OPTIMIZATION OF CONICAL ANISOTROPIC SHELLS

G.V.Vasiliev and G.N. Adam
Institute of Aviation - INCREST
Bucharest, Romania

Abstract

A computerized method based on the general momentum theory of thin shells for optimizing circular conical anisotropic shells, with the properties of orthotropy on main directions of their middle surfaces, in terms of their geometrical, mechanical and elastical characteristics, is developed.

From the general system of equilibrium equations, expressed in terms of displacements, a single governing equation is obtained, with respect to the potential function Φ, through which all characteristics of the stress-strain state of the shell can be expressed. This 8-th order differential equation with partial derivatives, may be solved by the development of the function Φ in trigonometric rows. The elementary stress-strain state corresponding to bending load is investigated and its axial and shear stress-flows are determined.

The optimization factors are established in terms of the chosen geometrical and elastical design parameters, in order to obtain a conical orthotropic shell with best qualities characterised by maximal stress-capacity reported to minimal weight.

1. Geometrical considerations

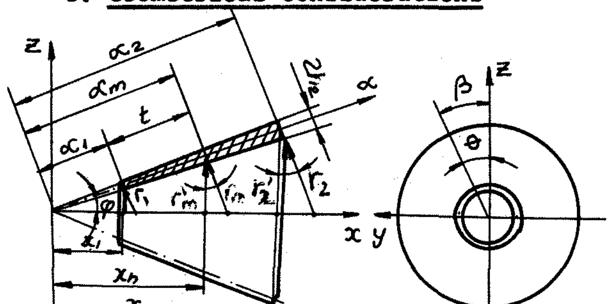

Fig.1. Geometrical characteristics of the shell

All geometrical characteristics of the shell are showr in the Fig. 1 and 2. The ratio thickness / radius of the shell in every section is constant, constituting a geometrical parameter :

$$\varkappa = \frac{h}{r} = const \qquad (1.1)$$

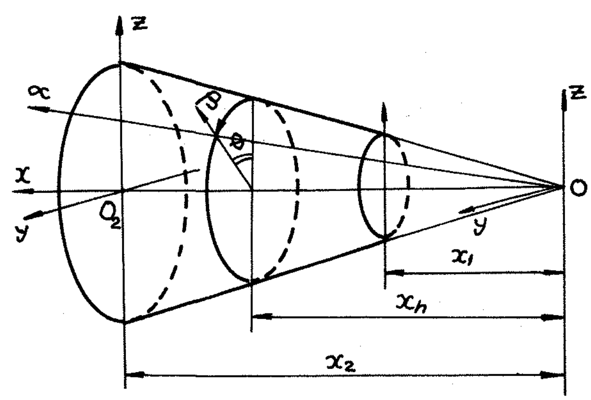

Fig. 2. Utilized orthogonal and curvilinear coordinate systems

The shell is reported to following coordinate systems :

- orthogonal x_1, y, z, with the origin in the apex of the cone;

- curvilinear α, β on the middle surface of the shell

- non-dimensional curvilinear "isotermic" coordonates t, θ being bound with α, β by the relations :

$$\alpha = \alpha_1 e^{mt} \text{ or } t = \frac{1}{m}\ln\left(\frac{\alpha}{\alpha_1}\right)$$
$$\beta = r_1\theta , \qquad (1.2)$$

where α_1 and r_1 are the lenght of the generator and the radius of the middle surface in the section x_1 ;

$$m = \sin\varphi , \qquad n = \cos\varphi , \qquad (1.3)$$

φ being the angle of conicity of the middle surface.

The material of the shell may be isotropic, with elastic constants E, μ and the density ρ, or orthotropic, characterized by elasticity moduli E_α, E_β and the Poisson's coefficients μ_α and μ_β, directed in the sense of coordinate lines α and β . Its density ρ_β may vary from $\rho_\alpha = \rho$ (in the case $E_\alpha = E_\beta$ and $\mu_\alpha = \mu_\beta$, i.e. isotropy) to ρ_β , when $E_\alpha \mu_\alpha \neq E_\beta \mu_\beta$.

In the case of an isotropic shell, reiforced by longitudinal and transversal stiffeners of the same material, their contribution may be taken into account by substituting the real heterogeneus shell by an equivalent homogeneous orthotropic one, characterized by elastical constants $E_\alpha, E_\beta, \mu_\alpha, \mu_\beta$ and of a "reduced" thickness (in the direction of α-lines

$$2h_\alpha = 2\sqrt[3]{\tfrac{3}{2}I_\alpha} , \qquad (1.4)$$

where I_α is the inertia moment of an unit element containing a stiffener in the direction of α - lines, which varies along the generator, so that the equality (1.4) would be satisfied. As the inertia moment in the transversal sense I_β is generally different from I_α it would result $h_\beta \neq h_\alpha$; in order to surpass this nonsense we assume that the thickness of the shell is overal $2h_\alpha$ (in both α and β directions), but the material is orthotropic, so that its moduli of elasticity maintain the proportionality with inertia moments :

$$\frac{I_\beta}{I_\alpha} = \frac{E_\beta}{E_\alpha} \qquad (1.4)'$$

Following relations are valid for the orthotropic material :

$$\left.\begin{aligned} & E_\alpha = E ;\ \lambda = E_\beta/E_\alpha ;\ \mu_\alpha = \mu/\sqrt{\lambda} ; \\ & \mu_\beta = \mu_\alpha\lambda = \mu\sqrt{\lambda} ; \\ & G = \frac{\sqrt{E_\alpha E_\beta}}{2(1+\sqrt{\mu_\alpha\mu_\beta})} = \frac{E_\alpha\sqrt{\lambda}}{2(1+\mu_\alpha\sqrt{\lambda})} ;\ \bar{\beta} = \frac{n}{3}\varkappa^2 ; \\ & a_1 = \frac{2E_\alpha\varkappa}{1-\mu_\alpha\mu_\beta} ;\ a_2 = \frac{\sqrt{E_\alpha E_\beta}\,\varkappa}{1+\sqrt{\mu_\alpha\mu_\beta}} ;\ \zeta = \frac{a_2}{a_1} \end{aligned}\right\} (1.5)$$

$$\rho_\alpha = \rho k \qquad (1.6)$$

where $k = \sqrt{\lambda}$, $\qquad (1.7)'$

or $k = \lambda$ $\qquad (1.7)''$

In order to obtain comparable results at the optimization factors (see later Ch.3) we shall investigate a family of conical shells of similarly geometrical characteristics, differing only by the conicity angle (φ), and the material parameter λ . All investigated shells are of the same length

$$x_k = x_2 - x_1 \qquad (1.8)$$

and the same radius r_m , in the section x_m which divides the volume of the shell V in two equal parts

$$V_I = V_{II}$$

(see Fig 1).

The volume of the shell is obtained by integration along the coordinate x of elementary ring sections

$$dV = 2\pi r'(x) \times 2h(\alpha)\, d\alpha$$

Using the substitutions :

$$r = \frac{r'}{n} = x\frac{m}{n^2} ;\ h = r\varkappa = x\frac{m}{n^2}\varkappa ;$$
$$d\alpha = \frac{dx}{n} \text{ and } r' = x\frac{m}{n} \qquad (1.9)$$

it results :

$$V = 4\pi\left(\frac{m}{n^2}\right)^2\varkappa\int_{x_1}^{x_2} x^2 dx = \frac{4\pi\varkappa}{3}\left(\frac{m}{n^2}\right)^2\left(x_2^3 - x_1^3\right) \qquad (1.10)$$

The weight of the conical shell is the product of (1.10) and (1.6).

$$G = V\,\rho_\beta \qquad (1.11)$$

In order to determine the unknowns x_1, x_2 and x_m, from the equality $V_I = V_{II}$, it results: $x_m^3 - x_1^3 = x_2^3 - x_m^3$

and $$h'_m = x_m \frac{m}{n} = \left(\frac{x_2^3 - x_1^3}{2}\right)^{1/3} \frac{m}{n} \qquad (1.12)$$

Then, from relations (1.8) and (1.12) it results a 3 rd degree equation to determine x_1:

$$x_1^3 + \frac{3}{2} x_h x_1^2 + \frac{3}{2} x_h^2 x_1 + \frac{x_h^3}{2} - \left(\frac{n}{m} h'_m\right)^3 = 0 \quad (1.13)$$

The coordinate x_1 is given by the real root of this equation:

$$x_1 = \left(-\frac{q}{2} + \Delta\right)^{1/3} + \left(-\frac{q}{2} - \Delta\right)^{1/3} - \frac{1}{2} x_h \quad (1.14)$$

where:

$$\Delta = \left[\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3\right]^{1/2};\ p = \frac{3}{4} x_h^2;\ q = -\left(\frac{n}{m} h_m\right)^3 \quad (1.15)$$

Unknowns x_2 and x_m are to be found from (1.8) and (1.12).

In establishing the geometrical characteristics of the shell (1.13), the condition: $d_p = \frac{x_h^3}{2} - \left(\frac{n}{m} h'_m\right)^3 < 0$ must be fulfilled. Otherwise the coordinate x_1 becomes negative. Being given the geometrical parameters x_h and h_m the conicity angle φ is limited by the condition:

$$\varphi_{cr} = \operatorname{arctg}\left(\sqrt[3]{2}\,\frac{h_m}{x_h}\right) = \operatorname{arctg}\left(1{,}25992\,\frac{h_m}{x_h}\right) \quad (1.16)$$

In practical computing the angle φ must be taken as : $\varphi_{lim} \leq 0{,}8\varphi_{cr}$ (1.17)

In such a way we obtain for different combinations of x_h and h_m the following values of the limit angle φ (see table T 1).

Table T 1

h_m/x_h	0,3	0,5	0,65	0,75	0,9	1	1,2
φ^o_{cr}	20,7	32,3	39,3	43,4	48,6	51,6	56,5
φ_{lim}	16,6	25,8	32,4	34,7	38,9	41,2	45,2

2. Teoretical considerations

2.1. Basic equations.

The system of internal forces and moments acting on an element of the conical shell subjected to an arbitrary external load, is shown in the Fig.3, where T_α, T_β, S_α, S_β, N_α, N_β are the internal forces (stress-flows) and G_α, G_β, H_α, H_β -are the internal moments.

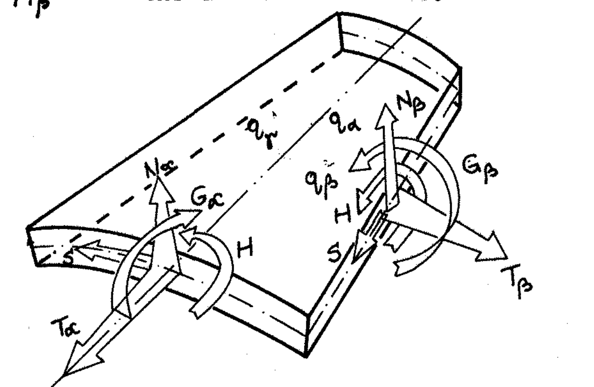

Fig 3. Systems of internal forces and moments

From the set of equilibrium quations in the terms of these internal forces and moments (I.2.1.) x) the equations which bind by themselves the displacement vector components u, v, w with the linear, shear and rotational strains $\varepsilon_\alpha, \varepsilon_\beta, \omega, \varkappa_\alpha, \varkappa_\beta, \tau$ (I.2.2.) x) and "constitutive" equations establishing the relations between the internal forces and strains (I.2.3.), x) a system of equilibrium equations expressed in terms of displacements (I.2.9) x) is obtained.

This system may be reduced to a single governing equation with respect to the potential function $\Phi(t,\theta)$:

$$\Delta\,\Phi(t,\theta) = 0$$

where Δ is the differential operator (I.2.11) x)

The equation (2.1.) is an equation with partial derivatives with respect to the variables t and θ, of 8^{th} order, with cons-

x) here and further; from the Work (5)

tant coefficients. Its complete expression is given in (5) (formulae I.2.1o).

The solution of the governing equation is obtained by the development of the function ϕ in trigonometric rows:

$$\phi(t,\theta)=\sum_{j=0}^{\infty}[\phi_j(t)\cos j\theta+\bar{\phi}_j(t)\sin j\theta] \quad (2.2)$$

where the terms $\phi_j \cos j\theta$ and $\bar{\phi}_j \sin j\theta$ represent the symmetric and antisymmetric parts of the function ϕ x)

By the substitution of (2.2) in (2.1) we obtain an infinite set of simple differential equations of the 8^{th} order

$$k_8 \phi_j^{(VIII)} + k_7 \phi_j^{(VII)} + \ldots\ldots + k_0 \phi_j = 0 \quad (2.3)$$
$(j=0,1,\ldots\infty)$

Each term j of the development represents an elementary stress-strain state, which may be treated independent.

Substituing $\phi_j(t)=e^{st}$ (2.4)

where s is an inknown number, one arrives to the algebrical "characteristic" equation x)

$$k_8 s^8 + k_7 s^7 + \ldots\ldots + k_1 s + k_0 = 0 \quad (2.5)$$

of the differential equation (2.3.)

The general solution of the equation (2.3) for the state j is:

$$\phi_j(t)=\sum_{\gamma=1}^{8} C_\gamma e^{s_\gamma t} \quad (2.6)$$

where C_γ are the constants of integration and s_γ the roots of the characteristic equation.

2.2. The investigation of the caracteristic wequation.

We shall concentate our attention on the term j = 1, corresponding to an elementary bending-shear stress-strain state. At j=0 and j = 1 the characteristic equation has 4 real roots $s_1 \ldots s_4$, corresponding to the general stress-strain state and 2 pairs of complex coots corresponding to edge effects. For j $\geqslant$ 2 the equation has 4 pairs of complex roots. In the case of j = 0 and j = 1 the equation (2.5) may be divided into two separate equations of 4^{th} degree.

For the general stress-strain state these equations are :

$$s^4+2ms^3-m^2s^2-2m^3s+\frac{\beta K_j}{n^2(\mu_\beta^2-\lambda)}=0 \quad (2.7)$$

where K_j is as (II.1.4) x). The roots of the equation (2.7) are:

As t $s_1 \cong 0$; $s_2=+m$; $s_3=-m$; $s_4=-2m$ (2.8)

As the result of relations (1.1), (2.4) and (2.8) the function ϕ_j x) for j = 1 symmetric, corresponding to a lateral load varying with $\cos\theta$, has following expression:

$$\phi_{(j=1)}=\left[C_1+\frac{r}{r_1}C_2+\frac{r_1}{r}C_3+\left(\frac{r_1}{r}\right)^2 C_4\right]\cos\theta \quad (2.9)$$

where $C_1 \ldots C_4$ are the constants of integration, r_1 - the radius in the section x_1 and $r(x)$ - the radius in the current section.

2.3. The expressions of the internal stress-flows.

All characteristic values of the stress-strain state, as internal forces, displacements, etc. may be expressed through the function ϕ.

The general expressions of the displacements and internal forces for the state j are given in the Appendices I 5 and I 6 of the work (5).

After the necessary substitutions and intermediary calculations the working formulae for the internal forces T_α and S, which present interest in our investigation, are as (I.6.2) x) :

$$T_\alpha = 2a_1 s(\lambda-\mu_\beta^2)m^2 n\left[C_3(j^2-m^2)\frac{r_1}{r}+C_4 3j\left(\frac{r_1}{r}\right)^2\right],$$
$$S = 6ja_2 C_4(\lambda-\mu_\beta^2)m^3 n\left(\frac{r_1}{r}\right)^2 \quad (2.10)$$

According to (1) $C_1 = C_2 = 0$ by virtue of the fact that at j=0 and j = 1 (symmetric and antisymmetric), from the 12 elementary states corresponding to the particular solutions of the equation (2.6), 6 refer to the displacements and 6 to the elementary stress-states due to 3 forces and 3 moments, applied in the sense of the axes x, y, z. At j = 1 C_1 and C_2 correspond to the displacements of translation (along O_z) and rotation (about Ox); C_3 and C_4 - to the stress-states due to the transversal forces (Pz)), and C_4(self)- to the moment My, applied at the frontal edge $x = x_1$.

2.4. Determination of integration constants.

In the general case of the loading characterised by j = 1, (1) - 1o equations (II.3.2)[x)] must be solved to determine the constants $C_1 \ldots C_8$ and the displacements α_I (on Ox, due to the rotation about Oy) and β_I (on Oz due to the movement of translation).

By applying of simplified edge conditions (1) (p.I, § 33) the number of unknowns reduces to 6: $C_1 \ldots C_4$, α_I and β_I.

In order to determine these unknowns we are obliged to solve following set of six equations :

a) on the edge x_1

- the equilibrium equations :

$$(1)\quad \left[T_\alpha m + S + N_\alpha n\right]_{x=x_1} = \frac{P_z}{\pi r_1},$$
$$(2)\quad \left[T_\alpha n - N_\alpha m + H_\alpha \frac{m}{r_1} - G_\alpha \frac{1}{r_1}\right]_{x=x_1} = \frac{M_y}{\pi r_1^2}; \qquad (2.11)$$

- the displacement equations:

$$(3)\quad [u]_{x_1} = \alpha_I n + \beta_I m ,$$
$$(4)\quad [v]_{x_1} = \beta_I , \qquad (2.12)$$

(4 bis) and an auxiliary equation:

$$[w]_{x_1} = \beta_I - \alpha_I m ;$$

b) on the edge x_2

$$(5)\quad [u]_{x_2} = 0$$
$$(6)\quad [v]_{x_2} = 0 \qquad (2.13)$$

The problem will be solved in two steps :

Firstly it will be determined the displacements α_I and β_I from the equations (1) and (2) where the internal forces and moments $T_\alpha, S, N_\alpha, H_\alpha, G_\alpha$ will be expressed in terms of displacements

$[u]_{x_1}$, $[v]_{x_1}$ and $[w]_{x_1}$.

Secondly - the integration constants $C_1 \ldots C_4$ will be calculated from the set of equations (3) (4), (5),(6).

Finally both constants $C_1 \ldots C_4$ and the stress and shear flows T_α and S may be expressed in terms of the external load:

$$C_1 = D_1 \xi_1 + D_2 \bar{\eta}_1 ; \quad C_2 = D_1 \bar{\xi}_2 + D_2 \bar{\eta}_2 ,$$
$$C_3 = D_1 \bar{\xi}_3 + D_2 \bar{\eta}_3 ; \quad C_4 = D_1 \bar{\xi}_4 + D_2 \bar{\eta}_4 \qquad (2.14)$$

and

$$T_\alpha = 2a_1 \zeta (\lambda - \mu_\beta^2) m^2 n \left\{ D_1 \left[\bar{\xi}_3 (j^2 - m^2) \frac{r_1}{r} + \bar{\xi}_4 3 j^2 \left(\frac{r_1}{r}\right)^2 \right] + D_2 \left[\bar{\eta}_3 (j^2 - m^2) \frac{r_1}{r} + \bar{\eta}_4 3 j^2 \left(\frac{r_1}{r}\right)^2 \right] \right\},$$
$$S = 6 j a_2 (\lambda - \mu_\beta^2) m^3 n \left(D_1 \bar{\xi}_4 + D_2 \bar{\eta}_4\right) \left(\frac{r_1}{r}\right)^2, \qquad (2.15)$$

where : $D_1 = P_z / \pi r_1$; $D_2 = M_y / \pi r_1^2$ (2.16)

The coefficients $\bar{\xi}_1 \ldots \bar{\xi}_4$ and $\bar{\eta}_1 \ldots \bar{\eta}_4$, as well as all other intermediate coefficients, are computed in the program KONUS (§ 3.3)

Now, using the relations (2.1o) or (2.15) we may determine the stress flows in different points of the interval $x_1 \ldots x_2$. Their greatest values appear at the free frontal edge $x = x_1$, where $r(x) = r_1$.

Edge effects, appearing in this stress-strain state are not taken into account in this computation.

Further we shall consider the average stress-flows and average stresses along the generators of the shell.

$$T_{av} = \frac{BB\, x_1 n}{x_h} (\ln x_2 - \ln x_1) + \frac{CC\, n x_1^2}{x_h^2} \qquad (2.17)$$
$$S_{av} = DD \frac{n x_1^2}{x_h^2}$$

where :

$$BB = 2a_1 \zeta (\lambda - \mu_\beta^2) m^2 n (j^2 - m^2) C_3$$
$$CC = 6a_1 \zeta (\lambda - \mu_\beta^2) m^2 n j^2 C_4 \qquad (2.18)$$
$$DD = 6 j a_2 (\lambda - \mu_\beta^2) m^3 n C_4$$

The average normal and tangential stress are : $\sigma_m = T_{av} / (2 \varkappa r_m)$; $\tau_m = S_{av} / (2 \varkappa r_m)$ (2.19)

All these computations are also performed in the program KONUS (§ 3.3).

3. Optimization program

3.1. Criteria for optimization

The influence of geometrical and physical parameters of the shell family described in Ch.1, on its "quality" of stress-capacity/weight, defined by the optimization factor ψ, is investigated.

The variable design parameters are :

a) The conicity angle φ ;

b) The parameter λ (1.5), which characterizes the elastical and mechanical properties of the shell and the density of its material.

The shells are subjected to the same external bending load (corresponding to j = 1) constituted of a force Pz, and moment My applied on the edge x_1.

3.2. Definition of the optimization parameters.

The optimization parameters ψ_1 and ψ_2 reported to the stress-components T_α and S are defined as non-dimensional ratios :

$$\psi_1 = FR1/GR \; ; \; \psi_2 = FR2/GR \qquad (3.1)$$

where FR 1 and FR2 are the "reserve-factors":

$$FR1 = \sigma_\ell/\sigma_m \; ; \; FR2 = \tau_\ell/\tau_m \qquad (3.2)$$

and GR is the ratio:

$$GR = G/GCL \qquad (3.3)$$

where G - is the weight of the shell (1.11) and GCL is the weitht of an equivalent cylindrical shell of radius r_m :

$$GCL = 2\pi \varkappa x_k r_m^2 \rho_\alpha \qquad (3.4)$$

The allowable stresses of the orthotropic material are considered:

$$\sigma_\ell = \sigma_\alpha \sqrt{\lambda} \; ; \; \tau_\ell = \tau_a \sqrt{\lambda} \qquad (3.5)$$

where σ_α and τ_a are the allowable (ultimate) stresses of the basical material.

The best quality of the shell is obtained at the greatest values of the optimization parameters.

The arrangement of the optimization parameters after their magnitude, allows to appreciate the combinations of design parameters in order to optimize the geometrical and mechanical characteristics of the conical structure, even in the phase of initial design.

3.3. The computer program to determine the optimization parameters.

A special computer program "KONUS" to determine the optimization parameters of the investigated class of conical shells, was developed and implemented for numerical computations.

The optimization parameters were computed for a family of conical orthotropic shells, differring by their conicity angles (from 5^o to 30^o) and the material parameter λ , varying from 0.5 to 1.5.

3.4. Numerical results.

The investigated family of conical shells were characterized by the following geometrical, elastical and mechanical parameters: $x_k = 100\,cm$; $r_m = 50\,cm$; $\varkappa = 0.002$

$\varphi = 5, 10, 15, 20$ and 25°

$E = E_\alpha = 7.2 \times 10^5\,daN/cm^2$; $\mu = 0.333$

$\rho_\alpha = 2.7 \times 10^{-3}\,daN/cm^2$;

$\sigma_\alpha = 4300\,daN/cm^2$; $\tau_a = 2700\,daN/cm^2$

$\lambda = 0.5$; 0.7 ; 1.0 ; 1.5

The applied load on the frontal edge $x = x_1$ (applied on all specimens), by the means of a stiff frame, is :

The transverse force Pz = 10^4 daN

the bending moment My = 10^6 daNcm

4. The analysis of obtained results

4.1. The variation $T_\alpha = T_\alpha(x)$ and $S = S(x)$.

Both T [x)] and S [x)] flows vary from their maximal values at the x_1 edge, to the minimal ones at the x_2 edge, following the quadratic low, resulted from their expressions (2.10). The major importance in the investigated stress-strain state have the axial T_α - flows, while the S shear flows have a smaller influence. Thus the parameter ψ_1 has a preponderent importance in establishment of the optimal design characteristics of the shell.

4.2. The influence of the parameter λ on the values of optimization parameter

The parameter λ has a significant influence on the values of the optimization parameters. This influence is evidenced in the table T 4-1, for a shell with $\varphi = 15^o$ conicity angle :

Table T. 4.1

λ_i	0.5	0.7	1	1.5
ψ_{1i}	0.1824	0.2168	0.2600	0.3193
ψ_{1i}/ψ_{11}	0.707	0.835	1	1.25

So, the optimization parameter ψ_1 grows with the growing of the factor λ in the proportion resulted in the table T.4.1.

This influence may be seen also in the fig 4.

4.3. The influence of the design parameters on the optimization parameters.

The dependence ψ_1 (or ψ_2) $= f(\varphi)$ for different values of the λ parameter is shown in the diagram - Fig 4., whence it results the following conclusions :

1°. The major influence on determining the design parameters in the bending stress-strain state of the investigated class of conical shells has the optimization parameter ψ_1 determined by the longitudinal stress-flow T_α . The ψ_2-parameters appear greater (in absolute values) than ψ_1-parameters,what denotes that the reserve factors with respect to the S-flow are significant greater than ones, with respect to T_α flow (i.e. smaller values of the S-flow in comparison with the T_α-flow, for the given load).

2°. The influence of the λ-parameter is important in all cases, on the ψ_1- and ψ_2-parameters. This influence is evidenced in the table T-4.1.
It is advantageous that the transversal-orthotropy properties (along β-coordinates) would be as great as possible in comparison with the elastical properties of the basic material.
This conclusion is valid for the orthotropic material with the density as in (1.6):

$$\rho_\beta = \rho_\alpha k \quad (k = \sqrt{\lambda})$$

In the case when $k \to \lambda$ this advantage decreases and vanishes at $k = \lambda$.

3°. An interesting dependence exists between the ψ_1- parameter and the conicity angle φ ($\psi_1 = f(\varphi)$). Firstly the ψ_1- parameter significantly grows with the φ- angle (this situation is valuable for every λ- parameter), till the value of the angle $\varphi_m \cong \varphi_\alpha/2$; then it begins to decrease till the values $\psi_{1\alpha} = 0$ at the angle φ_α. This shows that at given x_h and r_m the best combination of the geometrical design parameters is obtained for the angle $\varphi_m \cong \frac{1}{2} \operatorname{arctg}(1{,}25992 \frac{r_m}{x_h})$ (4.1)

4°. The optimization coefficients ψ_2 degrending exclusively from the transverse load Pz, have their maximal values at the smallest values of the conicity angles (i.e. at $\varphi = 5^\circ$), decreasing appoximative linearly with the growing of the - angle and becoming $\psi_2 = 0$ at the angle $\varphi = \varphi_\alpha$

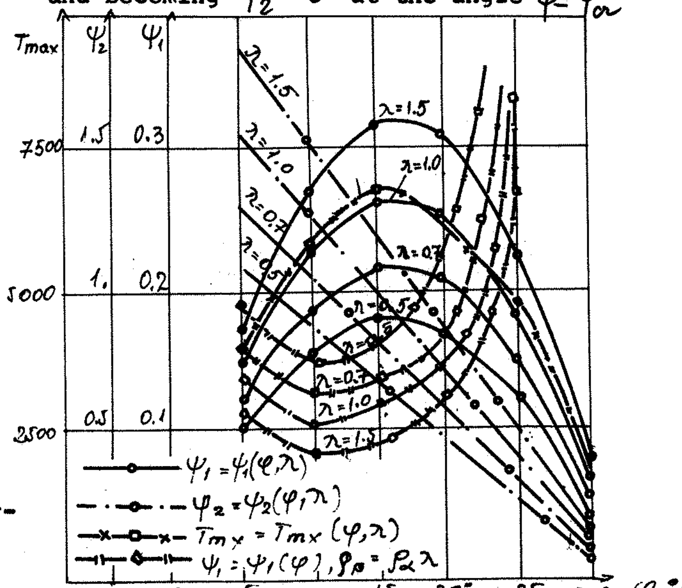

Fig 4. Dependences $\psi_1 = \psi_1(\varphi, \lambda)$ and $\psi_2 = \psi_2(\varphi, \lambda)$

5°. Conclusions

The performed analysis demonstrates that from the point of view of chosen optimization criteria for the investigated class of circular conical orthotropic shells, subjected to the bending load, the best quality is obtained for following combination of design elastical and geometrical parameters.

The ratio between the elastical moduli of the orthotropic material E_β and E_α:

$$\lambda = E_\beta / E_\alpha$$

must be as great as possible.

. When the density of the ortotropic material is of the form (1.6)-(1.7)" -the optimization parameters ψ_1 and ψ_2 become independent from λ.

2° The conicity angle must be chosen in such a way that its magnitude should be approximatively equal with the half of the critical angle, defined by the relation (1.16).

- The influence of the optimization factor ψ_2, defined by the shear flow S is of secondary order.

- The dependence $T_{MAX} = f(\varphi, \lambda)$ shows us that the minimal values of the T_{MAX} stress-flow (2.1o) are obtained for the angle:

$$\varphi_{(T_{MAX})_{min}} \cong \frac{1}{3} \varphi_{cr} . \qquad (5.1)$$

These indications will help us to obtain even at the initial design phase, the best shapes of conical components of aerospace vehicles, characterised by maximal stress-capacity reported to minimal weitht.

The analysis performed in present paper represents in the meantime the first step in the elaboration of a general procedure to optimize the conical anisotropic shells, of constant or variable thickness, subjected to arbitrary load. The presented procedure and the program KONUS will be extended and generalized to cover all practical situations.

6. Bibliography

1. Al. Goldenweizer - "Theory of elastic thin shells" - Gostehizdat, Moskow, 1953 (In Russian).
2. S.A.Ambartsumian - General theory of anisotropic shells - Nauka, Moskow,1974 (in Russian)
3. L.Librescu - "Elastoplastics and kinetiks of anisotropic and heterogeneous shell-type structures"- Noordhoff Internat,Publishing,Leyden,1975
4. L.Librescu - Refined Geometrically Nonlinear Theories of Anisotropic-Laminated Shells-Quarterly of Appl. Math. XLV,April 1987,p.p.1-22.
5. G.V.Vasiliev-"Elastic equilibrium of circular conical orthotropic shells of linearly-variable thickness" - Parts I, II, III (in Russian). Revue Roumaine des sciences techniques.Mécanique Appliquée -16, Nr.4, 5,6, pp. 841 - 866 ; 1o89-11o5; 1337-135o; Bucureşti, 1971
6. A.P.Zaharova - "Stress Analysis of Circular Cylindrical Shells,Loaded on Free Edges by Transversal Shear Forces" - Stress Analysis of Cylindrical Shells Oborongiz,Moskow, 1959 (in Russian).
7. V.P.Malcov, A.G. Ugodchikov - "Optimization of Elastic Systems", Nauka, Moskow, 1984(in Russian).
8. T.E.Shoup - A Practical Guide to Computer Methods for Engineers"- Prentice-Hall, INC, Englewood Cliffs, N.J., 1979
9. G.V.Vasiliev - "On Characteristic Equations of Conical and Cylindrical Thin Shells" (in Romanian) Studii Cercet.Mec.Appl.Bucharest, nr.4,1964