\text{where } \mu_{i}^{} = 1, \quad \left(\partial \sigma / \partial t \right)_{o}^{} > 0 \\ &= 1 / [1 + (t_{i}^{} - t_{io}^{}) / t_{io}^{})], \quad \left(\partial \sigma / \partial t \right)_{io}^{} < 0 \end{split}$$

C. BUCKLING CONSTRAINT

The buckling constraint is considered as fiber and matrix failure subject to biaxial and shear stress state. The stability equations are commonly used in the design of buckled structure.

D. GRID TRANSFORMATION

The grid transformation module (GTRAN) performs data transformation from any n x n input matrix or grid to m x m output matrix or grid. therefore, the GTRAN module supports transformations of any data between structure, aerodynamics, weights, and dynamics grids. The program establishes an automated patterning distribution technique to bridge a vector from a particular point to specified points in another model for single and multi surfaces. Once the bridge pattern has been established the vector then is distributed using the Multipoint Constraint Equations for 6 Degrees of Freedom (MCE6DF). The program defines a rigid linear relationship between a single degree of freedom (dof) and or more other dof's.

RESULTS AND DISCUSSION

The flow condition is M =0.9, α =0.0, Q=1.054 psi. The aerodynamic forces are calculated on the statically deformed wing with one of the control surfaces deflected. The range of control surface deflection was from δ =-5 to δ =+5 degrees.

For the first step, aerodynamic forces for the clean wing are computed. Then, control surfaces are deflected. The total lift coefficients, $\mathbf{C_L}$, vs $\delta's$ and the root bending moment coefficient vs $\delta's$ are shown in figure 5. This figure shows that trailing edge control surfaces deflection will yeild nonlinear bending moment, even if $\mathbf{C_L}$ is close to linear. On the other hand, both leading edge control surfaces (LEI and LEO) do not contribute much to either lift or bending moment.

Pressure profiles of this case with $\delta_{\mbox{TEO}}$ deflection are shown in figure 6. Discrepancies near the leading edge and near the hinge line are due to some separation. The full potential method predicted a typical inviscid solution. The full potential method, however, agrees well with experimental data.

The structural design sub-optimization results are presented in figure 7. Figure 8 shows the change in structural design parameters versus iteration number. The minimum weight design is achieved when the wing laminate is oriented such that the zero degree fiber is 5 degrees forward of the torque box center spar. However, the change in ply orientation is minor compared to the change in ply thicknesses which turned out to be the design drivers.

The complex wing structure is reduced by GTRAN down to 4 SIC (structural influence coefficients) locations. At these locations (shown in figure 3), local aerodynamic forces (ie, C_{ρ} , C_{η}) and sensitivities (ie, $\partial C_{\rho}/\partial \delta_{\dot{\eta}}$, $\partial C_{\rho}/\partial u$) are known. The suboptimization for number of plies and ply orientation was accomplished at these points. In addition, the local structural sensitivities (ie, $\partial u/\partial t$, $\partial u/\partial C_{\varrho}$) are calculated at these locations. Eqn (5) is used to generate the global sensitivities which are then used in a mathematical optimization process. The objective function is the root bending moment due to the lift force at the four span locations. The absolute value of the lift at each station multiplied by its moment arm is used in the total moment calculation. This eliminates minimization of the total bending moment by offsetting a positive sectional moment at one station with a negative value at another station. The constraint equation requires the total of the

$$M = 0.9$$
, $\alpha = 0.0$, $Q = 1.054$ PSI

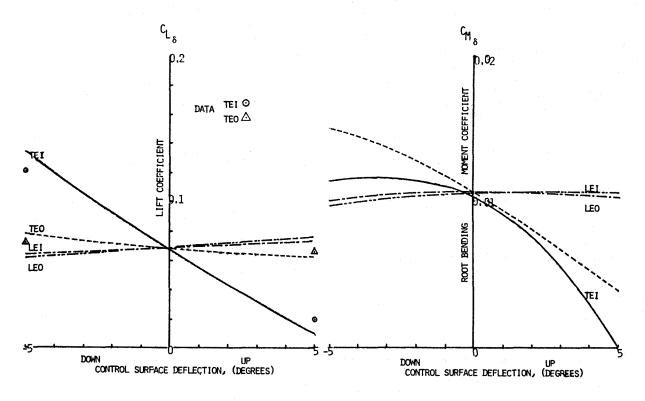


Figure 5. $^{\rm C}{\rm L}_{\delta}$ and $^{\rm C}{\rm M}_{\delta}$ for M =0.9, α =0.0, Q=1.054 psi

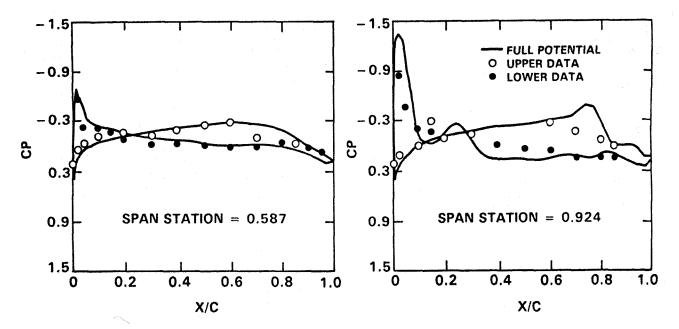


Figure 6. Pressure profiles for trailing edge outboard control surface deflection at M =0.9, α =0.0, Q=1.054 psi, δ =+5.0.

lift at the four locations be greater than or equal to the initial value. For the constraint equation, the inclusion of the sign of the force is required.

The results of the first pass of the mathematical optimization process are shown in figure 9. These results were accomplished using first-order Taylor series expansions of the objective and constraint function including a factor developed for frequency optimization 10 . The actual optimization was performed using ADS 11 program. Within ADS, the "no strategy" option was imployed. The optimizer option used was Method of Feasible Directions (MFD) for constrained minimization. The one-dimensional search option used finds the minimum of a constrained function by first finding bounds and then using polynomial interpolation. The optimizer predicted approximately a 9% decrease in bending moment. Figure 10 shows a plot of constraint value versus iteration. At the optimized state, the total lift is essentialy the same as the initial value. Comparisons of the initial and optimized distributions of lift and moment between the four locations are shown in figures 11 and 12, respectively. A decrease in lift at the three

0.9 8.0 0.7 0.6 Wf/Wi 0.5 0.4 0.3 02 0.1 10 11 12 13 2 3 4 5 6 7 8 9 Iteration Number

Figure 7. Weight ratio versus iteration number for sub-optimization problem.

outboard stations is offset by an increase in lift at the inboard station, in order to maintain the required lift force. Due to the longer moment arms at the outboard stations, the decrease in lift at these stations translates into an even larger

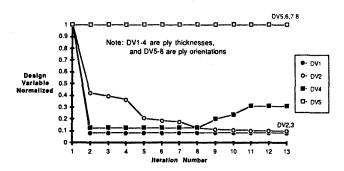


Figure 8. Design variables versus iteration number for sub-optimization problem.

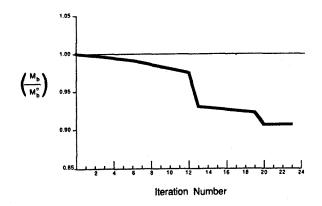


Figure 9. Objective function versus iteration number for multi-disciplinary optimization.

percentage decrease in bending moment. The percentage increase in bending moment due to the increase in lift at the inboard station is reduced because of its short moment arm which translates into an overall decrease in bending moment. The four control surface deflections versus iteration are shown in figure 13a-d. In addition, the change in element layer thichnesses are shown in figures 14-17. Although the results can only be examined on a qualitative basis due to the simplicity of the model, the capabilities of the methodology are well demonstrated with the present model.

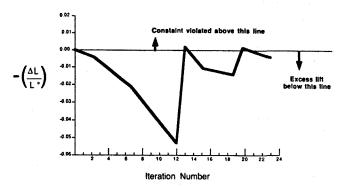


Figure 10. Constraint function versus iteration number for multi-disciplinary optimization.

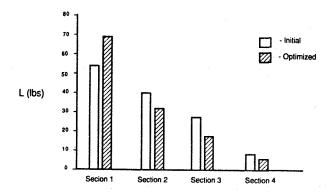


Figure 11. Comparison of initial and optimized sectional lift.

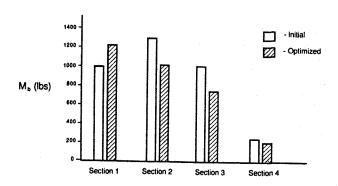
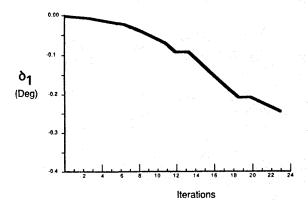
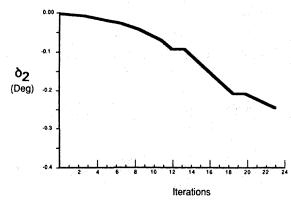


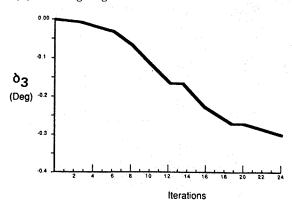
Figure 12. Comparison of initial and optimized sectional bending moment.



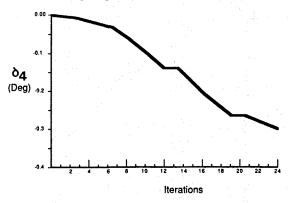
(a) leading edge inboard



(b) leading edge outboard



(c) trailing edge inboard



(d) trailing edge outboard

Figure 13. Control surface deflection versus iteration number for multi-disciplinary optimization.

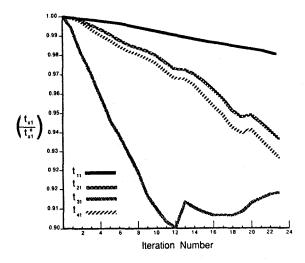


Figure 14. Comparison of layer #1 thicknesses versus Figure 17. Comparison of layer #4 thicknesses versus iteration number for each station.

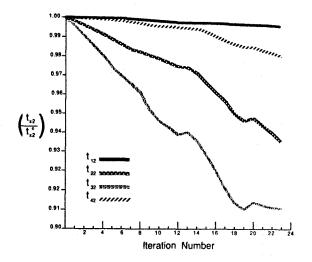


Figure 15. Comparison of layer #2 thicknesses versus iteration number for each station.

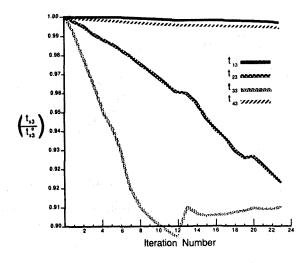
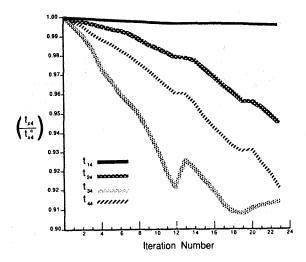


Figure 16. Comparison of layer #3 thicknesses versus iteration number for each station.



iteration number for each station.

CONCLUSIONS

A multi-level optimization technique based on a full potential concept and Rapid Structural Optimization Program (RSOP) has been presented for a static flexible fighter type wing configuration. This concept can be utilized for all types of aircraft design problems such as: maneuver load control, aileron reversal and lift effectiveness.

The aerodynamic results indicated that the nonlinearities are indeed important in the design of aircraft whose flight envelope includes the transonic regime. Furthermore, flexibility has a very significant effect on the aerodynamic forces, which in turn effect the integrated structure considerably.

The structural sub-optimization produces a significantly lighter wing by reducing ply thicknesses. The ply orientation, however, has an insignificant role in reducing the structural weight for the load case used. The multi-disciplinary optimization reduced the root bending moment, while maintaining the required lift. The aerodynamic load on the wing is shifted towards the root (or inboard) section to produce these results. This is accomplished by optimizing the control surface deflections, while including the ply thicknesses as additional design parameters.

Future plans include increasing the complexity of the multi-disciplinary optimization by increasing the number of stations (or SIC locations), optimizing for multiple load conditions, and increasing the order of the Taylor series expansions used to update the objective and constraint functions.

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